

Krasovsky Institute of Mathematics and Mechanics
Sobolev Institute of Mathematics
Melentiev Energy Systems Institute
Higher School of Economics, Nizhny Novgorod
Ural Federal University
Novosibirsk State University



XVIII International Conference
**Mathematical Optimization Theory
and Operations Research**
(MOTOR - 2019)
Ekaterinburg, Russia
July 8-12, 2019

Abstracts

Ekaterinburg Russia 2019

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1 . Plenary Lectures

Prof. Olga Battaia. *Decision under ignorance: a comparison of existing criteria in a context of linear programming.*

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Decision or optimization problems often arise in an uncertain context. Depending on available information, several approaches have been proposed to model this uncertainty. In this talk, we focus on the case of low knowledge on possible states, namely decision under ignorance. In this case the decision-maker is able to give the set of possible values of optimization problem parameters but she/he is not able to differentiate them. We compare a set of criteria that can be used in this case on the example of a linear programming problem and discuss some possible applications.

Prof. Oleg Burdakov. *Node partitioning and cycles creation problem.*

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We present a new class of network optimization problems, which extend the classical NP-hard traveling salesman problem. It is formulated as follows. Given a graph with a certain time associated with each node and each arc, a feasible partition of the nodes in subsets is such that, for each subset, there exists a tour whose traveling time is below the time associated with each node in the tour. It is required to find a feasible partitioning which minimizes the number of tours. Such problems are typical in numerous applications, where services are repeatedly provided for a set of customers. For each customer, there is a critical time within which a service must be repeated. Given the traveling time between the customers, the set of customers is partitioned so that each subset is served by one agent in a cyclic manner without violating any individual critical time requirement. The number of agents is minimized. As an example, we consider a problem, in which a fleet of unmanned aerial vehicles is used for area patrolling. We introduce an integer programming formulation of the node partitioning and cycles creation problem, and also heuristic algorithms for solving this problem. Results of numerical experiments are presented.

Prof. Christoph Dürr. *Bijective analysis of online algorithms.*

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In the online computing framework the instance arrives in form a request sequence, every request must be served immediately, through a decision, which generates some cost. Think at the paging problem for memory caches. The goal in this research area is to identify the best strategy, also called online algorithm. Classically this is done through the competitive analysis, i.e. the performance

of an online algorithm is compared with the optimal offline solution. The goal is to find an algorithm which minimizes this ratio over the worst case instance. You would say that algorithm A is better than algorithm B if it has a smaller ratio. However there are situations where two algorithms have the same ratio, still in practice one is better than the other. So people came up with a different technique to compare online algorithms directly with each other, rather than through the optimal offline solution. The bijective analysis is one of them. I would do a survey on this technique, and talk about a related personal work: Best-of-two-worlds analysis of online search, with Spyros Angelopoulos and Shendan Jin.

Prof. Alexander Grigoriev. *A survey on possible and impossible attempts to solve the treewidth problem via ILPs.*

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We survey a number of integer programming formulations for the pathwidth and for the treewidth problems. The attempts to find good formulations for the problems span the period of 15 years, yet without any true success. Nevertheless, some formulations provide potentially useful frameworks for attacking these notorious problems. Some others are just curious and interesting fruits of mathematical imagination.

Prof. Mikhail Kovalyov. *No-idle scheduling of unit-time jobs with release dates and deadlines on parallel machines.*

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While the problem of scheduling unit-time jobs with release dates and deadlines on parallel machines is polynomially solvable via a reduction to the assignment problem, the no-idle requirement destroys this reduction and makes the problem challenging. In the presentation, a number of properties of this problem are reported, and heuristic and optimal algorithms based on these properties are described.

Prof. Vadim Levit. *Critical and Maximum Independent Sets Revisited.*

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A set of vertices of a graph is independent if no two its vertices are adjacent. A set is critical if the difference between its size and the size of its neighborhood is maximum. Critical independent sets define an important area of research due to their close relationships with the well-known NP-hard problem of finding a

maximum independent set. Actually, every critical independent set is contained in a maximum independent set, while a maximum critical independent set can be found in polynomial time. If S is an independent set such that there is a matching from its neighborhood into S , then it is a crown. It is known that every critical independent set forms a crown. A graph is König-Egervary if every maximum independent set is a crown. Crowns are also accepted as important tools for fixed parameter tractable problems. For instance, the size of the vertex cover can be substantially reduced by deleting both the vertices of a crown and its neighborhood. In this presentation, we discuss various connections between unions and intersections of maximum (critical) independent sets of graphs, which lead to deeper understanding of crown structures, in general, and König-Egervary graphs, in particular.

Prof. Bertrand M.T. Lin. *An Overview of the Relocation Problem.*

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The relocation problem is formulated from a municipal redevelopment project in east Boston. In its abstract form, the relocation problem incorporates a generalized resource constraint in which the amount of the resource returned by a completed activity is not necessarily the same as that the activity has acquired for commencing the processing. We will first introduce the connection of the relocation problem to flow shop scheduling. Several traditional scheduling models with the generalized resource constraints have been proposed investigated. We will review existing results, suggest new models and present several open questions.

Prof. Natalia Shakhlevich. *On a New Approach for Optimization under Uncertainty.*

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Research on decision making under uncertainty has a long history of study. Still theoretical findings have strong limitations: stochastic programming requires probability distributions for uncertain parameters which are often hard to specify; robust optimisation essentially relies on worst-case scenarios which can be over-pessimistic and far from realistic scenarios; stability analysis explores optimal solutions which can be hard to find even for well predicted scenarios. As an alternative approach, we propose a new system model based on the concept of resiliency. Resilient solutions are not required to be optimal, but they should keep quality guarantees for the widest range of uncertain problem parameters. The talk illustrates key steps of resiliency analysis considering examples of 0/1 combinatorial optimisation problems.

Prof. Angelo Sifaleras. *Exterior Point Simplex-type Algorithms for Linear and Network Optimization Problems.*

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Two decades of research led to the development of a number of efficient algorithms that can be classified as exterior point simplex-type. This type of algorithms can cross over the infeasible region of the primal (dual) problem and find an optimal solution reducing the number of iterations needed. Thus, such approaches aim to find an efficient way to get to an optimal basis via a series of infeasible ones. In this lecture, we present the developments in exterior point simplex-type algorithms for linear and network optimization problems, over the recent years. We also present other approaches that, in a similar way, do not preserve primal or dual feasibility at each iteration such as the monotonic build-up Simplex algorithms and the criss-cross methods, and also discuss some open research problems.

Prof. Vitaly Strusevich. *Design of Fully-Polynomial Approximation Schemes for Non-linear Boolean Programming Problems.*

United Kingdom, University of Greenwich

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The talk is aimed at describing various techniques used for designing fully-polynomial approximation schemes (FPTAS) for problems of minimizing and maximizing non-linear non-separable functions of Boolean variables, either with no additional constraints or with linear knapsack constraints. Most of the reported results are on optimizing a special quadratic function known as the half-product, which has numerous scheduling applications. Besides, problems with a more general objective and nested linear constraints are considered and a design of an FPTAS based on the K-approximation calculus is discussed.

This lecture reviews the state of the art for probably the most common computational operation in applied mathematics — projection, which can be also considered as the problem of finding the least norm element (LNE) in a given subset of a linear vector space. The special attention in the lecture will be given to Euclidean or orthogonal projection, but we plan to discuss another norms as well. Projection is computationally intensive operation even for relatively simple sets like canonical simplexes and special algorithms are a way more efficient than off-the-shelf quadratic programming methods especially for large-scale problems. Large-scale projection problems can be decomposed in different sequential or parallel manner as extension of celebrated Kaczmarz sequential projection procedure and block-row action methods. We discuss also the problem of numerical instability of projection operation which is quite common in such applications as new optimization algorithms, linear programming, machine learning and automatic classification.

2. Tutorials

Prof. Tatjana Davidović. *Distributed memory based parallelization of metaheuristic methods.*

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Metaheuristics represent powerful tools for addressing hard combinatorial optimization problems. However, real life instances usually cannot be treated efficiently by the means of computing times. Moreover, a major issue in metaheuristic design and calibration is to provide high performance solutions for a variety of problems. Parallel metaheuristics aim to address both issues. The main goal of parallelization is to speed up the computations by dividing the total amount of work between several processors. Parallelization of stochastic algorithms, such as metaheuristics may involve several additional goals. Besides speeding up the search (i.e., reducing the search time), it could be possible to: improve the quality of the obtained solutions (by enabling searching through different parts of the solution space); improve the robustness of the search (in terms of solving different optimization problems and different instances of a given problem in an effective manner; robustness may also be measured in terms of the sensitivity of the metaheuristic to its parameters); and solve large-scale problems (i.e., solve very large instances that cannot be even stored in the memory of a sequential machine). A combination of gains may also be obtained: parallel execution can enable an efficient search through different regions of the solution space, yielding an improvement of the quality of the final solution within a smaller amount of execution time. The objective of this talk is to present a state-of-the-art survey of the main ideas and strategies related to the parallelization of metaheuristic methods. Various paradigms related to the development of parallel metaheuristics are explained. Among them, communications, synchronization, and control aspects are identified as the most relevant. Implementation issues are also discussed, pointing out the characteristics of shared and distributed memory multiprocessors as target architectures. All these topics are illustrated by the examples from recent literature related to the parallelization of various meta-heuristic methods, with the focus on distributed memory parallelization of Variable Neighborhood Search (VNS) and Bee Colony Optimization (BCO) using Message Passing Interface (MPI) communication protocol.

Prof. Stephan Dempe. *Bilevel optimization: The Model and its Transformations.*

Germany, TU Bergakademie Freiberg

Bilevel (or hierarchical) optimization problems aim to minimize one function subject to (a subset of) the graph of the solution set mapping of a second, parameter dependent optimization problem. The parameter is the decision

variable of the so-called leader, the optimization problem describing the constraints is the problem of the follower. These problems have a large number of applications in science, engineering, economics. To investigate and solve them, they need to be transformed into a single-level optimization problem. For that different approaches can be used. 1) If the follower's problem is regular and convex, it can be replaced using the Karush – Kuhn – Tucker conditions. The result is a so-called Mathematical Program with Equilibrium Constraints. In these nonconvex optimization problems, the Mangasarian – Fromovitz constraint qualification is violated at every feasible point. Solution algorithms converge (under suitable assumptions) to stationary points which are, in general, not related to stationary points of the bilevel optimization problem. To overcome this unpleasant situation, a certain regularization approach can be used. Another approach uses the transformation to a mixed integer (nonlinear) optimization problem. 2) If the optimal value function of the follower's problem is used, a nonconvex, nonsmooth optimization problem arises. Again, the (now nonsmooth) Mangasarian – Fromovitz constraint qualification is violated at every feasible point. If the optimal value function is convex or concave, its approximation is helpful to describe a solution algorithm. Optimality conditions can be derived using partial calmness or a certain penalization approach. 3) The problem can be reformulated as a generalized Nash equilibrium problem. Topic of the lecture is the introduction of the model together with some surprising properties and a short overview over promising accesses to investigate and solve it.

Prof. Oleg Khamisov. *The fundamental role of concave programming in continuous global optimization.*

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A comprehensive description of connections between concave programming and other branches of global optimization like Lipschitz optimization, d.c. optimization etc. is given. It is shown that in general solution of almost every global optimization problem can be reduced to solution of a sequence of concave programming problems. Modern concave optimization technology including cuts, branch and bounds, branch and cuts and so on as well as the corresponding extensions to different global optimization problems are presented. A part of the talk is devoted to the connection between concave and mixed 0-1 linear programming.

Prof. Alexander Kononov. *Primal-dual Method and Online Problems.*

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The primal-dual method is a powerful tool in the design of approximate algorithms for combinatorial optimization problems. In our tutorial we discuss how this method can be extended to develop online algorithms. The tutorial is based on the survey by N. Buchbinder and J. Naor and the web-presentation by N. Bansal.

Prof. Nenad Mladenovic. *Solving nonlinear system of equations as an optimization problem.*

Serbia, Mathematical Institute SANU

The Nonlinear System of Equations (NSE) problem is usually transformed into an equivalent optimization problem, with an objective function that allows us to find all the zeros. Instead of the usual sum-of-squares objective function, the new objective function is presented as the sum of absolute values. Theoretical investigation confirms that the new objective function provides more accurate solutions regardless of the optimization method used. In addition, we achieve increased precision at the expense of reduced smoothness. In this paper, we propose the continuous variable neighborhood search method for finding all the solutions to a NSEs. Computational analysis of standard test instances shows that the proposed method is more precise and much faster than two recently developed methods. Similar conclusions are drawn by comparing the proposed method with many other methods in the literature.

Joint work with: Jun Pei, Zorica Drazic, Milan Drazic, Panos M. Pardalos.

Prof. Evgeni A. Nurminski. *Projection Problems and Problems with Projection.*

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This lecture reviews the state of the art for probably the most common computational operation in applied mathematics — projection, which can be also considered as the problem of finding the least norm element (LNE) in a given subset of a linear vector space. The special attention in the lecture will be given to Euclidean or orthogonal projection, but we plan to discuss another norms as well. Projection is computationally intensive operation even for relatively simple sets like canonical simplexes and special algorithms are a way more efficient than off-the-shelf quadratic programming methods especially for large-scale problems. Large-scale projection problems can be decomposed in different sequential or parallel manner as extension of celebrated Kaczmarz sequential projection procedure and block-row action methods. We discuss also the problem of numerical instability of projection operation which is quite common in such applications as new optimization algorithms, linear programming, machine learning and automatic classification.

Prof. Alexander Strekalovsky. *Modern methods of nonconvex optimization.*
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We address the nonconvex optimization problem with the cost function and equality and inequality constraints given by d.c. functions. The linear space of d.c. functions possesses a number of very attractive properties. For example, every continuous function can be approximated at any desirable accuracy by a d.c. function and any twice differentiable function belongs to the DC space. In addition, any lower semicontinuous (l.s.c.) function can be approximated at any precision by a sequence of continuous functions. Furthermore, provided that for the optimization problem under study we proposed the new Global Optimality Conditions (GOCs), which have been published in the English and Russian languages. The natural question arises: is it possible to construct a computational scheme based on the GOCs (otherwise, what are they for?) that would allow us not only to generate critical points (like the KKT-vectors) but to escape any local pitfall, which makes it possible to reach a global solution to the problem in question? First of all, we recall that with the help of the Theory of Exact Penalization, the original d.c. problem was reduced to a problem without constraints. Moreover, it can be readily seen that this penalized problem is a d.c. problem as well. Furthermore, special Local Search Methods (LSMs) were developed and substantiated in view of their convergence features. In addition, the GOCs were generalized for the minimizing sequences in the penalized problem. A special theoretical method was proposed and its convergence properties were studied. We developed a Global Search Scheme (GSS) based on all theoretical results presented above, and, moreover, we were lucky to prove that the sequence produced by the GSS turned out to be minimizing in the original d.c. optimization problem. Finally, we developed a Global Search Method (GSM), combining the special LSM and the GSS proposed. The convergence of the GSM is also investigated under some natural assumptions. The first results of numerical testing of the approach will be demonstrated.

3 . Mathematical Programming

Maxim Demenkov. *From concave programming to polytope projection.*

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We consider a class of oracle-based algorithms for inner polyhedral approximation of convex sets with application to the polytope projection problem. The origin of the method, called polyhedral annexation, lies in concave programming. The method builds a sequence of inner polytopic approximations of a convex set, adding new points by optimizing linear functions (constructed from their facets) over the set. In this paper we apply the method to the polytope projection problem. For this, we study special kind of polytopes, called zonotopes. We lift the system of linear inequalities, adding new variables, so as to represent the projection as an inclusion of some vectors into a zonotope represented by a system of linear inequalities. This zonotope could be constructed using simplified polyhedral annexation method.

Stephan Dempe. *Computing local optimal solutions of the bilevel optimization problem using the KKT approach.*

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To investigate and solve the optimistic smooth bilevel optimization problem it needs to be transformed into a single level optimization problem. For doing that the lower level problem is often replaced by its Karush-Kuhn-Tucker conditions. The resulting problem is a nonconvex, irregular, nonsmooth optimization problem. Unfortunately, local optimal solutions of this problem do not need to be related to local optimal solutions of the bilevel optimization problem. To overcome this unpleasant situation, a sequence of local optimal solutions of certain approximations of this problem is computed which can be shown to converge to a local optimal solution of the bilevel optimization problem.

Boris Dobronets, Olga Popova. *Computational probabilistic analysis for random linear programming.*

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The article discusses a new approach to linear optimization problems with random input data. This approach uses computational probability analysis to construct probabilistic extensions and solutions of systems of linear algebraic equations. This allows us to estimate the probability density functions of the objective function on the set solutions to the optimization problem.

Anton Ereemeev¹, Nikolay Tyunin¹, Alexander Yurkov². *Non-Convex Quadratic Programming Problems in Short Wave Antenna Array Optimization.*

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In this paper, we describe a non-convex constrained quadratic programming problem arising in short wave transmitting antenna array synthesis and provide preliminary computational results. We consider problem instances for three different antenna designs including up to 25 radiators. In our preliminary computational experiments, BARON package is compared to the gradient optimization method, applied to the unconstrained problem formulation using the penalty function method. Global optimality of the obtained solutions was established using BARON package only for the smallest instances of 4 radiators. On the larger instances, both methods demonstrate similar results. The work was partially funded in accordance with the state task of the Omsk Scientific Center SB RAS (project number FWEF-2019-0006).

Vladimir Erokhin. *Regularization and matrix correction of improper linear programming problems.*

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The results of the study of improper linear programming problems are presented, in which the duality theory is essentially used and the approaches of I.I. Eremin (correction of incompatible constraints) and A.N. Tikhonov (creation of compatible systems of constraints equivalent in accuracy to given incompatible constraints).

Vladimir Erokhin¹, Alexander Krasnikov², Vladimir Volkov³. *Using matrix correction of improper linear programming problems in the problem of pattern recognition with intersecting classes.*

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Present paper is devoted to the application of optimal matrix correction to the pattern recognition problems. The solution of the pattern recognition problem with intersecting classes using the separation hyperplane method is considered. The initial problem is reduced to the problem of matrix correction of a pair of dual linear programming problems minimal with respect to the

Euclidean norm. The results of computational experiments confirming the efficiency of the proposed method are given.

Yury Evtushenko, Alexander Golikov, Igor Kaporin. *Penalty and regularization methods for solving systems of linear equations and inequalities.*

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The problems of finding solutions to underdetermined linear equations systems with nonnegative variables and linear inequalities systems do not belong to the scope of computational linear algebra classical problems. As a rule, these problems have non-unique solutions. They are reduced to the optimization problems. To solve such problems it is expedient to utilize the penalty method, regularization, duality theory and various unconstrained optimization methods, e.g. the Newton's method. Using the theory of duality, the connection between the penalty and regularization methods is shown. The optimization methods provide an opportunity to select a single solution from a set of linear system solutions (for example, a normal solution, a given point projection). The Newton's method performance was tested using the sample data from the NETLIB sparse matrix collection as well as the quasirandom data.

Sergey Ivanov, Irina Zhenevskaya. *Estimation of the necessary sample size for approximation of stochastic optimization problems with probabilistic criteria.*

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We consider stochastic optimization problems with probabilistic and quantile objective functions. The probability objective function is defined as the probability that the value of losses does not exceed a fixed level. The quantile function is defined as the minimal value of losses that cannot be exceeded with a fixed probability. We formulate sample approximations of the considered problems. We describe a method to estimate the accuracy of the approximation of the probability maximization and quantile minimization for the case of a finite set of feasible strategies. Based on this method, we estimate the necessary sample size to obtain (with a given probability) an epsilon-optimal strategy to the original problems by solving their approximations in the cases of finite set of feasible strategies. Also, we obtain necessary sample size for the probability maximization in the case of a bounded set of feasible strategies and a Lipschitz continuous probability function, we obtain the sample size.

Miloica Jacimovic, Nevena Mijajlovic. *Dynamical systems and quasi-variational inequalities.*

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We propose and analyze projected dynamical systems associated with quasi-variational inequalities by using the techniques of the projection and Fejer operators. We prove the globally asymptotic stability of these dynamical systems.

Yuri Kan, Sofia Vasil'eva. *Deterministic approximation of stochastic programming problems with probabilistic constraints.*

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Supported by the Russian foundation of basic research (project No.18-08-00595).

The work is devoted to the development of a method for solving the linear stochastic programming problem with a deterministic objective function and individual probabilistic constraints. Each probabilistic constraint is a constraint on the probability of an inequality for a certain loss function that is linear in random parameters. In this case, the considered function may be nonlinear in strategies. It is proposed to replace each probabilistic constraint by an equivalent inequality for the quantile function [1]. This inequality is approximated using the notion of the kernel of a probability measure [1]. The kernel of a probability measure of a given level for the distribution of a random vector is defined as intersection of all closed half-spaces, the probability measure of which is greater than or equal to the given one [1]. It is known that in the case where the kernel satisfies the regularity property and the loss function is linear in random parameters, its quantile can be found as maximum in realizations of random parameters belonging to the probability measure kernel. To find quantiles, the method [2] is used, based on the external polyhedral approximation of the probability measure kernel. When replacing a kernel by its approximation, the maximum mentioned above is an upper estimate of the exact value of the quantile function. With such a replacement, each quantile constraint is replaced by several deterministic constraints, in which random variables are replaced by their implementations, which are the vertices of the constructed polyhedral approximations. Note that the parameters of the polyhedral approximation of the kernel do not depend on the optimized strategies. A theorem on the convergence of the obtained deterministic approximation to the exact solution by the criterion value is proved. The properties of the kernel of a probability measure are described in details in [1]. It should be noted that the probability measure of the kernel of a probability measure of a given level generally does not have the same value of probability. In [3], new properties of the probability measure kernel are obtained, namely a theorem

on an interior point and a sufficient condition for the kernel to be non-empty. These results allow us to extend the applicability of methods based on the kernel of a probabilistic measure.

1. Kibzun A.I., Kan Yu.S. Zadachi stokhasticheskogo programmirovaniya s veroyatnostnymi kriteriyami (Stochastic programming problems with probabilistic criteria) – Moscow, Fizmatlit, 2009. –372 p. [in Russian] 2. S. N. Vasil’eva, Yu. S. Kan A method for solving quantile optimization problems with a bilinear loss function // Autom. Remote Control, 2015, no. 76:9, pp. 1582–1597. 3. S. N. Vasil’eva, Yu. S. Kan Approksimaciya veroyatnostnyh ogranichenij v zadachah stokhasticheskogo programmirovaniya s ispol’zovaniem yadra veroyatnostnoj mery // Avtomatika i telemekhanika (in print) [in Russian]

Mikhail Khvostov¹, Vladimir Erokhin². *About matrix correction of a dual pair of improper linear programming problems with a minimum weighted Euclidean norm.*

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The paper focused on problem of matrix correction of a dual pair of improper linear programming problems with respect to the minimum weighted Euclidean norm. This problem is reduced to the auxiliary problem of unconstrained differentiable minimization. The Broyden-Fletcher-Goldfarb-Shanno quasi-Newton algorithm is considered as a tool for the numerical solution of this problem. Mentioned algorithm uses analytical formulas for calculating the partial derivatives of the objective function, which are obtained in the paper. The results of computational experiments algorithm convergence in terms of the objective function and the argument are given.

Igor Konnov, Olga Pinyagina. *Splitting method with adaptive step-size.*

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We suggest the modified splitting method for mixed variational inequalities and prove its convergence under rather mild assumptions. This method maintains the basic convergence properties, but does not require any iterative step-size search procedure. It involves a simple adaptive step-size choice, which takes into account the behavior of the problem along the iterative sequence.

The key element of this approach is a given majorant step-size sequence converging to zero. The next decreased value of step-size is taken only when the current iterate does not give a sufficient descent of the goal function. This descent value is estimated with the help of an Armijo-type condition, similar to the rule used in the inexact step-size linesearch. If the current iterate gives

a sufficient descent, we can even take an increasing step-size value at the next iterate. Preliminary results of computational experiments confirm efficiency of the proposed method.

Olga Murav'eva. *Matrix Correction of Inconsistent Systems of Linear Inequalities Using the Matrix l_1 -Norm.*

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The problem of determining the minimal change in the coefficients of a inconsistent system of linear inequalities that makes the system consistent is considered. There are two cases: the right-hand sides of inconsistent systems of linear inequalities coefficient are adjuted or fixed. A criterion for the correction magnitude is the sum of the moduli of all elements of the correction matrix or matrix l_1 -norm. The matrix correction of inconsistent systems of linear inequalities written in different forms (with the condition that some of the variables or all of them are nonnegative) are considered. Optimization problems of this type can arise when infeasible linear programming problems are corrected. For an improper linear programming problem with inconsistent constraints we consider the minimal correction of the constraint matrix under restriction to the value of the objective function. Formulated problems of minimal matrix correction of inconsistent systems of linear inequalities or infeasible linear programming problems are reduced to a collection of finitely many linear programming problems.

Leonid Popov. *Methods for matrix games with mixed strategies and quantile payoff function.*

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Matrix games with mixed strategies are investigated, where the payment function is defined not as the mathematical expectation of a random gain in a long series of parties, but as its VaR-estimate for a given level of risk. The properties of such games are studied, and the methods for their solution are suggested.

Mikhail Posypkin, Andrey Gorchakov. *A high performance method for constructing an outer approximation of a set defined by a system of equations.*

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In practice it is often needed to approximate a set defined a system of equations. Such approximations can help to estimate a volume of a set or construct a path between two points of the set. We show how traditional global optimization techniques can be used for obtaining an approximation. We compare a well-known approach based on interval analysis with a different approach proposed by us based on Lipschitzian optimization. We show that local solution methods can significantly improve the quality of the approximation. Unfortunately the performance is a key issue preventing an efficient utilization of the method for many practical cases. We show how this issue can be mitigated by parallelization. We propose a parallel approximation algorithm aimed at multicore shared memory machines. Computational experiments demonstrate that parallelization can significantly improve the performance of the method.

Igor Prudnikov. *Accelerated method of finding for the minimum of arbitrary convex function.*

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The goal of the paper is development of an optimization method with the superlinear convergence rate for an arbitrary convex function. For optimization an approximation is used that is similar to the Steklov integral averaging. The difference is that averaging is performed over a variable-dependent set, that is called a set-valued mapping (SVM) satisfying simple conditions. Novelty approach is that with such an approximation we obtain twice continuously differentiable convex functions, for optimizations of which are applied methods of the second order. The estimation of the convergence rate of the method is given.

Vladimir Semenov, Yana Vedel, Viacheslav Dudar. *Convergence of the optimistic mirror-prox method for saddle point problems and variational inequalities.*

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A convex-concave saddle point problems and variational inequalities with pseudo-monotone operators are considered. Variants of the method of optimistic gradient descent are proposed for solving these problems. Convergence theorems are proved and non-asymptotic estimates of the efficiency of the methods are obtained.

Vladimir Skarin. *On the application of the quasi-solution method for the correction of inconsistent problems of convex programming.*

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The method of quasi-solutions which is one of the standard regularization procedures for ill-posed optimization problems is applied to an improper convex programming problem. The inconsistent constraints for the typical scheme of the quasi-solution method are eliminated by the some penalty function. The estimates characterising the connection between solution of the problem with penalty and appropriate problem for approximation of initial improper model are obtained.

Fedor Stonyakin. *Some adaptive algorithms for strongly convex-concave saddle point problems.*

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We consider the problem a strongly convex-concave saddle point problems for functionals with a special generalized smoothness condition. For such problems we introduce some analogue of the known concept of inexact oracle by O. Devolder - F. Gliner - Yu.E. Nesterov. For considered class of problems adaptive analogues of the methods by A.S. Nemirovsky and Yu. E. Nesterov proposed by us. Theoretical estimates of the rate of convergence for these methods are obtained. The question of the influence on the final estimate errors of solving auxiliary problems arising at the iterations of methods is discussed.

Fedor Stonyakin^{1,2}, Darina Dvinskikh³, Pavel Dvurechensky³, Alexey Kroshnin^{2,4}, Olesya Kuznetsova², Artem Agafonov², Alexander Gasnikov^{2,4}, Alexander Turin⁷, Cesar Uribe⁵, Dmitry Pasechnyuk⁶, Sergei Artomonov⁷. *Gradient Method for Problems with Inexact Model of the Objective.*

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We consider optimization methods for convex optimization problems under inexact information on the objective function. We introduce inexact model of the objective, which as a particular cases includes inexact oracle [1] and relative smoothness condition [2]. We analyze gradient method which uses this inexact model and obtain convergence rates for convex and strongly convex problems. To show potential applications of our general framework we consider three particular problems. The first one is clustering by electoral model introduced in [3]. The second one is approximating optimal transport distance, for which we propose a Proximal Sinkhorn algorithm. The third one is devoted to approximating optimal transport barycenter and we propose a Proximal Iterative Bregman Projections algorithm. We also illustrate the practical performance of our algorithms by numerical experiments.

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Fedor Stonyakin^{1,2}, Mohammad Alkousa², Alexander Titov², Victoria Piskunova¹. *On Some Methods for Strongly-Convex Optimization Problems With One Functional Constraint.*

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We consider the classical optimization problem of minimizing a strongly convex, non-smooth, Lipschitz continuous function with one Lipschitz-continuous constraint. We develop the approach [<https://arxiv.org/abs/1806.09071>] and propose two methods for considered problem with adaptive stopping rules. The main idea of the methods is using the Dichotomy method and solving an auxiliary one-dimensional problem each iteration. Theoretical estimates for the proposed methods are obtained. Partially, for smooth functions (with Lipschitz-continuous gradient) we prove the linear rate of convergence of the methods. We also consider theoretical estimates in the case of non-smooth functions. The results for some examples of numerical experiments illustrating the advantages of the proposed methods and the comparison with adaptive optimal method for non-smooth strongly convex functions are also given.

Tatiana Tchemisova¹, Olga Kostyukova². *Phenomenon of Immobility in study of convex Optimization problems.*

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We are concerned with convex problems of infinite Optimization, namely problems of convex Semi-Infinite Programming (SIP), linear problems of Semi-definite Programming (SDP), and linear Copositive Programming (LCP) problems that are closely related.

In this talk, we present our recent results on optimality for convex Semi-Infinite Programming and apply them to problems of linear SDP and LCP. Our approach is based on the notions of immobile indices and their immobility orders for problems of LSIP and LCP and of a subspace of immobile indices for problems of linear SDP. We show how these concepts can be used to obtain new CQ-free optimality conditions for the considered classes of Optimization problems.

**Dragan Urošević¹, Yiad Ibrahim Yousef Alghoul²,
Zhazira Amirgaliyeva³, Nenad Mladenović⁴.** *Less is more: Tabu search for Quadratic Bipartite Programming Problem.*

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Having defined complete bipartite graph G , with weights associated with both vertices and edges, the Bipartite Quadratic Programming problem (BQP) consists in selecting a sub-graph that maximizes the sum of the weights associated with the chosen vertices and the edges that connect them. Applications of the BQP arise in mining discrete patterns from binary data, approximating matrices by rank-one binary matrices, computing the cut-norm of a matrix, etc. In addition, BQP is also known in the literature under different names such as: maximum weighted induced sub-graph, maximum weight bi-clique, matrix factorization or maximum cut on bipartite graphs. Since the problem is NP-hard, many heuristic methods have been proposed in the literature to solve it. In this paper we apply the recent Less is more heuristic approach, whose basic idea is to design a simple as possible, i.e., a method that would use a minimum number of ingredients, but to provide solutions of better quality than the current state of the art. To reach that goal, we propose a simple hybrid heuristic based on Tabu search, that uses two neighborhood structures and relatively simple rule for implementation of short-term memory operation. In addition, a

simple rule for calculating tabu list length is introduced. Computational results compare favourable with the current state-of-the-art heuristics. Despite of its simplicity, our heuristic was able to find 8 new best known solutions on very well studied test instances.

Rashid Yarullin. *Proximal Bundle Method with Periodically Discarding Cutting Planes.*

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It is proposed a proximal bundle method for minimizing a convex function on a polyhedron. The minimization method is characterized by periodically discarding cutting planes.

Vitaly Zhadan. *Variant of Simplex Method for Second-order Cone Programming.*

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The linear second-order cone programming problem is considered. For its solution the variant of primal simplex-type method is proposed. This variant of the simplex method is generalization onto cone programming of the standard method for linear programming. At each iteration the dual variable and dual slack are defined, and passage from the moving extreme point to another one is realized. Finite and infinite convergence of the method to solution of the problem, having a special form, is discussed.

Valery Zorkaltsev. *History and prospects of interior point method.*

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History of creation of family of efficient algorithms by interior point method is presented in the report. Estimation of Lagrange multipliers under non optimal plan by least square method (L. Kantorovich 1965) is the main idea for creation of these algorithms. On the base of the idea I. Dikin developed and was studying the algorithm from 1966 till 1972. The algorithm was assumed the name "affine scaling method". The algorithms of such type were being progressed in Russia in Siberian Energy Institute (I. Dikin, V. Zorkal'tsev) and in Computation Centre (Y. Evtushenko, V. Zhadan) of Academy of Sciences of the USSR before the mid-eighties. A wealth of experience of using of the algorithms in energy models is accumulated. The algorithms are got a wide popularity in different countries

after promising article by N. Karmarkar in 1984. We are going to consider of conceptually connection of these algorithms with method of logarithm penalty function developed by R. Frish in 1955. Axiomatic defined sets of primal and dual interior point algorithms will be presented. These algorithms originally combine procedures of entering and optimization in feasible region in one computational process. As a result of theoretical researches we extracted subsets of algorithms with different useful properties such as linear and superlinear convergence, getting of relative interior points of set of optimal solution, faster convergence of dual estimates. The last property allows to recommend using of dual algorithms for faster finding of solution of primal problem with required accuracy. Results of experimental researches of more effective variants of algorithms will be presented. We will also discuss about perspective algorithms witch allow to increase speed of optimization process and to find cases of nonexistence of solution arising by reason of inconsistency of constraints or unboundedness of objective function on the feasible set. The special consideration will be devoted of practical using of property of interior point algorithms to produce relative interior points of set of optimal solutions under analysis of stability of solutions to errors of original data, under solving of multicriteria problems of lexicographic optimization on example of the model of reliability analyzing of electricity supply. In particular, this property plays important role in algorithm of searching of Chebyshev projection of point to polyhedron including to linear manifold. It allows to make of computations without Haars condition.

Anna Zykina, Olga Kaneva, Victoria Munko. *Multicriteria model curriculum.*

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The curriculum process of formation is a key problem of the educational process of the University. Most of the research on this problem use information technology in the curriculum formation without analyzing the requirements of Federal State Educational Standards. At the same time, the curriculum formation is characterized by a significant influence of the higher education workers contingent and, as a result, it needs a serious restructuring. This is especially true in the context of increasing requirements for the quality of training. A primitive curriculum with a random distribution of disciplines by modules leads to a decrease in the quality of the basic educational program. In consequence of that, there is a decrease in the quality of professional skills of graduates. The inclusion of optimization procedures in the construction of the curriculum allows us to evaluate the choice of specific disciplines that fully provide competence.

The formation of the curriculum is implemented under the condition of the fulfillment of many requirements that defined in the regulatory documents. According to the mathematical modeling, they are fuzzy. Requirements, on the one hand, can be represented as restrictions that imposed on the curriculum formation, on the other hand, can be represented as the target functions, the value of which must be maximized (minimized). As a result, the task of the curriculum formation is reduced to a class of poorly formalized problems with indistinct restrictions, incomplete and fuzzy data. The feature of this problems is that it is impossible to find the only acceptable optimal solution. These disadvantages leads to a variety of curriculum formation models. Moreover, the choice of a particular solution depends on subjective factors and during their formalization depends on the models and algorithms adopted. The developed approach to the formation of the curriculum provides a link not only disciplines and competencies, but also teachers. The obtained results can be used to solve tasks for improving the efficiency of compiling and verifying the compliance of the curricula quality criteria.

4 . Global Optimization

Maria Barkova. *On generating nonconvex optimization test problems.*

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This paper addresses a technique for generating two types of nonconvex test problems. We study quadratic problems with d.c. inequality constraints and sum-of-ratios programs where both numerators and denominators are quadratic functions. Based on the idea of P. Calamai and L.Vicente, we propose the procedures for constructing nonconvex test problems with quadratic functions of any dimension, where global and local solutions are known. The implementation of the procedures does not require any complicated operations and solution of auxiliary problems, except for elementary operations with matrices and vectors.

Maria Barkova, Alexander Strekalovskiy. *Computational testing of the local search method with penalty update procedure.*

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This paper addresses a nonconvex optimization problem with the goal function and inequality constraints given by d.c. functions. Due to the exact penalization theory, the original problem is reduced to a penalized problem, the goal function of which is presented as d.c. function. The main idea of the developed local search method (LSM) is a consecutive solution of the partially linearized problem. These problems linearized with respect to the basic nonconvexity and turn out to be convex. Convergence properties of the local search scheme are also investigated, which, in particular, yield that the sequence, produced by LSM, converges to a solution of the problem linearized at the limit point. Moreover the proposed LSM contains the procedure for choosing and updating the penalty parameter. Finally, a computational testing of the LSM has been performed on test nonconvex quadratic problems with known global solution.

Olga Druzhinina¹, Olga Masina², Alexey Petrov². *Algorithms for global optimization in the problems of technical systems motion modeling and control.*

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The issues of creating and implementing global optimization algorithms in design problems of the technical systems motion control models are considered. A method for studying switching dynamic models using polynomial approximation and intelligent control components is proposed. The problem of search the optimal motion parameters using unconditional optimization with scalar

ranking criteria is studied. To solve this problem, optimization algorithms based on the particle swarm optimization method and the random search method are proposed. The generalized models are considered taking into account nonstationary perturbations and the possibility of finding in some cases analytical solutions. Indicated models can be used to verify the obtained results. The proposed algorithms are implemented in the form of software libraries that are a part of a specialized software package. A comparative analysis of the algorithms efficiency is performed. The effect of the algorithms use is associated with time and energy resources minimizing. The obtained results can be applied in the problems of design and the search for optimal parameters of the autopilot vehicles motion models, as well as of technical systems with switching operation modes.

Victor Gergel, Alexander Sysoev, Maria Kocheganova, Evgeny Kozinov. *Some approaches for visual evaluation of global optimization method efficiency.*

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In order to evaluate efficiency of some global optimization method or compare efficiency of different methods, it is necessary to select a set of test problems, fix method parameters (if any), define comparison measures, and, finally, chose a way of visual presentation of the computational results. In this paper, a wide set of test optimization problems is considered. Main performance measures and comparative criteria of efficiency are presented. The ways of visual presentation of computational results are suggested. Such visual methods are given both for evaluating any chosen method and for performing comparative analysis of several methods.

Aleksander Gornov, Tatiana Zarodnyuk, Anton Anikin, Pavel Sorokovikov. *The Stochastic Coverings Algorithm for Solving Applied Optimal Control Problems.*

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The paper considers a heuristic method for a global extremum search in an optimal control problem based on the idea of covering a reachable set by n-dimensional balls, including the built-in mechanisms for Lipschitz constant estimating of the objective functional. A step-by-step description of the coverage algorithm and the proposed method for generating start and auxiliary controls

are presented. Proposed technique was used for solving applied optimal control problems: the problem of investment programs in Buryatia Republic and the problem of restoring the Black Lands in Kalmykia.

Igor Kandoba, Alexander Uspenskiy. *On one applied problem of vector optimization.*

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The paper is devoted to mathematical simulations of an autonomous network of wholesale at a local market. The question of optimization of prices at outlets is discussed. The problem of price optimization at interconnected outlets under additional restrictions is the focus. The suggested mathematical model is classified as a linear problem of vector optimization. The main properties of the multicriteria problem are studied. The optimal plan is defined. The necessary and sufficient conditions for the existence and uniqueness of the optimal plan are formulated. The finite iterative procedure for the problem solution is developed. The numerical algorithm is based on specific variations of the parameters. The results of the extensive numerical experiments show that the suggested numerical solution algorithm for the vector optimization problem is faster and more robust than the traditional methods. The results are illustrated by examples of numerical solutions using real data. Practical applications are presented in the form of specialized algorithms and software.

Oleg Khamisov. *Cutting plane methods for global minimization of a quasiconcave function over a box.*

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Two types of cutting planes are suggested to use in the problem of global minimization of a quasiconcave function over a box. The first type is formed by well-known concavity cuts, the second by so called second order concavity cuts. If the objective function is differentiable and concave then some procedures of construction deep cuts by means of best concave extensions is suggested. Results of numerical experiments are given.

Valeriy Marakulin. *Spatial equilibrium in a multidimensional space: an immigration-consistent division into countries centered at barycenter.*

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The problem of immigration proof partition for communities (countries) in a multidimensional space is studied. This is an existence problem of Tiebout type equilibrium, where migration stability suggests that every inhabitant has no incentives to change current jurisdiction. In particular an inhabitant at every frontier point has equal costs for all possible for him jurisdictions. It is required that inter-country border is represented by a continuous curve. The paper presents the solution for the case of the costs described as the sum of the two values: the ratio of total costs on the total weight of the population plus transportation costs to the center presented as a barycenter of the state. In the literature, this setting is considered as a case of especial theoretical interest and difficulty. The existence of equilibrium division is stated via an approximation reducing the problem to the earlier studied case, in which centers of the states never can coincide: to do this proved earlier a generalization of Krasnosel'skii conic fixed point theorem is applied.

Ilya Minarchenko. *On minimization of a quadratic function with one negative eigenvalue.*

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It is well known that a quadratic programming minimization problem with even one negative eigenvalue is NP-hard. However in practice we may expect such problems to be not so difficult to solve. For convenience let us reduce the objective function to the principal axes by a linear transformation (separable function form). Then for minimization we suggest to use a branch and bound procedure with partitioning in concave variable only. In order to compute a lower bound for every partition set, we minimize partially linearized objective function with linearized concave term. Results of computational experiment are presented.

Evgeni Nurminski¹, Natalia Shamray². *Discrete Time Lyapunov-Type Convergence Conditions for Recursive Sequences in Optimization.*

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We present here the set of conditions for recursive optimization-like processes which guarantee their convergence to a given solution set. These conditions simplify studies of convergence for such processes by essentially reducing them to the analysis of the processes local behavior at arbitrary small vicinity of points outside the solution set. They also implicitly implement rather complicated part of the logic of convergence proofs when there is no strict monotony of Lyapunov function along the process trajectory. The new development in the

area of convergence conditions is specifically directed to the study of approximate solutions of optimization and similar problems and demonstrated for a quasi-gradient method for non-convex non-differentiable (weakly convex) functions.

Andrei Orlov, Tatiana Gruzdeva. *The Local and Global Searches in Bilevel Problems with a Matrix Game at the Lower Level.*

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This work addresses the simplest class of the bilevel optimization problems (BOPs) with equilibrium at the lower level. We study linear BOPs with a matrix game at the lower level in their optimistic statement. First, we transform this problem to a single-level nonconvex optimization problem with the help of the optimality conditions for the lower level problem. Then we apply the special Global Search Theory (GST) in general d.c. optimization problems to the reduced problem. Following this theory, the methods of local and global searches in this problem are constructed. These methods take into account the structure of the problem in question.

Lyubov G. Shagalova. *Piecewise linear minimax solution of Hamilton – Jacobi equation with nonhomogeneous Hamiltonian.*

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The terminal value Cauchy problem is considered for Hamilton – Jacobi equation with nonhomogeneous Hamiltonian. The Hamiltonian and the terminal function are piecewise linear, and the dimension of state space is two. This equation reduces to an equation with a homogeneous Hamiltonian in three-dimensional state space. A finite algorithm for the exact construction of the minimax and/or viscosity solution is proposed and justified. The algorithm consists of a finite number of consecutive stages, at each of which elementary problems of several types are solved and the continuous gluing of these solutions are carried out. The solution built by the algorithm is a piecewise linear function. Structural matrices are a convenient form for representation of the solution.

Evgenii Sopov, Alexey Vakhnin. *A Decomposition-based Approach for Constrained Large-scale Global Optimization.*

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Many real-world global optimization problems are too complex for comprehensive analysis and are viewed as “black-box” (BB) optimization problems. Modern BB optimization has to deal with growing dimensionality. Large-scale global optimization (LSGO) is known as a hard problem for many optimization techniques. Nevertheless, many efficient approaches have been proposed for solving LSGO problems. At the same time, LSGO does not take into account such features of real-world optimization problems as constraints. The majority of state-of-the-art techniques for LSGO are based on problem decomposition and use evolutionary algorithms as the core optimizer. In this study, we have investigated the performance of a novel decomposition-based approach for constrained LSGO (cLSGO), which combines cooperative coevolution of SHADE algorithms with the ε constraint handling technique for differential evolution. We have introduced some benchmark problems for cLSGO, based on scalable separable and non-separable problems from IEEE CEC 2017 benchmark for constrained real parameter optimization. We have tested SHADE with the penalty approach, regular ε -SHADE and ε -SHADE with problem decomposition. The results of numerical experiments are presented and discussed.

Vladislav Sovrasov. *Comparison of several stochastic and deterministic derivative-free global optimization algorithms.*

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In this paper popular open-source solvers are compared against Globalizer solver, which is developed at the Lobachevsky State University. The Globalizer is designed to solve problems with black-box objectives satisfying the Lipschitz condition and shows competitive performance with other similar solvers. The comparison is done on several sets of challenging multi-extremal benchmark functions. Also this work considers a method of heuristic hyperparameters control for the Globalizer allowing to reduce amount of initial tuning before optimization. The proposed scheme allows substantially increase convergence speed of the Globalizer by switching between “local” and “global” search phases in runtime.

Vladimir Stanovov, Shakhnaz Akhmedova, Eugene Semenkin. *Genetic Algorithm with Success History Based Parameter Adaptation.*

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The development of modern and efficient optimization methods is an important direction of research, because these methods could find their application

in the area of various technical, engineering and scientific problems. Today, heuristic methods, which do not use any information about specific properties of the problem at hand, have gained a lot of attention from the research community as they are capable of solving any type of optimization problems, including binary, integer, real-valued, combinatorial, constrained, multi-objective optimization problems and many others. One of the earliest heuristic methods is the genetic algorithm (GA), which uses the idea of natural evolution to generate new solutions via operators of selection, crossover, mutation and population update. Modern genetic algorithms usually rely on a set of different types of genetic operators, presented above, and many of them implement self-adaptation or self-configuration schemes. The self-configuration is required because the efficiency of GA depends on the type of operators used: each operator has its own properties, which could be helpful at different stages of the search process. Moreover, some genetic algorithm variants use parameter adaptation mechanisms to change the probabilities of selection, crossover and mutation operators' application. In this study the success history adaptation (SHA) mechanism is applied to genetic algorithm to improve its performance. The SHA method was originally proposed for another class of evolutionary algorithms, namely differential evolution (DE). The application of DE's adaptation mechanisms for genetic algorithm allowed significant improvement of GA performance when solving different types of problems including binary optimization problems and continuous optimization problems. For comparison, in this study, a self-configured genetic algorithm is implemented, in which the adaptive mechanisms for probabilities of choosing one of three selection, three crossover and three mutation types are implemented. The comparison was performed on the set of functions, presented at the Congress on Evolutionary Computation for numerical optimization in 2017. The results demonstrate that the developed SHAGA algorithm outperforms the self-configuring GA that proves the importance of the presented modification.

Alexander Strekalovsky. *Minimizing sequences and global search in d.c. optimization problem.*

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We consider the general optimization problem with the goal function and equality and inequality constraints given by d.c. functions. The original problem is reduced to a problem without constraints with the help of the exact penalization theory. Besides, the goal function of the auxiliary penalized problem turns out to be d.c. function. We study the case when the sets of minimizing sequences of both problems coincide. Further we prove necessary and sufficient conditions for the sequence to be minimizing in the penalized problem. In

addition we propose a theoretical method that generates a minimizing sequence for the penalized problem. We also develop a global search scheme and prove that it generates a minimizing sequence in the penalized problem.

Sergey Trofimov, Alexey Ivanov. *An Infinitesimal Approach to the Construction of Optimality Criterion for Unconstrained Optimization Problems in Polar Coordinates.*

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Traditionally, when checking stationary points for optimality in unconstrained optimization problem, first-order and second-order optimality conditions are used, which imply finding the gradient and the Hessian of the optimized function. In the case of unconstrained programs with a semidefinite Hessian, directions, for which the coefficient of the quadratic component is zero, play an important role in verifying the optimality of stationary points. Thus, to clarify the optimality of a point, it is necessary to investigate the behavior of the coefficients of the third, fourth, etc. orders of smallness. In this paper, we propose an infinitesimal approach to the construction of optimality criterion for unconstrained optimization problems. This approach involves the transition from the Cartesian coordinate system to the polar coordinate system. With this approach, each term in the Taylor series is defined by the order of smallness of this term and the set specified in the polar coordinate system. Moreover, the dimension of these sets coincides with the dimension of the variables. New components are the product of the modulus of the vector in the first or second power by the proportionality coefficient of an infinitely small quantity in the direction of this vector. We show that the coefficient function has an analytical expression in the form of a linear combination of harmonics depending on polar direction angles. Logarithmic algorithm is used for finding the proportionality coefficients of the infinitely small quantities with the first, second, etc., order of smallness. We give examples of expansion of the infinitely small quantities with two and three variables, for which graphs of coefficients up to the third order of smallness are plotted. We propose an optimality criterion, in which algebraic curves are used in space, the dimension of which is one less than the dimension of the variables of the original function. For the function of two variables, the criterion is expressed in terms of real or complex roots of polynomials. For the function of three variables, algebraic criteria on the plane are used.

Ushakov Vladimir, Lebedev Pavel. *Iterative methods for optimal packing approximations constructing for non convex polygons.*

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The paper proposes algorithms for iteratively constructing optimal coverings of non-convex flat figures with sets of circles. They are based on the procedure of dividing the figure into the areas of influence of points that serve as the centers of the elements of the initial packing, and finding the Chebyshev centers of these zones. To generate the initial array of points, stochastic procedures are used that use the synthesis of optimal hexagonal grids and random vectors.

Simeon Vom Dahl, Andreas Löhne. *On polyhedral d.c. optimization problems.*

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We investigate a class of non-convex optimization problems where the objective function is a difference of two convex functions. One of these two functions is assumed to be polyhedral convex. This problem is called polyhedral d.c. optimization problem. We characterise the existence of optimal solutions for polyhedral d.c. optimization problems. The polyhedral d.c. optimization is transformed into a quasi-concave optimization problem and solved by a respective algorithm.

5 . Integer Programming

Vladimir Beresnev, Andrey Melnikov. *Algorithm to compute an upper bound for the competitive facility location problem with prescribed choice of suppliers.*

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We consider the competitive facility location problem with the prescribed choice of supplier. The problem can be formalized as a pessimistic bi-level program. It is relative to a family of competitive location models built on the basis of the Stackelberg game, where both the upper and the lower level problems are facility location problems with orders. For the problem under consideration, we suggest the approach to compute an upper bound for values taken by the objective function on subsets of a feasible region. The method consists in constructing estimating problems in a form of MIP obtained from a high-point relaxation (HPP) of the bi-level program. To improve the quality of the upper bound, we introduce a new family of additional constraints satisfied by any pessimistic feasible solution and reducing a feasible region of the HPP. Additionally, we describe a new procedure to generate these constraints allowing to cut-off an optimal solution of the estimating problem when it is infeasible in a bi-level sense. The procedure relies on an auxiliary optimization problem aiming to generate cuts which are as strong as possible.

George Bolotashvili. *Expansion (m, k) facets, in the case of $k \geq 4$, k -even, $m = 3k - 1$, for a linear ordering polytope.*

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Is it possible to construct a polyhedron, using linear equalities and inequalities, corresponding to some NP-hard problem? This question is relevant to the author. In this paper, for the NP-hard linear order polytope, a new class of facets is built. In 1987 we built the so-called (m, k) facets, where $m = \top k - 1$. When these facets are expanded, below certain values of $\top$ and k , we obtain fundamentally different facets from each other. Therefore, given the difference and the complexity of individual classes of cells, they are studied separately. When $k \geq 3$, k is odd, $\top = 3$, Bolotashvili G., Demidenko V., Pisaruk N. built the facet in 2014; when $k \geq 3$, k is odd, $\top \geq 4$, Kovalev M., Bolotashvili G. built the facet in 2012; when $k \geq 4$, k is even, $\top = 3$, the facet is built in this work. Also facets are built separately: when $k \geq 5$, k is odd, $\top = 3$; when $k \geq 5$, k is odd, $\top \geq 4$; and when $k \geq 4$, k is even, $\top \geq 4$.

Maksim Barketau. *Representation and properties of the optimal solutions of several discrete optimization problems with incomplete input.*

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We consider the general problem formulation as the minimization of the sum of the numbers of the feasible subset of cardinality n out of the set of N numbers. Asymmetric travelling salesman problem, assignment problem and the problem of minimization of the sum of n numbers out of the set of N numbers can be formulated in these terms. The input of all these problems can be represented as the point from the non-negative part of the unit sphere after some intuitive scaling. In this settings we assume that the retrieval of the input data is difficult due to the communication complexity. We propose to skip the retrieval of some input numbers and substitute them with the zeros. We call this problem the problem with incomplete input. We estimate the quality of the optimal solution for the problem with incomplete input used as the solution for the problem with the complete data.

**Anton Ereemeev¹, Alexander Kelmanov², Mikhail Y. Kovalyov³,
Artem Pyatkin^{2,4}.** *Maximum Diversity Problem with Squared Euclidean Distance.*

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In this paper, we consider the Maximum Diversity Subset problem with optimization criterion to maximize the squared Euclidean distances between the chosen M points, given a set of points in Euclidean space. We propose an exact dynamic programming algorithm for the case of integer input data. If the dimension of the Euclidean space is bounded by a constant, the algorithm has a pseudopolynomial time complexity. Using this algorithm, we develop an FPTAS for the special case where the dimension of the Euclidean space is bounded by a constant. We also propose a new proof of strong NP-hardness of the problem in the general case.

Aigul Fabarisova¹, Vadim Kartak². *An integer programming approach to the irregular polyomino tiling problem.*

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In this paper new integer programming models to the problem of irregular polyomino tiling are introduced. We consider tiling of finite, square, $N \times N$ -sized structure with L-shaped trominoes without any restriction on their number. Each polyomino can be rotated 90° , so there are four orientations of L-tromino. Developed models are effective for small-size instances. For medium- and large-size instances we suggest dividing the initial structure into several equally sized parts and combine the solution of optimized tilings. We tried to apply new models to the existing information-theoretic entropy based approach. We conduct computational experiments using IBM ILOG CPLEX package. The problem of irregular polyomino tiling can be applied to the design of phased array antennas where polyomino-shaped subarrays are used to reduce the cost of the array antenna and to reduce the undesired sidelobes radiation. Computational results along with antenna simulation results are presented in the paper.

Dmitry Griбанov, Dmitry Malishev. *Integer Conic Function Minimization Based on the Comparison Oracle.*

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Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be a conic function and $x_0 \in \mathbb{R}^n$. In this note, we show that the shallow separation oracle for the set $\{x \in \mathbb{R}^n : f(x) \leq f(x_0)\}$ can be polynomially reduced to the comparison oracle of the function f . Combining these results with known results of D. Dadush et al., we give an algorithm with $O(n)^n \log R$ calls to the comparison oracle for checking the non-emptiness of the set $\{x \in \mathbb{Z}^n : f(x) \leq f(x_0)\} \cap R B_2^n$, where B_2^n is the unit euclidean ball. Additionally, we give a randomized algorithm with the expected oracle complexity $O(n)^n \log R$ for the problem to find an integral vector that minimizes values of f on $R B_2^n$. It is known that the classes of convex, strictly quasiconvex functions, and quasiconvex polynomials of a nonzero degree are included into the class of conic functions. Since any system of conic functions can be represented by a single conic function, the last facts give us an opportunity to check the feasibility of any system of convex, strictly quasiconvex functions, and quasiconvex polynomials by an algorithm with $O(n)^n \log R$ calls to the comparison oracle of the functions. It is also possible to solve a constraint minimization problem with the considered classes of functions by a randomized algorithm with $O(n)^n \log R$ expected oracle calls.

Maximilian John, Andreas Karrenbauer. *Dynamic Sparsification for Quadratic Assignment Problems.*

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We present a framework for optimizing sparse quadratic assignment problems. We propose an iterative algorithm that dynamically generates the quadratic part of the assignment problem and, thus, solves a sparsified linearization of the original problem in every iteration. This procedure results in a hierarchy of lower bounds and, in addition, provides heuristic primal solutions in every iteration. This framework was motivated by the task of the French government to design the French keyboard standard, which included solving sparse quadratic assignment problems with over 100 special characters; a size where many commonly used approaches fail. The design of a new standard often involves conflicting opinions of multiple stakeholders in a committee. Hence, there is no agreement on a single well-defined objective function that can be used for an extensive one-shot optimization. Instead, the process is highly interactive and demands rapid prototyping, e.g., quick primal solutions, on-the-fly evaluation of manual changes, and prompt assessments of solution quality. Particularly concerning the latter aspect, our algorithm is able to provide high-quality lower bounds for these problems in several minutes.

Tatiana Levanova, Alexander Gnusarev. *Development of Ant Colony Optimization Algorithm for Competitive p-Median Facility Location Problem with Elastic Demand.*

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In this paper, we consider the p-median competitive facility location and design problem that we have formulated based on the problem with elastic demand and the classical p-median problem. The situation that arises in a new company planning to enter the existing market of goods and services is considered. The firm wants to place its businesses in p points, capturing as much of the profits from competitors as possible. The problem has a mathematical model with a non-linear objective function. Searching the optimal solution to constructed problem is difficult. The CPU-time of commercial software is significant even for not too large dimension. For the new model, we have previously proposed variants of local search algorithms, and created a series of test instances based on real data. In this paper, an ant colony algorithm is developed, and an artificial ant algorithm is proposed. The algorithm parameters are adjusted taking into account the specifics of the problem. Experimental studies and comparison of the ant colony optimization algorithm with the simulated annealing are carried out.

Nadezhda Muravyova¹, Oksana Pichugina². *The Polyhedral-Surfaced Cutting-Plane Method for Linear Combinatorial Optimization.*

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For Linear Optimization on vertex-located sets (VLSs, i.e., coinciding with vertex sets of their hull), a Polyhedral-Surfaced Cutting-Plane Method (PSCM) is presented. It generalizes two linear optimization techniques, valid for VLSs, namely, the Method of Combinatorial Cuttings and the Method of Surfaced Cuttings. PSCM uses a representation of a VLS as an intersection of a circumscribed hypersurface. It causes that no feasible points in the interior of the polytope and all its faces of positive dimension, as well as on most of the circumsurface. A core of PSCM is in iterative refining feasible sets of two continuous relaxations of the original problem, polyhedral and surfaced, where LP relaxation is solved on each iteration first and yields a polyhedral cone, which intersection with the circumsurface underlies constructing right cuts. Depending on the type of the circumsurface, polyhedral or strictly convex, different versions of PSCM are obtained. For polyhedral-spherical and polyhedral-ellipsoidal sets, explicit solutions to auxiliary problems are derived. PSCM can be applied to a wide variety of real-world problems modeled as linear Boolean programs or permutation-based problems allowing embedding into Euclidean space.

Andrei Nikolaev. *On vertex adjacencies in the polytope of pyramidal tours with step-backs.*

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We consider the traveling salesperson problem in a directed graph. The pyramidal tours with step-backs are a special class of Hamiltonian cycles for which the traveling salesperson problem is solved by dynamic programming in polynomial time. The polytope of pyramidal tours with step-backs $PSB(n)$ is defined as the convex hull of the characteristic vectors of all possible pyramidal tours with step-backs in a complete directed graph. The skeleton of $PSB(n)$ is the graph whose vertex set is the vertex set of $PSB(n)$ and the edge set is the set of geometric edges or one-dimensional faces of $PSB(n)$. The main result of the paper is a necessary and sufficient condition for vertex adjacencies in the skeleton of the polytope $PSB(n)$ that can be verified in polynomial time.

Artem Ripatti, Vadim Kartak. *Bounds for non-IRUP instances of Cutting Stock Problem with minimal capacity.*

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We consider the well-known cutting stock problem in order to find non-IRUP instances with minimal parameters. We have found a non-IRUP instance with integer sizes of items having capacity $L=16$, while a previously known instance of such kind had capacity $L=18$. All instances with capacity $L \leq 10$ are IRUP.

Yaroslav Salii. *One Branch-and-Bound Scheme for Dynamic Programming in Precedence-Constrained Traveling Salesman Problem.*

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The precedence constrained traveling salesman problem (TSP-PC) or sequential ordering problem (SOP) consists of finding an optimal TSP tour that satisfies the namesake constraints. Dynamic programming (DP) is one its solution method, viable for “heavily constrained” problem instances; we attempt to make tractable the less constrained instances through a Morin–Marsten branch-and-bound scheme that would inherit the abstract travel cost aggregation feature of the DP, permitting its usage with both the ordinary and bottleneck (min-max instead of min-sum) versions of TSP-PC. This scheme prunes the DP state graph based on some feasible solution and a lower bound heuristic checked for each state.

We conduct a preliminary testing of this scheme as a method of proving upper bounds’ optimality, with lower bound produced by the first stage of Edmonds–Chu–Liu minimum spanning arborescence algorithm: for every city, a minimum-cost arc entering it is taken, if it does not directly contradict precedence constraints, with conventional min-sum and bottleneck (min-max) objective functions.

Alexander Semenov. *Merging variables: one technique of search in pseudo-Boolean optimization.*

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In the present paper we describe new heuristic technique, which can be applied to the optimization of pseudo-Boolean functions including Black-Box functions. This technique is based on a simple procedure which consist in transition from the optimization problem over Boolean hypercube to the optimization problem of auxiliary function in a specially designed metric space. It is shown that there is a natural connection between the points of the original Boolean hypercube and points from new metric space. For a Boolean hypercube

with fixed dimension it is possible to construct a number of such metric spaces. The proposed technique can be considered as a special case of Variable Neighborhood Search, which is focused on pseudo-Boolean optimization. Preliminary computational results show high efficiency of the proposed technique on some reasonably difficult problems. Also it is shown that the described technique in combination with the well-known (1+1)-Evolutionary Algorithm allows to decrease the upper bound on the runtime of this algorithm for arbitrary pseudo-Boolean functions.

Sergey Semenov, Nikolai Zolotykh. *A dynamic algorithm for constructing the dual representation of a polyhedral cone.*

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We propose a dynamic variation of the double description method for generating the extreme rays of a polyhedral cone. The dynamic variation of the algorithm supports online input of inequalities. Some modifications of the method were implemented and the results of computational experiments are presented. On a series of problems, our implementation of the algorithm showed higher performance results in comparison with the known analogues.

Vladimir Servakh, Kseniya Chernykh. *Research of an optimum solution to a machine problem combinatorial structure.*

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This study aims at the problem of minimization of the equal-length jobs performed on the same machine within the set jobs arrival time and with the possibility of jobs interruption. This problem is one of a few that still have an unknown computational complexity. The present article studies the combinatorial structure of possible optimum solutions for this case. The following approach is being proposed. The working weight is assigned parametrically for the deadline jobs with the set jobs arrival time. The algorithm developed in the working process turns into a basis for a finite subset of schedules that definitely contains the optimum solution. The linear program calculates a proper combination of weights that make each of the created schedules optimum. Otherwise, the absence of this combination is proven. Therefore, it gets possible to study the structure of optimum schedules as well as the reasons why some of the schedules have a dearth of them. This research studies some of the key examples, fathoms the main combinatorial features of optimum solutions, analyze the problem and distinguishes polynomially solvable cases.

Inna Urazova¹, Ruslan Simanchev¹, Yury Kochetov². *Polyhedral attack on the graph approximation problem.*

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In the clique partition problem (CPP), we need to find a spanning family of pairwise vertex-disjoint cliques of minimum total weight in a complete edge-weighted graph. In this paper, we consider the special case of the CPP, so-called graph approximation problem (GAP), where the weights of edges are 1 or -1. It is one of the most computationally difficult case of the CPP. We present our polyhedral approach to this problem based on the facet inequalities and the branch and cut framework. Computational experiments on the randomly generated instances indicate simple and hard classes of the GAP and maximal dimension for exact and an approximate solution with a given accuracy.

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A local branching MIP heuristic for a real-world Curriculum-Based Course Timetabling Problem.

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Automated timetabling is a challenging area in the timetabling and scheduling theory and practice, intensively addressed in research papers in the last two decades. There are three main classes of problems, which are usually studied: school timetabling, course timetabling and examination timetabling. In this report, we address a case study of the Curriculum-Based Course Timetabling (CB-CTT) problem, arising at Engineering Department of Sannio University. In general, the problem consists of finding a feasible weekly assignment of course lectures to rooms and time periods while respecting a wide range of constraints, which have to be either strictly satisfied (hard constraints) or satisfied as much as possible (soft constraints). The case study here addressed has many special requirements due to local organizational rules. We were able model the complex requirements by an Integer Programming formulation. The solution approach consists of using a MIP solver, integrated with two local branching heuristics tailored on the problem. The effectiveness of the proposed approach is illustrated by the computational results on two real instances

Igor Vasilyev, Anton Ushakov, Tatiana Gruzdeva.

A bi-level r -interdiction p -median problem.

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In this report we address a variant of well-know location p -median problem with the vulnerability to interdiction, i.e. the facilities can be subject to failure due the natural disasters of terrorism strikes. The objective of the p -median is to open p facilities (medians) on a network, assign each customer a median so as to minimize the overall assignment cost. The problem with the vulnerability to interdiction is considered as a static Stackelberg game. The leader solves the p -median problem taking into account that some r open facilities interdicted by the follower. The customers are reassigned from the interdicted facilities increasing the leader's assignment cost. For this bi-level problem, an one-level mixed integer linear formulation with nonpolynomial number of constraints is proposed. The study of some cases of this formulation is illustrated with the preliminary computational results on test instances.

Lidia Zaozerskaya. *Analysis of Integer Programming Model of Academic Load Distribution.*

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The bicriteria problem of academic load distribution (ALD) and its model of integer linear programming (ILP) are considered. Earlier, it was showed that the search to a feasible solution of this problem is NP-hard, and the cardinality of the complete set of alternatives is polynomial. The finding a Pareto-optimal solution can be formulated as a weighted bin packing problem with color constraints. In this problem, the number of bins is given and items have volume and color. For bins, there are upper bounds on the number of different colors. These bounds depends on volume of bin. For each item, there are set coefficients of the effectiveness (or weights) of placing in any bin. In this paper, we study the ILP model for finding a Pareto-optimal solution. Parametric families of ALD problems are constructed and the L-covering of these problems are studied. These problems have a small duality gap, in particular it can be equal to one. We investigate the complexity of solving these problems by the Land and Doig algorithm at some widely used branching rules. It is shown, that the iterations number grows exponentially with increasing number of bins.

6 . Algorithms Design and Analysis

René Van Bevern¹, Till Fluschnik², Oxana Tsidulko^{1,3}. *On $(1 + \varepsilon)$ -approximate data reduction for the Rural Postman Problem.*

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Given a graph $G = (V, E)$ with edge weights $\omega: E \rightarrow \mathbb{N} \cup 0$ and a subset $R \subseteq E$ of required edges, the Rural Postman Problem (RPP) is to find a closed walk W^* of minimum weight $\omega(W^*)$ containing all edges of R . We prove that RPP is WK[1]-complete parameterized by the number and cost $d = \omega(W^*) - \omega(R) + |W^*| - |R|$ of edges traversed additionally to the required ones, that is, presumably cannot be polynomial-time reduced to solving instances of size polynomial in d . In contrast, denoting by $b \leq 2d$ the number of vertices incident to an odd number of edges of R and by $c \leq d$ the number of connected components formed by the edges in R , we show how to reduce any RPP instance I to an RPP instance I' with $2b + O(c/\varepsilon)$ vertices in $O(n^3)$ time so that any α -approximate solution for I' gives an $\alpha(1 + \varepsilon)$ -approximate solution for I , for any $\alpha \geq 1$ and $\varepsilon > 0$. That is, we provide a polynomial-size approximate kernelization scheme (PSAKS). We make first steps towards a PSAKS for the parameter c .

Valentina Bykova, Aleksandr Soldatenko. *Polynomial Time Approximation Algorithm for Resource Constrained Shortest Path problem.*

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The paper we considers the Resource Constrained Shortest Path problem (RCSP). This problem is NP-hard extension of a well-known shortest path problem in the directed graph $G = (V, E)$. In the RCSP problem each arc e from E has a cost $w(e)$ and additional weight functions $r_i(e), i = 1, \dots, k$, which specifying its requirements from a finite set of resource. The RCSP problem has various practical applications, including design and operation of multi-service network. Nowadays, multi-service networks grow at a rapid pace. Therefore, it is relevant to search for a new approximation algorithms that can solve the RCSP problem quickly. This paper reviews existing approximation algorithms for the RCSP problem. A polynomial time ϵ -approximation algorithm RevTree based on node labeling method is presented in the paper. The main advantage of the RevTree algorithm over existing ones is its ability to produce ϵ approximation of the RCSP problem in $\mathcal{O}(|V|^2)$ time. For real networks ϵ can be calculate using values of $w(e)$ and $r_i(e), e \in E$. The present paper provides a description of the RevTree algorithm and results of computational experiments, which justify the effectiveness of proposed algorithm. Main ideas of the RevTree algorithm can be used to solve problems that extend the RCSP problem with the following

difficulties: resource recovering, probability of failure of network elements and time windows.

Ilya Chernykh¹, Antonina Khramova². *Another algorithm for the two-machine open shop and its application to one routing problem.*

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The two-machines open shop problem is proved to be solvable in linear time by Teofilo Gonzalez and Sartaj Sahni in 1976. Several algorithms for solving this problem have been proposed since that time. We introduce another optimal algorithm for that classical problem with an interesting property: it allows to process jobs in almost arbitrary order, unlike the Gohzalez-Sahni algorithm where jobs have to be partitioned into two specific subsets. While our algorithm has the same linear running time as algorithms known from literature, its properties help us to solve a much more general problem: the easy-TSP version of the routing open shop with variable depot, in which unmovable jobs are located at the nodes of a transportation network (with optimal cyclic route known), and mobile machines have to travel between the nodes to process jobs in the open shop environment. The initial location of the machines is not fixed but has to be chosen, and all machines have to return to that location — the depot — to minimize finish time. We also consider the generalization of this problem in which travel times are individual for each machine. We describe polynomially solvable subcases for this general problem. This research contributes to the discussion on the difference between various scheduling models with transportation or setup delays: classic transportation delays (in our terms, with no depot at all), with variable depot and with fixed depot. It turns out that adding the depot into the equation is crucial, and fixing the location of the depot makes the problem harder to solve.

Edward Kh. Gimadi^{1,2}, Alexander Shevyakov². *A polynomial-time algorithm for a three-level facility location problem on tree-like networks.*

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A three-level facility location problem on a tree-like network is considered under the restriction that the transportation costs for a unit of production from one node to another is equal to the sum of the edges in the path connecting these nodes. Some exact algorithm with polynomial time complexity is suggested for this problem, where n is the number of the production demand points and, m is an upper bound on the number of possible facility location sites of each level.

Edward Kh. Gimadi^{1,2}, Ekaterina Shin¹. *On random MST problem with given diameter.*

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We consider MST problem with given diameter of a tree constructed. This problem is NP-hard. For solving the problem a polynomial approximation deterministic algorithm is presented. Conditions of asymptotic optimality are given in the case of random inputs.

Edward Kh. Gimadi^{1,2}, Alexander Shtepa². *On some Implementations of solving the Restricted Capacitated Facility Location Problem.*

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Consider a graph $G = (V; E)$. At the vertices of G there are consumers of some product and the possible places of its production. For each vertex i in V the demand volume $b(i)$, the cost $f(i)$ for opening a facility and the restriction $a(i)$ on the facility's capacity are given. For each edge e in E , there are given the cost of the transportation of the product unit c_e and the maximum quantity q_e of a product that can be transported along this edge. It is required to place the facilities in a way they satisfy all demand with minimal total cost of opening facilities and delivering the product to consumers. We propose a polynomial time algorithm solving the problem and give aposterior guarantees as a result of computer experiments on some classes of random input data.

Edward Kh. Gimadi^{1,2}, Oxana Tsidulko^{1,2}, Ivan A. Rykov¹. *On PTAS for the Geometric Maximum Connected k -Factor Problem.*

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We consider the maximum Connected k -factor problem (k -CFP): given a complete edge-weighted n -vertex graph, the goal is to find a connected k -regular spanning subgraph of maximum weight. The problem is called geometric, if the vertices of a graph correspond to a set of points in a normed space R^d and the weight of an edge is the distance between its endpoints. The k -CFP is a natural generalization of the well-known Traveling Salesman Problem, which is equivalent to the 2-CFP. We complement the known $(1 + O(1/k^2))$ -approximation algorithm for the max k -CFP from [Baburin, Gimadi, 2006] with a polynomial-time approximation algorithm for the geometric maximum k -CFP, and prove that the latter algorithm is asymptotically optimal for $k=o(n)$

and constant d . Together these two algorithms can be transformed into a PTAS for the geometric maximum k -CFP in an arbitrary normed space of fixed dimension.

Aleksey Glebov. *A 5/6-approximation algorithm for the maximization version of the pseudo-metric TSP.*

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In 1985, Kostochka and Serdyukiv presented a polynomial algorithm with the guaranteed approximation ratio $5/6$ for the maximization version of the metric Travelling Salesman Problem (TSP). In this problem, one need to find a Hamiltonian cycle of the maximum weight in a complete weighted graph whose weights of edges satisfy the triangle inequality. We extend the result of Kostochka and Serdyukov to the case of an incomplete graph having large enough minimum degree. More precisely, we construct a $5/6$ -approximation algorithm with the cubic running-time for the maximization version of the TSP in an incomplete weighted graph G having N vertices and minimum degree at least $17N/18$, where the triangle inequality holds for every triangle contained in G . Based on this algorithm, we give a simplified description of the polynomial $5/6$ -approximation algorithm (previously developed by Glebov and Gordeeva) for the maximization version of the metric m -Peripatetic Salesman Problem (m -PSP) which consists in finding m edge-disjoint Hamiltonian cycles of the maximum total weight in a complete weighted graph satisfying the triangle inequality.

Natalia Grigoreva. *Single Machine Scheduling with Precedence Constraints, Release and Delivery times.*

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The goal of this paper is to propose algorithms for scheduling problem, where set of jobs performed on a single processor. Each job has a release time, when it becomes available for processing, a processing time and a delivery time. We study the case in which there exist precedence constraints among jobs and preemption is not allowed. The objective is to minimize the time, by which all jobs are delivered. The single machine scheduling problem is one of the classic NP-hard optimization problems, and it is useful in solving flowshop and jobshop scheduling problems. We develop branch and bound algorithm, which can find an optimal solution for the single processor scheduling problem. In this paper, we propose an approximate algorithm to find an upper bound. We solve the preemptive version of the problem to provide a lower bound and use a binary branching rule, where at each branch node, a complete schedule is generated.

To illustrate the effectiveness of our algorithms we tested them on randomly generated set of jobs.

Victor Il'ev^{1,2}, Svetlana Il'eva¹, Alexander Morshinin². *A 2-approximation algorithm for the graph 2-clustering problem.*

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We study a version of the graph clustering problem equivalent to the well-known 2-correlation clustering. In this version, for a given undirected graph, one has to find a nearest 2-cluster graph, i.e., the graph on the same vertex set with exactly 2 non-empty connected components each of which is a complete graph. The distance between two graphs is the number of noncoinciding edges. The problem under consideration is NP-hard. In 2004, Bansal, Blum, and Chawla presented a simple polynomial time 3-approximation algorithm for the similar correlation clustering problem in which the number of clusters doesn't exceed 2. In 2008, Coleman, Saunderson, and Wirth presented a 2-approximation algorithm for this problem applying local search to every feasible solution obtained by the 3-approximation algorithm presented by Bansal, Blum, and Chawla. Unfortunately, the method of proving the performance guarantee of the Coleman-Saunderson-Wirth's algorithm is not suitable for the graph 2-clustering. Coleman, Saunderson, and Wirth used so-called "switching technique" to reduce clustering of any graph to the equivalent problem whose optimal solution is the complete graph, i.e., the cluster graph consisting of the single cluster. In the graph 2-clustering problem any optimal solution has to consist of exactly 2 clusters, so we need another approximation algorithm and other method of proving a bound on its worst-case behaviour. We present a modified 2-approximation algorithm for the graph 2-clustering. In contrast to the proof of Coleman, Saunderson, and Wirth, our proof of the performance guarantee of this algorithm doesn't use switchings.

Alexander Kelmanov, Vladimir Khandeev. *The problem K -means and given J -centers: polynomial solvability in one dimension.*

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We consider the strongly NP-hard problem of partitioning a finite set of N points in Euclidean space into $K + J$ clusters so as to minimize the sum over all clusters of the intracluster sums of the squared distances between clusters elements and their centers. The centers of J clusters are given as an input, while the other K centers are defined as centroids (geometrical centers). We

present an exact polynomial-time algorithm for the one-dimensional case of this problem. Our algorithm has $O(N^2 K J)$ running time.

Vladimir Khandeev. *Approximation Scheme for a Problem of Searching for the Largest Subset.*

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The paper is addressed to one strongly NP-hard problem of searching for the largest subset in the finite set of points in Euclidean space. We present an approximation scheme for this problem.

Michael Khachay, Katherine Neznakhina. *Generalizations of the asymmetric TSP and CVRP: does the triangle inequality implies constant-ratio polynomial time approximation?*

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The Traveling Salesman Problem (TSP) and Vehicle Routing Problem (VRP) are well-known combinatorial optimization problems having numerous valuable practical applications in operations research. Both problems are strongly NP-hard and remain intractable even in the Euclidean plane. Although, these problems are hardly approximable in general case, they admit polynomial time approximation within fixed ratios for metric settings and even polynomial time approximation schemes for the Euclidean spaces of fixed dimension. In many cases, for symmetric versions of the problems, these algorithms relies on the celebrated Christofides-Serdyukov 3/2-approximation algorithm for the metric TSP. The similar result obtained in 2018 by O.Svensson et al. for the ATSP with triangle inequality opens new opportunities of algorithmic design for asymmetric versions of CVRP, Generalized ATSP and Cycle Cover problems. In this paper, we present our first results on a way of extending this seminal result to generalizations of asymmetric TSP and VRP.

Michael Khachay, Yuri Ogorodnikov. *Approximation scheme for the Capacitated Vehicle Routing Problem with Time Windows and non-uniform demand.*

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The Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) is the well-known combinatorial optimization problem having numerous valuable applications in operations research. Unlike the classic CVRP (without time

windows constraints), approximability of the CVRPTW (even in the Euclidean plane) in the class of algorithms with theoretical guarantees is much less studied. In this paper, perhaps for the first time, we propose an approximation scheme for the planar CVRPTW with non-uniform splittable demand combining the well-known instance decomposition framework by A. Adamaszek et al. and QPTAS by L Song et al. Actually, the scheme proposed finds a $(1 + \varepsilon)$ -approximate solution for the problem in polynomial time provided the capacity q and the number p of time windows does not exceed $2^{\log^\delta n}$ for some $\delta = O(\varepsilon)$. For any fixed p and q the scheme is Efficient Polynomial Time Approximation Scheme with subquadratic time complexity.

Konstantin Kobylkin^{1,2}, Irina Dryakhlova². *Approximation algorithms for piercing special families of hippodromes: an extended abstract.*

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Polynomial time approximation algorithms are proposed with constant approximation factors for a problem of computing the smallest cardinality set of identical disks whose union intersects each segment from a given set E of n straight line segments on the plane. This problem has important applications in operations research, namely in wireless and road network analysis. It is equivalent to finding the least cardinality piercing (or hitting) set for the corresponding family of n Euclidean r -neighbourhoods of straight line segments of E on the plane, which are called r -hippodromes in the literature. When the number of distinct orientations is upper bounded by k of segments from E , a simple $O(n \log n)$ -time $4k$ -approximate algorithm is known for this problem. Besides, when E contains arbitrary straight line segments, overlapping at most at their endpoints, $O(n^4 \log n)$ -time 100-approximate algorithm is given recently. In the present paper simple approximation algorithms are proposed with small approximation factors for E being edge set of some special plane graphs of interest in road network applications; here the number of distinct orientations of straight line segments from E can be arbitrarily large. More precisely, $O(n^2)$ -time approximation algorithms are constructed for edge sets of either Gabriel or relative neighbourhood graphs or of Euclidean minimum spanning trees with factors of 14, 12 and 10 respectively. These algorithms are much faster, more accurate and conceptually much simpler than the aforementioned 100-approximate algorithm for the general case of the problem on edge sets of arbitrary plane graphs.

Yurii Mezentsev, Pavel Pavlov. *Approximate Efficient Algorithms for Solving One Class Mixed Integer Programming Problems Using Semidefinite Relaxation.*

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A universal mathematical economic model is discussed, which is designed to find optimal strategies for controlling the logistics subsystems (subsystem components) of a company. The declared universal character of the model allows a systematic consideration of both production components, including constraints associated with how raw materials and components are converted into goods for sale, and resource-based and logical constraints on input-output material flows. The model and the generated control problems are developed within a single approach allowing the implementation of logical conditions of any complexity and the formulation of the corresponding formal optimization problems. An explanation is provided for the meaning behind the criteria and constraints. An approximate efficient algorithm is proposed for solving the formulated mixed programming optimization problems of actual dimension. The results are presented of testing the algorithm for problem instances over a wide range of dimensions.

Dmitry Mokeev. *Effective algorithms for the k-path packing and k-path vertex cover problems on graphs of some classes.*

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A k-path packing problem is to find the maximum set of pairwise vertex-disjoint paths each on k vertices in a graph. A k-path vertex cover problem is to find the minimum set of vertices, such that each path of k vertices consists at least one of them. A graph is Konig for a k-path if every its induced subgraph has the solutions of the k-path packing and k-path vertex cover problems of equal cardinalities. We give polynomial algorithms for solving the both problems on Konig graphs for a k-path. Several cases of k is considered.

Anna Panasenکو. *A PTAS for One Cardinality-Weighted 2-Clustering Problem.*

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We consider one strongly NP-hard problem of clustering a finite set of points in Euclidean space. In this problem, we need to partition a finite set of points into two clusters minimizing the sum over both clusters of the weighted intracluster sums. Each of these sums is the sum of squared distances between

the elements of the cluster and their center. The center of the one cluster is unknown and determined as the centroid, while the center of other one is fixed at the origin. The weight factors for both intracluster sums are the given sizes of the clusters. In this paper, we present an approximation algorithm for the problem and prove that it is a polynomial-time approximation scheme (PTAS).

Anatoly V. Panyukov, Yuliya F. Leonova. *Cycle Merging Algorithm for MAX TSP Problems.*

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The paper proposes an algorithm for the approximate solution of MAXTSP consisting of search a 2-factor of extreme weight in a given graph, and then applying the operation of optimally combining the cycles into one Hamiltonian cycle. The computational complexity of the algorithm does not exceed $O(|V|^3)$. Accuracy of the algorithm not less $5/6$. The software implementation of the algorithm is described and the results of a computational experiment are presented. The study of the joining cycles algorithm allowed us to put forward the hypothesis that algorithm is asymptotic accuracy on a class of problems with a uniform distribution of edge weights.

Vladimir Shenmaier. *A Structural Theorem for Center-Based Clustering in High Dimensional Euclidean Space.*

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A general framework for center-based clustering in high-dimensional Euclidean space is given. We prove that, for any finite set X in this space and any fixed $\varepsilon > 0$, there exists a polynomial-cardinality set of points which can be constructed in polynomial time and which contains a $(1 + \varepsilon)$ -approximation of every point of space in terms of the distances to all the elements of X . This allows to get approximation schemes PTAS for a lot of clustering problems reducible to finding centers of optimal clusters. One of such problems is, given a finite set of points and a positive integer l , find an l -element subset of this set and select a fixed number of points in space (centers) to minimize an arbitrary objective function depending on the Euclidean distances from the input points to the selected centers and satisfying some natural continuity-type property.

Gennady Zabudsky¹, Natalia Veremchuk². *On the One-Dimensional Space Allocation Problem with Partial Order and Forbidden Zones.*

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In this paper we consider a generalization of the well-known optimization problem, namely the One-Dimensional Space Allocation Problem (ODSAP). The classical formulation of the problem is as follows. It is necessary to place rectangular connected objects (linear segments) on the line with the minimum total cost of connections between them. The structure of connections between objects is defined using a graph. The generalization of the problem is that there are fixed objects (forbidden zones) on the line. The centers of the placed objects are connected among themselves and with the centers of the zones. In addition, between the objects a partial order of their placement on the line is given. Then the structure of connections between objects and the partial order will be determined by the oriented acyclic graph. A similar situation arises, for example, when designing the location of technological equipment of petrochemical enterprise. Technological scheme of production specifies the order of processing of raw materials. It is necessary to place units equipment so that the total cost of the pipeline ties was minimal and the sequence of processing of raw materials was carried out.

A review of the formulations and methods for solving the classical ODSAP is given. We propose a polynomial algorithm for finding a local optimum for the case when the graph of connections between objects is a composition of rooted trees and series-parallel graphs.

7. Heuristics and Metaheuristics

Angie Lizeth Blanco Cañon¹, Lindsay Alvarez Pomar¹, Heriberto Garcia Reyes². *Hybrid Algorithm for Open-Shop Scheduling Problem.*

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The open shop scheduling problem (OSSP) is one of the most complex combinatorial optimization problems. Due to its complexity is considered NP-Hard, this means that the solution time increases quickly when the size of the instances grows. Therefore, the development of methods to reduce the solution time is very relevant, especially in the current competitive environment. This work proposes a hybrid algorithm which use a genetic algorithm with an operator called two-edge recombination and simulated annealing among other solution approaches. The algorithm was tested considering the total processing time (makespan) as performance measure. Eight scenarios were defined combining different ways to build the initial population and executing several selection and replacement mechanisms for comparison purposes. Results were compared with those obtained from proved algorithms. The comparison shows smaller solution time using the proposed algorithm in considerable scenarios. The analysis of the scenarios allows identifying the best combination of parameters and search methods for the use of the proposed algorithm, as well as the next steps to generalize the conclusion obtained with more than fifty instances.

Vladislav Chulkov. *Hybrid local search heuristic for a scheduling problem in flexible manufacturing systems.*

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This paper addresses a new production schedule problem in flexible manufacturing systems. We have some identical machines or production lines. A finite set of jobs must be produced on these machines. For each job, we know the processing time and sequence depending setup times. Interruptions of jobs are allowed. All jobs must be produced on a given time horizon. The goal is to find a scheduling for these machines with minimal total setup times (total delay). We present two mixed integer linear programming models which can be used to find optimal solutions for small test instances. To obtain near optimal solutions for large scale instances, we design a local search approach with a sequence of jobs for coding solutions. Greedy heuristics are used for generating a starting solution. Dynamic programming is applied as a decoding procedure. Computational results for real test instances originated from the food industry are discussed.

Alexander Chentsov, Alexey Grigoryev, Alexey Chentsov. *Procedures of local optimization in routing problems with constraints.*

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The problems of sequential bypass of megalopolises with constraints and cost functions admitting the task list dependence are considered. It is supposed that the initial problem has sufficiently large dimension. Therefore, construction of precise solution of the arising routing problem is practically impossible. It is supposed that we have some admissible solution in the form of pair route – trajectory (we keep in mind the admissibility with respect to full system of constraints). We consider the question about local improvement of this admissible solution by optimizing insertions and multi-insertions. We use constructions for which the employment of dynamic programming (DP) under insertions construction is realized. The constructions using individual insertions can be combined with iterated procedures for which localization of realized insertion may vary.

Another way of insertion employment is connected with realization of multi-insertion. Namely, we create a system of individual insertions. In every individual insertion, the local DP procedure is used. In addition, localizations of individual insertions are separated; we assume cross connections between individual insertions. This stipulation permits to use parallel algorithms for performing calculations. Namely, every individual insertion is realized by one processor.

Given approach was used in problem connected with optimization of the dose loading of workers under realization of mantling of radiating elements. The opposite approach is connected with construction of iterated procedures using only individual insertions.

Dimitrije D. Čvokić¹, Yury Kochetov², Alexander Plyasunov², Aleksandar Savić³. *The competitive hub location under the price war.*

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Two transportation companies want to enter the market and they are aware of each other. The objective for both of them is to maximize their respective profits by finding the best hub and spoke networks and price structures, each for itself. One company wants to establish r hubs and the other wants to establish t hubs. It is assumed that the customers choose the route by price and the logit model is used to estimate how the demand is shared. After setting their networks, the competing companies engage in the price war. We propose

a new model for finding a Stackelberg strategy, that includes a price game, as a bi-level nonlinear mixed-integer program called the (r|t) hub-centroid problem under the price war. It is shown that there is a unique finite Bertrand-Nash price equilibrium. On the basis of this result, the we show the solution existence, propose a new equations for the best response pricing, and address the computational complexity of the problem. Finally, we discuss some possible future research directions that concern the solution approach and some other competitive scenarios which can include prices.

Ivan Davydov, Petr Gusev. *VNS based heuristic for the (r/p)-centroid problem under l_1 metric.*

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Topics: heuristics and metaheuristics

In the (r|p)-centroid problem, two players, called the Leader and the Follower, open facilities to service customers. We assume that customers are identified with their location on the plane, and facilities can be opened anywhere in the plane. The Leader opens p facilities. Later on, the Follower opens r facilities. Each customer patronizes the closest facility. The distances are calculated according to l_1 -metric. The goal is to find the location of the Leader's facilities maximizing her market share. We provide the results on the computational complexity of this problem and develop a local search heuristic, based on the VNS framework. Computational experiments on the randomly generated test instances show that the proposed approach performs well.

Damir Gainanov^{1,2}, Nenad Mladenovic^{1,3}, Varvara Rasskazova².

Simplicial Vertex Heuristic in Solving the Railway Arrival and Departure Paths Assignment Problem.

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This paper considers a fast solving the practical problem in railway planning and scheduling, i.e., the problem of assigning given arrival and departure railway paths to routs. This problem is to execute as fully as possible the train traffic across the railway station, using a fixed amount of the resources. It appears that the problem may be solved by using any efficient maximum Independent set algorithm, which is known to be NP-hard. On the other hand, Simplicial vertex test is known heuristic that gives good quality solutions on sparse graphs.

So, for solving the maximum independent set on sparse graphs, we propose an efficient heuristic based on the extended simplicial vertex test.

Evgenii Goncharov. *Variable Neighborhood Search for the Resource Constrained Project Scheduling Problem.*

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We consider the resource-constrained project scheduling problem (RCPSP) with respect to the makespan minimization criterion. The problem accounts for technological constraints of activities precedence together with resource constraints. Activities preemptions are not allowed. The problem with renewable resources is NP-hard in the strong sense. We propose a variable neighborhood search algorithm with two versions of neighborhoods. Numerical experiments based on standard RCPSP test dataset J120 from the PCPLIB library demonstrated that the proposed algorithm produces better results than existing algorithms in the literature for large-sized instances. For some instances from the dataset j120 the best known heuristic solutions were improved.

Egor Grishin, Alexander Lazarev, Semen Galakhov, Elena Musatova, German Tarasov. *Algorithms of the organization of locomotive's maintenance.*

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The algorithms of locomotive's maintenance scheduling in the depot are described. Algorithms of constraint programming, dynamic programming and heuristic algorithm are presented. The presented statement of the problem corresponds to the work of the Eastern polygon of Russian Railways. Numerical experiments on real data for real depot configurations were carried out to verify the efficiency. The presented algorithms are compared.

Lev Kazakovtsev, Ivan Rozhnov. *Application of algorithms with variable greedy heuristics for k-medoids problems.*

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Progress in location theory methods and clustering algorithms is mainly targeted at improving the performance of the algorithms. The most popular clustering models are based on solving the p-median and similar location problems (k-means, k-medoids). In such problems, the algorithm must find several points called cluster centers, centroids, medoids, depending on the specific

problem which minimize some function of distances from known objects to the centers. In the the k-medoids problem, the centers (medoids) of the cluster must coincide with one of the clustered objects. The problem is NP-hard, and the efforts of researchers are focused on the development of compromise heuristic algorithms that provide a fairly quick solution with minimal error. In this paper, we propose new algorithms of the Greedy Heuristic Method which use the idea of the Variable Neighborhood Search (VNS) algorithms for solving the k-medoids problem (which is also called the discrete p-median problem). In addition to the known PAM (Partition Around Medoids) algorithm, neighborhoods of a known solution are formed by applying greedy agglomerative heuristic procedures.

According to the results of computational experiments, the new search algorithms (Greedy PAM-VNS) give more accurate and stable results (lower average value of the objective function and its standard deviation, smaller spread) in comparison with known algorithms on various data sets.

Sergey Khapugin, Andrey Melnikov. *Local Search Approach for the Medianoid Problem with Multi-purpose Shopping Trips.*

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We consider a modification to the classic medianoid problem, where facilities of different types are present on the market. A newcomer firm opens facilities providing a specific type of products and competes with existing facilities of that type. Each customer requires multiple products of different types and chooses the shortest route visiting facilities providing the needed types of products. A local search approach to maximize the market share of the newcomer firm is proposed, utilizing upper and lower bounds for the customers' trip lengths to avoid time-consuming computations.

Yuri Kochetov¹, Natalia Shamray². *Genetic Algorithm for Optimizing of Ambulance Fleet Allocation.*

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Prompt delivery of emergency medical service (EMS) is an important part of the health care system. In this paper, we consider the problem of the best allocation of ambulance crews between base stations in a city for each time period of a day. The intensity of service requests and the congestion of the transportation network vary a lot during the day. These factors have a strong impact on the efficiency of the delivery of EMS and are taken into account in

the optimization process. Our objective is the minimization of the average time of ambulance crew arrived at the clients. We propose a genetic algorithm for optimizing of ambulance fleet allocation. The simulation model is constructed to provide input data for the algorithm: the queue of requests and the service time. The nearest crew is assigned for the first request in the queue taking into account the current congestion of the transportation network. Based on real EMS statistics of Vladivostok and the travel time between city zones, we have implemented the simulation model and have conducted numerical experiments. Computational results of the experiments and comparison with the real allocation of ambulance crews between base stations are reported.

Polina Kononova, Igor Kulachenko. *The VNS Approach for a Consistent Capacitated Vehicle Routing Problem under the Shift Length Constraints.*

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We consider a new real-world application of vehicle routing planning in a finite time horizon. A company has a fleet of identical vehicles with limited capacity in some depots and must service a set of clients. There is a frequency for each client stating how often this client must be visited. Time intervals between two consecutive visits must be the same but the visiting schedule is flexible. To get some competitive advantage, a company tries to increase the service quality. To this end, each client should be visited by one driver only. The goal is to minimize the total number of vehicles over the planning horizon under the frequency constraints and driver shift length constraints. We present an integer linear programming model for this new consistent capacitated vehicle routing problem. To find near optimal solutions, we design the Variable Neighborhood Search metaheuristic with nine neighborhood structures. The driver shift length constraints are penalized and included into the objective function. Empirical results for real test instances from Orenburg region in Russia with up to 900 clients and four weeks in the planning horizon are discussed.

Yulia Kovalenko¹, Aleksey Zakharov². *Pareto-based Hybrid Algorithms for the Bicriteria Asymmetric Travelling Salesman Problem.*

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We consider the bicriteria asymmetric travelling salesman problem (bi-ATSP): Given a complete directed graph where each arc is associated to a couple of positive weights, the aim is to find the Pareto set, consisting of all non-dominated Hamiltonian circuits. We propose new hybrid algorithms for

the bi-ATSP using the adjacency-based representation of solutions and the operators that use the Pareto relation. Our algorithms are based on local search and evolutionary methods. The local search combines principles of the well-known Pareto Local Search procedures and Variable Neighborhood Search approach, realizing the search in width and depth. A genetic algorithm with NSGA-II scheme is applied to improve and extend a set of Pareto local optima by means of evolutionary processes. The experimental evaluation shows, that the algorithms yield competitive results on various structures of the bi-ATSP instances generated randomly and constructed from benchmark asymmetric instances with single objective. Performance of the algorithms is estimated by hyper volume, two set coverage metric, spread measure, and generational distance.

Anna Kozlova, Andrei Nikolaev. *Simulated annealing approach to verify vertex adjacencies in the traveling salesperson polytope.*

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We consider 1-skeletons of the symmetric and asymmetric traveling salesperson polytopes whose vertices are all possible Hamiltonian tours in the complete directed or undirected graph, and the edges are geometric edges or one-dimensional faces of the polytope. It is known that the question whether two vertices of the symmetric or asymmetric traveling salesperson polytopes are nonadjacent is NP-complete. A sufficient condition for nonadjacency can be formulated as a combinatorial problem: if from the edges of two Hamiltonian tours we can construct two complementary Hamiltonian tours, then the corresponding vertices of the traveling salesperson polytope are not adjacent. We consider a heuristic simulated annealing approach to solve this problem based on finding a vertex-disjoint cycle cover and a perfect matching. The algorithm has a one-sided error: the answer “not adjacent” is always correct, and was tested on random and pyramidal Hamiltonian tours.

Luka Matijević¹, Tatjana Davidović¹, Vladimir Ilin², Panos M. Pardalos³. *Matheuristic Approach to Asymmetric Vehicle Routing Problem with Time Window.*

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The problem of delivering online ordered and possibly perishable goods from a single depot (warehouse) to the multiple customers can be modeled as

an asymmetric vehicle routing problem. The problem consists of visiting and serving all customers within the prespecified time window and using a limited number of homogeneous vehicles. The objective function to be minimized is the total distance traveled with an assumption that the distance matrix is asymmetric (as it is usual case in real life applications). Starting with a mixed-integer linear programming (MILP) formulation of the problem, we apply three matheuristic methods, namely, Variable Neighborhood Branching (VNB), Variable Neighborhood Decomposition Search for 0-1 MIP (VNDS-MIP) and Variable Intensity Neighborhood Search (VINS). The methods are compared on real life instances from a hyper market company in Serbia.

Alexander Petunin^{1,2}, Efim Polishchuk¹, Stanislav Ukolov¹. *A novel algorithm for construction of the shortest path between a finite set of non-intersecting contours on the plane.*

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The optimization problem of the shortest path between a set of non-intersecting contours on a plane is considered. These types of problems arise, in particular, at the tool path design for the CNC sheet cutting machines. It is shown that this problem is equivalent to a subproblem with a set of contours that do not contain internal contours. A heuristic algorithm for solving the problem based on the combinatorial approach is proposed. The results of the computational experiment are described. A lower bound for the optimal value of the objective function is proposed.

Roman Plotnikov, Adil Erzin. *Constructive Heuristics for Min-Power Bounded-Hops Symmetric Connectivity Problem.*

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We consider a Min-Power Bounded-Hops Symmetric Connectivity problem that consists of the construction of communication spanning tree on a given graph, where the total energy consumption spent for the data transmission is minimized and the maximum number of hops between two nodes is bounded by some predefined constant. We focus on the planar Euclidian case of this problem where the nodes are placed at the random uniformly spread points on a square and the power cost necessary for the communication between two network elements is proportional to the squared distance between them. Since this is an NP-hard problem, we propose different polynomial heuristic algorithms for the approximation solution to this problem. We perform a posteriori comparative

analysis of the proposed algorithms and present the obtained results in this paper.

Ivan Rozhnov, Victor Orlov, Lev Kazakovtsev. *Variable neighborhood search algorithms for the k-means clustering problem.*

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The k-means algorithm with the corresponding problem formulation is one of the first methods that researchers use when solving a new automatic grouping (clustering) problem. In this research, we propose new algorithms of the Greedy Heuristic Method, which use an idea of searching in variable neighborhoods for solving the classical cluster analysis problem k-means, which allows to obtain a more accurate and stable result of solving in comparison with the known algorithms including known algorithms of the same method. The Variable Neighborhood Search (VNS) algorithms work in a pre-defined set of the neighborhoods of some known solution, and try to find a better solution in one of these neighborhoods. If the search is unsuccessful, the algorithm switches to the next neighborhood. The Greedy Heuristic Method offers various greedy heuristic procedures which can be used as the crossing-over operators in the genetic algorithms. In this research, we propose new greedy procedures and use them for forming the specific neighborhoods of the known solution. Our computational experiments show that the new algorithms allows us to obtain results with better values of the objective function (sum of squared distances) in comparison with classical algorithms such as k-means, j-means and genetic algorithms on various practically important datasets. In addition, we present the first results for the GPU realization of the Greedy Heuristic Method.

Hanan Shabana^{1,2}, Mikhail Volkov¹. *Using Sat solvers for synchronization issues in partial deterministic automata.*

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We approach the task of computing a carefully synchronizing word of minimum length for a given partial deterministic automaton, encoding the problem as an instance of SAT and invoking a SAT solver. Our experimental results demonstrate that this approach gives satisfactory results for automata with up to 100 states even if very modest computational resources are used.

Konstantin Speshilov¹, Valeriy Khabarov². *Heuristic approaches to create a model of the behavior of tutoring agents in simulator complexes.*

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Simulator complexes are used in many areas of professional activity related to the use of unique skills. Implementation of an artificial tutoring agent in a simulator complex improves the efficiency and speed of a learning process. One of the main tasks arising in the developing of a tutoring agent is the formation of its behavior model in the domain. The paper proposes an approach to the formation of this model based on heuristic behavior algorithms. The article contains a conceptual model of such an agent, an example for a tutoring agent of the simulator complex of a railroad hump yard is given.

Anastasia Tavaeva¹, Dmitry Kurennov², Vladimir Krotov³, Alexander Petunin². *A Cost Minimizing at Laser Cutting of Sheet Parts on CNC machines.*

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The problem of cost minimizing at laser cutting of sheet parts on CNC machines is considered. As a objective function, the cost function of the cutting process is used, depending on the number of frames in the NC program. To solve the optimization problem, it is proposed to construct the tool path of the CNC machine by using special cutting techniques, i.e: common edge, chain cutting, multi-contour and multi-segment cutting. The results of the computational experiment, which show a statistically significant improvement in the value of the objective function compared to using the standard cutting technique, are presented.

Aida Valeeva, Yuliya Goncharova, Ruslan Valeev. *On the problem of finding rational routes of delivery of a homogeneous cargo to different customers.* Russia, Ufa, State Aviation Technical University

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The actual direction of transport logistics is the development of effective methods for solving routing problems that are NP-hard. The most popular from a practical point of view is the task of drawing up rational routes for the delivery of cargo placed in containers to various customers, taking into account its loading into vehicles. At the same time, the following restrictions were fulfilled: the company engaged in the transportation of cargo has several

warehouses (depots) where the cargo is stored; the route starts and ends at the depot; in drawing up the route, the quality and cost of roads, the type of roads, and the speed limit on the roads are taken into account. In addition, there may be a cargo in the vehicle intended for several customers; set the time period during which the goods must be delivered to customers; during the route the vehicle is allowed to stop at certain intervals; the demand of each client may exceed the carrying capacity of the vehicle, and the requirements of the client may be taken into account in other routes; every customer has the opportunity to return the goods. In the process of compiling delivery routes, the results of rational distribution of cargo within the vehicle are taken into account. To solve the problem under consideration, a method based on an ant colony algorithm based on population has been developed, using algorithms to build a road graph, distribute customers to the vehicle, and a probabilistic strategy of updating the population to save the best solution. The proposed method was used by Bitel LLC (Ufa) for the delivery of equipment to various regions of the Republic of Belarus and the Russian Federation. The work was supported by RFBR grant No. 13-07-00579.

Oleg Zaikin, Stepan Kochemazov. *Black-box optimization in an extended search space for SAT solving.*

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The Divide-and-Conquer approach is often used to solve hard instances of the Boolean satisfiability problem (SAT). In particular, it implies splitting an original SAT instance into a series of simpler subinstances. If this split is performed by choosing a small subset of variables and varying all possible assignments of their values, then it is possible to use a stochastic pseudo-Boolean black-box function to estimate the time required for solving an original SAT instance with the chosen decomposition. This function's value is computed via the Monte-Carlo method by launching a SAT solver over a randomly chosen subinstances forming a random sample. One can use black-box optimization methods to minimize the function over the space of all possible subsets of variables. In the present study, we make use of two peculiar features of the Boolean satisfiability problem to improve this general approach. The first feature consists in the fact that there are many ways to reduce an original problem to SAT resulting in different propositional formulas with clear interconnection between their Boolean variables. Another feature is that since the majority of state-of-the-art SAT solver algorithms are heuristic in nature they have a number of parameters available for tuning. Thus it is actually possible to extend the search space over which the function is minimized by adding solver parameters and SAT encoding parameters into it. In computational experiments, the SMAC algorithm was used for black-box minimization. In the role of

hard SAT instances, we used the cryptanalysis instances for several keystream generators.

Alexander Zyryanov, Yury Kochetov, Sergey Lavlinskii. *A randomized matheuristic for the bilevel public-private partnership problem.*

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We present a new bi-level linear integer programming model for the strategic planning of the public-private partnership. In this model, we need to select and schedule some ecological, infrastructure, and production projects within a finite planning horizon. Two players, the leader (government) and the follower (private companies) make own decisions sequentially under the budget constraints. The players try to maximize own total profits. We claim that this bi-level problem is $\sum_2^p$ -hard in cooperative and non-cooperative cases. To find near-optimal solution, we design a stochastic tabu search metaheuristic according to the upper level variables. The optimal solution for the lower level is obtained by CPLEX solver. To reduce the running time, we use randomized flip and move neighborhoods. To evaluate the neighboring solutions, we solve the lower level problem approximately. Computational results for real world instances for the Transbaikalian polymetallic fields and comparison with a matheuristic based on the high point relaxation are discussed.

8 . Scheduling

Ilya Chernykh¹, Ekaterina Lgotina². *How the difference in travel time affects the optima localization for the routing open shop.*

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The routing open shop problem, being a generalization of the metric TSP and the open shop scheduling problem, is known to be NP-hard even in case of two machines with a transportation network consisting of two nodes only. We consider a generalization of this problem with unrelated travel times of each machine. We determine a tight optima localization interval for the two-machine problem in the case when the transportation network consists of at most three nodes. As a byproduct of our research, we present a linear time $5/4$ -approximation algorithm for the same problem. We prove that the algorithm has the best theoretically possible approximation ratio with respect to the standard lower bound.

Adil Erzin, Roman Plotnikov. *The Convergecast Scheduling Problem on a Regular Triangular Grid.*

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The problem of conflict-free data aggregation in an arbitrary graph is NP-hard. On a square unit grid, in each node of which a sensor is located, the problem is polynomially solvable. For the case when the graph is a regular triangular grid, the upper bound on the length of the schedule of conflict-free data aggregation was previously known. In this paper, the refined estimates are given for the length of the schedule of conflict-free aggregation of data on a triangular grid, as well as polynomially solvable cases are found and algorithms for constructing optimal and approximate schedules are proposed.

Eugeny Goncharov¹, Dmitriy Mishin¹, Nina Plyaskina^{2,3}, Daria Orlova⁴. *On the Project Scheduling Problem with maximum profit for given target dates and limited resources.*

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We consider the Project Scheduling Problem with maximum profit for given target dates and limited resources. An approximation algorithm is presented.

Alexander Kononov¹, Julia Memar², Yakov Zinder². *Scheduling with limited storage - a polynomial-time algorithm and efficient heuristics.*

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The paper is concerned with the two-machine flow shop with a limited storage (buffer), where each job needs a certain storage space from the start till the end of its processing. The storage requirement varies from job to job. The goal is to minimise the time needed for the completion of all jobs. This scheduling problem is NP-hard in the strong sense even for very restricted cases such as the case with a given order of jobs processing on one of the machines. The paper contributes to the efforts of establishing the borderline between the NP-hard and polynomial-time solvable cases by proving that there exists a polynomial-time algorithm which constructs an optimal schedule if the duration of each operation does not exceed one fifth of the buffer capacity. The presented polynomial-time algorithm is used as a basis for a heuristic for the general case. This heuristic is complemented by a Lagrangian relaxation based heuristic and a bin-packing based constructive heuristic. The heuristics are tested by computational experiments.

Yulia Kovalenko, Alexander Kononov. *Approximation Algorithms for Speed Scaling Scheduling of Parallel Jobs.*

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We consider a problem of energy efficient scheduling rigid parallel jobs on speed scalable processors. Each job is specified by release date, deadline, processing volume and the number of processors, which are required for execution of the job. A processor may vary its speed dynamically. Running a job at a slower speed is more energy efficient, however it takes longer time and may affect the performance.

We provide the ideas of reducing an energy efficient scheduling to the makespan minimization one, and transforming an optimal schedule for single-processor jobs to an approximate schedule for rigid jobs. As a result, new polynomial time algorithms achieving constant factor approximation guarantees are developed for such interesting settings as agreeable jobs without migration and preemptive instances.

Ketevan Kutkhashvili. *On a probabilistic model of scheduling theory.*

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Based on a scheduling theory, an original mathematical model of scheduling problems has been constructed which can perform its sub-tasks with a single-stage multiprocessor system. It is assumed that the processors of this system are partially interchangeable, and the set of additional resources and the set of partial order are empty. The times of tasks receipting in the system are not fixed and considered as random values.

The following criteria are chosen for investigating in the model:

- a) the average system weight or the average system cost;
- b) the criterion of the minimum schedule.

The constructed mathematical model and the corresponding algorithm are of practical importance. They can be used in various sectors of the economy. In particular, it can be used to build an optimal schedule in the conditions of transporting a large amount of cargo when using various vehicles (ships, trains, containers), to plan work when building a building complex, to schedule the power system or consistent repair work in several large industries. The applications of this algorithm in computing are possible too.

Yurii Mezentsev¹, Igor Estraykh¹, Nikita Chubko². *Practical aspects of implementation of an efficient parametric algorithm for optimal scheduling on unrelated parallel machines with release dates.*

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An original statement and solution algorithms are presented for one of the key problems in the scheduling theory. The problem of optimal scheduling for a parallel system consists in the generation or control of schedules to minimize the schedule length or the losses from schedule disruptions in the completion of jobs on machines. This problem is NP-hard and cannot be solved exactly for any real-life number of dimensions. A series of modifications of the efficient parametric algorithm are proposed to find an approximate solution, which are an extension of a similar algorithm for optimal scheduling on unrelated parallel machines with release dates by the performance criterion (Cmax). Software implementations of the algorithm modifications have been tested on the data of a generating problem by the Cmax criterion; the corresponding statistics is provided.

Artem Pyatkin^{1,2}, Mikhail Golovachev¹. *Routing Open Shop with two nodes, unit processing times and equal number of jobs and machines.*

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In the Routing Open Shop problem n jobs are located in the nodes of an edge-weighted graph G and m machines must process all jobs in such a way that each machine processes only one job at a time and each job is processed by only one machine at a time. The goal is to minimize the makespan, i. e. the time when the last machine comes back to the initial node called a depot. This problem is NP-hard even when the graph contains only two nodes. In this paper we consider the case of $G = K_2$ when all processing times and a travel time are unit. We pose the conjecture that the problem is polynomially solvable in this case, i. e. that the makespan depends only on the number of machines and the loads of the nodes and can be calculated in time $O(\log mn)$. We provide some bounds on the makespan for the case of $m = n$ depending on the loads distribution.

Anna Romanova. *Minimizing Resource Cost in Project Scheduling Problem with Accumulative Resources of Time-dependent Cost.*

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We consider a project with a set of jobs $J = \{1, \dots, n\}$. Precedence constraints are given by a directed acyclic graph $G = (J, E)$, where vertices correspond to jobs and an arc (i, j) belongs to E if and only if job i is a direct predecessor of job j . All jobs have to be completed before the due date T of the project. To be processed, the jobs require accumulative resources that are purchased. Let K denote the set of resources. While being processed, job $j \in J$ requires q_{kj} units of resource $k \in K$ during every period of its non-preemptable duration p_j . Let c_{kt} be the purchase cost of a resource unit k in the time period t . The goal is to find a feasible schedule and a planning of purchasing and using of resources so that the total resource cost is minimized. For unlimited storage, when each resource can be stored in any amount, we prove polynomial solvability of the problem. However limited storage is more interesting from the practical point of view. In this case, the surplus of a resource k , that is, the resource amount which may be spent later should not exceed the value V_k^{cont} , $k \in K$, in each period of time. In the case of unlimited storage we prove that the problem is NP-hard in strong sense. Dynamic programming algorithms and some heuristics are also developed.

Sergey Sevastyanov. *Some Positive News on the Proportionate Open Shop Problem.*

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The special case of the open shop problem in which every job has equal length operations on all machines is known as a proportionate open shop problem. The problem is NP-hard in the case of three machines, which makes topical such traditional research directions as designing efficient heuristics and searching for efficiently solvable cases. In this paper we found several new efficiently solvable cases (wider than known) and designed linear-time heuristics with good performance guarantees (better than those known from the literature). Besides, we computed the exact values of the power of preemption for the three-machine problem, being considered as a function of a parameter γ (the ratio of two standard lower bounds on the optimum: the machine load and the maximum job length). We also found out that the worst-case power of preemption for the m -machine problem asymptotically tends to 1, as m tends to infinitely. Finally, we established the exact complexity status of the three-machine problem by presenting a pseudo-polynomial algorithm for its solution.

9 . Optimal Control and Games

Alla Albu, Vladimir Zubov. *Application of the Fast Automatic Differentiation for Solving Inverse Coefficient Problems by Second-Order Methods.*

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The inverse problem of determining the thermal conductivity coefficient depending on temperature is considered and investigated. The consideration is based on the initial boundary value problem for the non-stationary heat equation. The inverse coefficient problem is reduced to the following variational problem: it is required to find such dependence of the thermal conductivity coefficient on the temperature, at which the temperature field and heat fluxes at the boundary of the object, obtained as a result of solving the direct problem, differ little from the data obtained experimentally. An algorithm for the numerical solution of the posed inverse problem for the one-dimensional and two-dimensional non-stationary heat equation was proposed by the authors in previous works. The present work is devoted to formulation the inverse problem in n-dimension case, to derivation the necessary conditions for appearance the nonuniqueness solutions of the inverse problem, and also to examine the possibility of applying the Fast Automatic Differentiation Technique to solve the optimal control problem by second-order methods. The examples of solving the inverse coefficient problem confirm the efficiency of the proposed algorithm.

Boris Ananyev. *Control Problem of Parabolic System with Incomplete Information.*

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We consider the controllable PDE system of parabolic type. The state of the system is unknown, but finite-dimensional signal is available. The time interval is divided by two subinterval. On the first one the observation process is provided. On the second one the minimax control problem is solved. Some examples are investigated.

Anatoly Antipin¹, Elena Khoroshilova². *A method of cross-sections of state constraints by convex programming problems.*

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A terminal control problem with linear dynamics on a fixed time interval and a moving right end of a state trajectory is considered. The right end implicitly

sets the terminal condition, which is defined as a solution of a boundary value problem of convex programming. The left end of the state trajectory is fixed. The set of controls is convex and closed. The main feature and difficulty of the problem under consideration is the presence of state constraints. State constraints are defined over the entire time interval. The talk addresses the development of evidence-based and reasonable methods for solving problems of terminal control in the presence of state constraints. The problem is investigated in the Hilbert function space. To obtain sufficient optimality conditions, the Lagrangian is used instead of the Hamiltonian. Slater's regularity condition is required for the existence of saddle points. But the interpretation of the Slater condition depends on the topology of the functional space and this creates additional difficulties. In the Hilbert function space, Slater's condition for state constraints is not satisfied. To overcome these difficulties, we introduce a discretization of state constraints. Cross-sectional state constraints generate polyhedra, on the basis of which convex (linear) programming problems are formed. Constraints (in sections) approximate a continuous "tube" of state constraints. The more sections, the more accurate is the approximation of a continuous tube. A saddle-point method of the extragradient type is proposed, in which continuous state and conjugate trajectories are recalculated (shifted) along the gradient only at the discretization points. The convergence of the proposed method is proved for all components of the optimal control problem. Namely, convergence in controls is weak, convergence in state, conjugate trajectories and in terminal variables is strong. The limiting state trajectory at the discretization points passes through all the constraints (sections) of finite-dimensional convex problems.

Eugene Barsky¹, Michael Barsky². *Entropic Optimization of Separation Processes.*

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An entropic method of unambiguous evaluation of the composition of complicated systems with any amount of components has been developed. At the separation of such systems into components in industrial conditions, mutual clogging of final products always takes place. Entropy criterion application allows us to obtain an unambiguous evaluation of the results of separation processes. On this basis, a methodology of optimization of the mentioned class of processes allowing to achieve a maximal efficiency has been worked out.

Aleksandr Buldaev, Ivan Burlakov. *Iterative Method with Exact Fulfillment of Constraints in Optimal Control Problems.*

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A new approach is proposed for constructing a relaxation sequence of admissible controls in the class of optimal control problems with constraints. The approach is based on the construction of a system of non-local conditions for improving the admissible control in the form of a fixed point problem of the control operator. To build the conditions for improving the admissible control, we apply the transition to an auxiliary optimization problem based on the well-known principle of extension. Sufficient conditions for the optimality of admissible control and the existence of a minimizing sequence of admissible controls in the considered class of problems with constraints are substantiated. A comparative analysis of the computational efficiency of the proposed iterative method of fixed points with the exact implementation of constraints in model and test optimal control problems is carried out.

Maxim Demenkov. *First-order linear programming algorithm with real-time applications.*

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We investigate first-order algorithm for linear programming based on the conversion of the problem into finding an intersection between a zonotope and a line (in case we have all problem variables constrained to a box). Zonotope is an affine transformation of multidimensional cube. If we know an interior point of the zonotope on the line, it is possible to derive a linearly convergent (in terms of projection steps) algorithm based on the bisection of an interval on the line. At each iteration we apply a projection onto a simple set (e.g. using Frank-Wolfe algorithm) to construct an oracle deciding if the point is inside or outside the zonotope. Due to the fact that the number of iterations can be computed in advance for the given accuracy, we investigate an application of the algorithm for dynamic optimization in automatic control.

Vasily Dikussar¹, Andrzej Yatsko². *Optimal movement of train with accounting profile of path.*

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Two optimal problems for movement of train with accounting profile of path are considered: minimum energetic inputs; minimum time problem. Maximum speed is constrained (state constraints). Two stage method of solution optimal control problems with state constraint is suggested. On the first stage we solve discrete problem using linear or nonlinear programming to result geometry of optimal path. Second stage is connected with verification of the discrete

solution on the base Dubovitski-Milyutin scheme. The methods of continuation the solutions, information processing, and parallel computations are used.

Yurii Dolgii¹, Alexander Sesekin¹, Oleg Tashlykov¹, Kien Trujng Tran². *Sequential optimal control of the nuclear fuel reload mechanism.*

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The BN-800 reactor overload system is designed to overload fuel assemblies and consists of a set of nodes that provide guidance for the reload mechanism at given coordinates, grabbing, lifting, lowering and rotating assemblies. On the throat of the BN-800 reactor there are three rotary plugs, the smaller of which is located inside the middle one and the middle one inside the large one. On a smaller tube placed the capture mechanism of the fuel assembly. The plugs, which are the reactor cover, perform the role of thermal and biological protection, as well as, guide the reload mechanism to the given coordinates of the core in order to capture the fuel assembly and move it to the required zone with the given coordinates. In this paper, we will consider the problem of minimal-time cast a gripper located on a smaller traffic jam to a given fuel assembly, assuming that the traffic jams will turn in series and at each moment only one traffic jam can be turned. The solution of such a task will contribute to the reduction of the stopping time of the power unit for carrying out operations on refueling. For this problem, a mathematical model was constructed that describes the movement of three connected plugs. Based on it, an algorithm for constructing optimal control is proposed under the assumptions made.

Alexander Dubanov. *Building models of the movement of objects in the pursuit problem. Solution in the system of computational mathematics MathCAD.*

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This article provides a description of the developed behavior models of objects in the persecution task, the objects is the pursuer and pursued. The idea of the research is to development an algorithm for autonomous robotic systems. In the proposed behavior models, local dynamic coordinate systems are introduced, which are formed by the direction of movement of objects. For a certain interval of time, the object must decide in which direction it should move depending on the result of the analysis of the coordinates of the second object. Due to the fact that an object cannot move instantly when moving in space, in our problems, "inertia" is modeled using the angular velocity of rotation.

According to the proposed models of the behavior of objects in the pursuit problem, programs are written in the computer math system MathCAD, which can be found on the website of the author. The results of the programs obtained animated images of the movement of objects, references to which are given in the text of the article.

Vladimir Dykhta, Stepan Sorokin. *Feedback minimum principle for optimal control problems in discrete-time systems and its applications.*

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The talk is devoted to a generalization of a necessary optimality condition in the form of the feedback minimum principle for a non-convex discrete-time free-endpoint control problem. The approach is based on an exact formula for the increment of the cost functional. This formula is completely defined through a solution of the adjoint system corresponding to a reference process. By minimizing that increment in control variable for a fixed adjoint state, we define a multivalued map, whose selections are feedback controls with the property of potential "improvement" of the reference process. As a result, we derive i) an iterative algorithm for discrete-time optimal control problems, and ii) a necessary optimality condition (optimal process does not admit feedback controls of a "potential descent" in the cost functional). In the case when the well-known discrete maximum principle holds, condition ii) can be further strengthened. Finally, we present sufficient optimality conditions and approximate feedback minimum principle for problems, where discrete maximum principle does not make sense.

Vladimir Fedorov¹, Dmitriy Gordievskikh². *An approximate controllability on infinite-dimensional degenerate fractional order systems in the sectorial case.*

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We consider a class of linear inhomogeneous equations in Banach spaces with a degenerate operator at the fractional Gerasimov - Caputo derivative. Such equations is called degenerate. We study the case of the existence of an analytical in a sector resolving operators family for the respective homogeneous equation. The existence of a unique solution of the Cauchy problem and of the Showalter - Sidorov problem to the inhomogeneous degenerate equation is proved. We derive the form of the solution also. The approximate controllability of infinite-dimensional control systems, described by the equations of the considered class, is researched. Initial state of the system is defined by the Showalter

- Sidorov conditions. We reduce the control system to two subsystems on mutually complement subspaces. One of subsystems is resolved with respect to the fractional derivative, another subsystem has an explicit solution. It is proved the equivalence of the approximate controllability of the original degenerate system and of every of two subsystems by the same control function. Based on this result, obtained criteria of the approximate controllability of the subsystems are used to get a criterion for the whole degenerate control system. The criterion is illustrated by the application to a system, which is described by an initial-boundary value problem for a partial differential equation, not solvable with respect to the time derivative.

Mikhail Gusev. *Estimates of the minimal eigenvalue of the controllability Gramian for a system containing a small parameter.*

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We consider a linear time-invariant control system with right-hand side depending on a small parameter. Assuming that the system is controllable, we study the asymptotics of the minimal eigenvalue of system's controllability Gramian and provide some lower bounds for the eigenvalue. These estimates are applied to the study of convexity properties of reachable sets for nonlinear control systems with integral constraints on control variables.

Igor' Izmet'sev, Viktor Ukhobotov. *On a single-type differential game with a non-convex terminal set.*

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We consider the problem of controlling a rod attached to a rotor. A rotating flywheel is attached to one end of the rod. The rotor is controlled by the first player. The flywheel is controlled by the second player. The goal of the first player is to bring the rotor to a vertical position at a given time. The goal of the second player is the opposite. This problem is an example of a more general linear differential game with an one-dimensional aim. Using a linear change of variables, this problem is reduced to a single-type one-dimensional differential game with a non-convex terminal set, for which we have found the necessary and sufficient conditions of termination and constructed the corresponding controls of the players.

Ivan Kamenev. *Application of the two-factor human model for modeling of the age dynamics of the human capital.*

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Our research shows that the age dynamics of the human mentality can be taken into account, assessing the human capital. Two factors (groups of factors) were formulated, determining the changes from the starting human capital formed in the education system: cognitive abilities and human awareness. Their influence is taken into account in the dynamic model, which is identified and researched by using the method of Metric data analysis (MMDA). The model separates (re)productive and creative human capitals, which play the different role in the economy and are the independent criteria of (two-factor) optimization.

It is shown that between these criteria established a complex relationship, the nature of which varies under the influence of age-related mortality rates. Thus, in conditions of high mortality the Pareto-frontier is formed, which degrades into the Slater-frontier for the moderate one, and into a single dominant solution for the low mortality. It indicates that countries with different life expectancy should use different priorities in building their education systems. In this context, the problems of optimization of human capital from the point of view of the government, which formulation are given in this publication, are of particular interest for the future research.

Alexander Karasev¹, Dmitriy Kuvshinov². *The connectedness of the set of admissible motions in the two-persons non-zero sum hierarchical differential positional game with linear dynamics.*

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In this article, the authors consider the differential game of two persons with continuous terminal quality functions. The formalization of the players' strategies and of the motions generated by them is based on the formalization of the positional differential game was developed by N.N. Krasovskii and a generalization for the non-zero-sum game proposed by A.F. Kleimenov. The dynamics of game be given by differential equations with linear right-hand side and the constraints on the player's controls are convex sets. Admissible movements in the Stackelberg hierarchical game with the first player as a leader are called the movements along which the second player's value function reaches a maximum at the ends of the trajectories. This set includes all solutions of the Stackelberg differential game and can be used to search for them in

algorithms. The conditions of connectedness of the set of admissible movements are researched.

Dmitry Khlopin. *General limit value for stationary Nash equilibrium.*

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We analyze the uniform asymptotics of the equilibrium value (as a function of initial state) in the case when its payoffs are averaged with respect to a density that depends on some scale parameter and this parameter tends to zero; for example, the Cesàro and Abel averages as payoffs for the uniform and the exponential densities, respectively. We also investigate the robustness of this asymptotics of the equilibrium value with respect to the choice of distribution when its scale parameter is small enough. We establish the class of densities such that the existence of the asymptotics of the equilibrium value for some density guarantees the same asymptotics for a piecewise-continuous density; in particular, this class includes the uniform, exponential, and rational densities. By reducing the general n -person dynamic games to mappings that assigns to each payoff its corresponding equilibrium value, we gain an ability to consider dynamic games in continuous and discrete time, both in deterministic and stochastic settings.

Dmitry Khlopin. *On necessary conditions and Clarke subdifferential in infinite-horizon control problem.*

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For infinite-horizon optimal control problems, we deal with necessary conditions of optimality in view of overtaking and weakly overtaking optimality as optimality criteria. In addition to the Pontryagin Maximum Principle, applying certain theorems on convergence of subdifferentials to the payoff functions, we obtain a necessary condition on the co-state arc—with some examples to enframe these. Also, under additional assumptions on the payoff function's asymptotic behavior, we obtain a complete system of relations, and this boundary condition points out the unique co-state arc through a Cauchy-type formula.

Elena Khoroshilova. *On application of Lagrange approach in a terminal control problem with intermediate state constraints.*

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The problem of terminal control with controlled dynamics, defined by a system of linear ordinary differential equations (ODE), is considered for a fixed time interval. The time interval is divided into a finite number of intervals, and then the problem of moving the trajectory from a point at the left end of the interval to a point at its right end is solved at each of these intervals. Moreover, these points are given implicitly as solutions of the boundary-value optimization problem in the presence of constraints on trajectories and controls (that is, a complex of convex programming problems is solved). The solutions of these problems determine the intermediate positions of the optimal trajectory. Saddle-point sufficient optimality conditions are formulated by primal and dual variables. An iterative method is proposed and its convergence to the solution of the problem is proved.

Yuri Kropotov, Valery Ermolaev. *Functional differential and differential-difference models of systems with acoustic feedback.*

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In this paper, mathematical models of telecommunication systems with acoustic feedback are investigated, stability issues are studied on the basis of the construction of functional differential and differential-difference models. The aim of the work is the construction, analysis and identification of linear and nonlinear functional differential models of echo, reverberation and acoustic feedback; sensitivity analysis of models, stability analysis of telecommunication systems with acoustic feedback. A mechanism for the formation of acoustic feedback is presented. When analyzing the propagation of sound in closed rooms using the method of geometric acoustics, the kernel equation was obtained, which is approximated by non-negative functions taking into account the reverberation model and the model of multiple reflections. The considered scheme of the formation of echo and reverberation is an element of the acoustic feedback model, characteristic of warning systems and technological communication. The model of acoustic systems with delayed feedback is investigated. The graph of the kernel of a functional differential equation describing a system with feedback is given. The models of systems with acoustic feedback presented in the paper provide an approximation to reality, while taking into account the possibilities of non-linear distortions in the sound propagation paths is considered as the next task.

Sergey V. Kruglikov. *Distributed Algorithm of Information Image Forming while Interaction.*

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The development of advanced technologies for cognitive robotics requires research into the mathematical foundations of a combination of autonomous, centralized, distributed and decentralized management processes, reflecting differences in the concepts of distributed artificial intelligence and artificial life. The development of effective algorithms for controlling the operation of intelligent robotic complexes in solving problems arising, in particular, during transport operations, monitoring and protection of objects and zones of special attention, is topical. A convenient means of formalizing meaningfully technical productions is a multi-agent approach that provides comparable opportunities for coordinated description of real and virtual agents, including intellectual ones. This feature allows us to consider the concept of a common information space of intellectual subsystems of a robotic complex based on methods of guaranteed control theory under uncertainty, as a mathematical model for tasks of interpretive navigation and autonomous management; coordination of actors' behavior. The report considers a discrete event algorithm of information image forming in the process of exchanging structured data and coordinating the information spaces of participants.

Evgeniy A. Krupennikov. *On estimates of the solutions of inverse problems of optimal control.*

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This talk is devoted to the problem of reconstruction of the normal control generating a realized trajectory of a dynamic control system by using known inaccurate measurements of this trajectory. A class of dynamic control systems with dynamics linear in controls and non-linear in state coordinates is considered. A new method, suggested in earlier publications, for solving such problems is discussed. This approach relies on necessary optimality conditions in an auxiliary variational problem on extremum of an integral discrepancy functional. The distinguishing feature of the method is using a functional which is convex in control variables and concave in state variables discrepancy. This form of the functional allows to obtain oscillating solutions. The estimates of the error of the discussed method are exposed and validated.

Ildus Kuchkarov, Ovanes Petrosian. *On a Class of Linear Quadratic Non-cooperative Differential Games with Continuous Updating.*

Russia, Saint-Petersburg State University

Most of real life conflicting processes evolve continuously in time, and their participants continuously receive updated information and adapt. For this kind of processes an approach was proposed that allows constructing more realistic models, namely, games with dynamic updating [1], [2]. Fundamental models considered in the theory of differential games previously are related to the problems defined on a fixed time interval (players have all the information on a closed time interval) [3], problems on an infinite time interval with discounting (players have information on an infinite time interval) [4], problems defined on a random time interval (players have information on a given time interval, but the terminating instant is a random variable) [5], also one of the first works in the theory of differential games are devoted to the pursuer and evader game (payoff of player depends on the time of catching opponents) [6]. In all the above models and suggested solutions, it is assumed that players at the beginning of the game know all the information about the dynamics of game (motion equations) and about the preferences of players (payoff functions). However, this approach does not take into account the fact that in many real life processes, players at the initial instant do not know all the information about the game. Thus, existing approaches cannot be directly used to construct a sufficiently large range of real life game-theoretic models. The aim of this work is to apply the approach to a special class of dynamic games, where the environment can be modeled by a set of linear differential equations and the objectives can be modeled as functions containing just affine quadratic terms. The popularity of these so-called linear quadratic differential games is caused on the one hand by practical considerations in engineering. To some extent these kinds of differential games are analytically and numerically solvable. On the other hand this linear quadratic problem setting naturally appears if the agents' objective is to minimize the effect of a small perturbation of their nonlinear optimally controlled environment. By solving a linear quadratic control problem, and using the optimal actions implied by this problem, players can avoid most of the additional cost incurred by this perturbation. The question of constructing Nash equilibrium for game models with continuous updating is studied. In the game models with continuous updating it is assumed that players 1. have information about the motion equations and payoff functions on the truncated time interval with length defined by the information horizon, 2. continuously receive updated information about the motion equations and payoff functions and as a result continuously adapt to the updated information. Obviously, it is difficult to obtain Nash equilibrium due to the lack of fundamental approaches for control problems with moving information horizon. Classical methods such as dynamic programming and Hamilton-Jacobi-Bellman equation do not allow to directly construct Nash equilibrium in problems with moving information horizon. Till now only the class of games with dynamic updating was studied

in the papers [1], [2], [7], [8], [9], [10], [11], where authors laid the foundation for further study in the class of games with dynamic updating. It is assumed that the information about motion equations and payoff functions is updated in discrete time instants, and the interval on which players know the information is defined by the value of information horizon. For the linear quadratic game models with continuous updating Nash equilibrium both in open-loop and in closed-loop form are constructed and it is proved that Nash equilibrium in the corresponding linear quadratic game with dynamic updating uniformly converges to the constructed controls. This approach allows to conclude that the constructed control indeed is optimal in the game model with continuous updating, i.e. in the case when the length of updating interval converges to zero. Similar procedure is performed for the optimal trajectory and corresponding payoffs.

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Konstantin Kudryavtsev¹, Vladislav Zhukovskiy². *Hybrid Equilibrium in N-persons Games.*

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In this article, for the game of N-persons in normal form the notion of hybrid equilibrium is introduced. This notion is synthesis of Nash and Berge equilibrium as well as Pareto-maximum. The properties of such equilibrium are discovered. The sufficient conditions whom satisfies the hybrid equilibrium are established and finally its existence under "usual" restrictions for mathematical game theory are established.

Oleg Kuzenkov, Elena Ryabova, Vladislav Ryabov. *Optimization of self-replicating systems.*

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In this work we propose a new formalization for self-replicating systems based on the existence indicator that we introduce, which is a dynamic measure or a distribution on the set of inherited elements. Such formalization is a generalization of the known equation with inheritance in the case of simple replication.

We compare selective advantages of various inherited elements based on the introduced principle. One element is better than the other, if it displaces the other from the system over time during the replication process. Thus, the order of preference is introduced on the set of different elements. The best element in the system of coexisting competing replicating elements is the one that is preserved in the system for unlimitedly long time, displacing other elements from it. The central fact here is the proved theorem, which states that the objects that have the maximum replication rate average during the selection process have the advantage over others.

As a result, an extremal principle is formulated, which makes it possible to determine and predict the evolutionarily stable behavior of a self-replicating system. In the course of time, only those elements that correspond to the maximum replication rate average coefficient remain in the self-replicating system. Therefore, it is possible to formulate optimization problems for self-replicating systems and solve it by using various methods.

The effectiveness of the presented approach is demonstrated by the example of forecasting evolutionarily stable daily vertical migrations of zooplankton.

Alexey Lamotkin. *Investigation of a problem of antagonistic wheel braking in the case of limited capabilities of the braking player.*

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The author presents the process of braking the wheel in the form of a zero-sum differential game. In this game, the first player, controlling the braking torque, tries to minimize the braking distance. The second player controls the coefficients of dry friction, while trying to maximize braking distance. It is assumed that the braking torque module is limited to small number, which reduces the capabilities of the first player.

Onesimo Hernandez Lerma. *Differential games with Pareto-optimal Nash equilibria.*

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Pareto optimality and Nash equilibrium are two standard solution concepts for cooperative and noncooperative games, respectively. At the outset, these concepts are incompatible. But, on the other hand, there are particular games (mainly, static games) in which Nash equilibria turn out to be Pareto optimal. In this talk we introduce several classes of deterministic and stochastic differential games with Pareto-optimal Nash equilibria. To identify these classes, our main tool are particular aspects of dynamic potential games. Interesting examples illustrate our results.

Nadezhda Maltugueva, Nikolay Pogodaev, Olga Samsonyuk.

Optimality conditions and numerical algorithms for hybrid systems.

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For an optimal control problem with intermediate state constraints, we construct an iterative descent algorithm and prove a related necessary optimality condition. Finally, we show how these results can be applied to measure-driven multiprocesses.

Mikhail Marchenko¹, Dmitry Smirnov¹, Merey Kenzhebayeva².

Two optimization methods for solving the inverse gravimetrical problem.

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Mineral exploration, in particular, increasing the accuracy of calculations up to now is one of the urgent problems of geophysics. It is necessary to restore the density of a given inhomogeneity after considering the results of measuring the potential and its gradient on the surface of the earth. One of the measured values, the potential or the gradient of the gravity field, is chosen beyond the boundary condition, and the second corresponds to the minimized functional, i.e. one inverse problem can be put in correspondence two different optimization problems. Both problems are solved on the basis of the same gradient method. Based on the numerical analysis, two methods for solving the inverse problem are compared.

Vladimir Mazalov¹, Elena Parilina². *Game of competition for opinion with two centers of influence.*

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The paper considers the model of opinion dynamics in the network having a star structure. An opinion about an event is distributed among network agents restricted by the network structure. The agent in the center of the star is influenced by all other agents with equal intensity. The agents located in non-center nodes are influenced only by the agent located in the center of the star. Additionally, it is assumed that there are two players who are not located in the considered network but they influence the agents' opinions with some intensities which are strategies of the players. The goal of any player is to make opinions of the network agents be closer to the initially given value as much as possible in a finite time interval. The game of competition for opinion is linear-quadratic and is solved using dynamic programming method. The Nash equilibrium in open-loop strategies is found. A numerical simulation demonstrates theoretical results.

Ovanes Petrosian, Anna Tur. *Hamilton-Jacobi-Bellman Equations for Non-cooperative Differential Games with Continuous Updating.*

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This paper formulates a new approach to analyze differential games with uncertainties and unknowns in the players' future payoff structures. The approach [1], [2] is used for constructing game theoretical models and defining solutions for conflict-controlled processes where information about the process updates continuously in time. Existing differential game models often rely on the assumption of time invariant game structures for the derivation of

equilibrium solutions. However, many events in the considerably far future are intrinsically unknown. Therefore, the behavior of players should as well be modelled using the assumption that they use only the truncated information about the game structure. It is supposed that players lack certain information about the motion equations and payoff function on the whole-time interval on which the game is played. At each time instant information about the game structure updates, players receive information about motion equations and payoff functions. This new approach for the analysis of differential games via information updating provides a more realistic and practical alternative to the study of differential games. Existing differential game models defined on the closed time interval [3], infinite time interval [4], random time interval [5] do not take into account the fact that in many real-life processes, players at the initial instant do not know all the information about the game. Thus, existing approaches cannot be directly used to construct a sufficiently large range of real-life game-theoretic models.

The aim of this work is to present an optimality conditions in the form of the Hamilton-Jacobi-Bellman equation for the feedback Nash equilibrium in the framework of continuous updating. In the game models with continuous updating it is assumed that players have information about the motion equations and payoff functions on the truncated time interval with length defined by the information horizon and they continuously receive updated information about the motion equations and payoff functions and as a result continuously adapt to the updated information. Obviously, it is difficult to obtain Nash equilibrium due to the lack of fundamental approaches for control problems with continuously moving information horizon. Classical methods such as dynamic programming and Hamilton-Jacobi-Bellman equation do not allow to directly construct Nash equilibrium in problems with moving information horizon. Taking into account the assumptions the two main problems arise:

1. How to define Nash equilibrium for a class of games with continuous updating?
2. How to derive the optimality conditions for the Nash equilibrium with continuous updating?

Both questions are addressed in this work. The game model with continuous updating and the solution technique besides the current time parameter uses an additional one. It is used to take in account truncated information available to players. Feedback Nash equilibrium in the game model with continuous updating is defined using the so-called generalized feedback Nash equilibrium as a strategy profile depending not only on current time t and state x , but also on the additional time parameter. Special transformation is introduced to obtain Nash equilibrium with continuous updating as a strategy profile depending only on current time t and state x . In order to define generalized Nash equilibrium, we to introduce the new type of Hamilton-Jacobi-Bellman equation for class of

games with continuous updating.

Till now only the class of games with dynamic updating was studied in the papers [1], [2], [6], [7], [8], [9], [10], where authors laid the foundation for further study in the class of games with dynamic updating. It is assumed that the information about motion equations and payoff functions is updated in discrete time instants, and the interval on which players know the information is defined by the value of information horizon.

Leon Petrosyan, Yaroslavna B. Pankratova. *Equilibrium and Cooperation in the Repeated Hierarchical Games.*

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In the paper a two-level infinitely repeated hierarchical game with one player (center) C on the first level and n subordinate players on the second level is considered. On each stage of the game player C selects vector x from a given set X , in which each component represents vector of resources delivered by C to one of subordinate players. At the second level, each subordinate player chooses his control depended upon the choice of player C.

In this game, a set of different Nash equilibrium also based on threat and punishment strategies is obtained.

In one case, the center enforces special behaviour of subordinate firms (vector of manufactured goods), threatening to deprive them of resources on the next steps if the subordinate firms refuse to implement the prescribed behaviour.

In another case, the subordinate firm can force the center to use a certain recourse allocation threatening to stop production.

Using different combinations of such behaviours on different stages of the game, we obtain a wide class of Nash equilibrium in the game under consideration.

The cooperative version of the game is also considered. The conditions are derived under which the cooperative behaviour can be supported by Nash Equilibrium or Strong Nash Equilibrium (Nash Equilibrium is stable against deviations of coalitions).

Lev Petrov. *Using Nonlinear Interactions To Control Oscillations Of Dynamic Systems.*

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The use of nonlinear effects in the control of stationary oscillations in nonlinear dynamic systems is considered. To find periodic solutions of corresponding ordinary differential equation systems, an interactive algorithm

is used, based on minimizing the solution deviation from the periodic form. The possibility the system behavior controlling due to the mutual nonlinear influence of various types of oscillations is considered. For nonlinear dynamical systems with one and several degrees of freedom, examples of various types oscillations control are given.

Marina Plekhanova¹, Guzel D. Baybulatova². *Problems of hard control for a class of degenerate fractional order evolution equations.*

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We find conditions of unique strong solution existence for the generalized Showalter — Sidorov problem to semilinear evolution equations with a degenerate operator at the highest fractional Gerasimov — Caputo derivative and with some constraint on the image of the nonlinear operator. Then we consider a class of optimal control problems for systems, whose dynamics is described by such equations endowed with the respective initial value conditions. Target functional is assumed not to take into account control costs. In such situation we used the additional condition of the admissible controls set boundedness. The obtained result of the initial problem unique solvability and properties of some functions spaces are applied to the proof of optimal control existence for such class of problems. Abstract results are applied to study of a control problem for a system, which is described by an initial-boundary value problem to a nonlinear partial differential equation, not solvable with respect to the highest time fractional derivative.

Liudmila Prokudina, Dmitrii Bukharev. *Simulation of flow regimes of non-isothermal liquid films.*

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For moderate Reynolds numbers, a nonlinear partial differential equation of the free surface state of a non-isothermal liquid film is presented. The algorithm was developed and the program was written in Matlab R2017b using the Symbolic Math Toolbox module. The wave characteristics of the liquid film under heat and mass transfer are calculated. The flow regimes of a vertical liquid film with a maximum perturbation growth rate are distinguished, and the effect of temperature gradients and surface viscosity on them is investigated.

Anna N. Rettieva. *Coalition Stability in Dynamic Multicriteria Games.*
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We consider a dynamic, discrete-time, game model where the players use a common resource and have different criteria to optimize. The coalition formation process is considered in the case of asymmetric participants. Internal and external stability concepts are adopted for dynamic multicriteria games to obtain new stability conditions. The notion of coalitional stability that takes into account the possible moves of players from one coalition to the other is presented. To illustrate the obtained approaches a multicriteria bioresource management problem with coalition structure is investigated.

Artem Ripatti. *Faster calculating of nim-values of Grundy's game.*
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We consider the well known Grundy's game and several so-called octal games. By Sprague-Grundy theorem, every heap size n of the Grundy's game can be mapped onto equivalent nim heap size $G(n)$. Berlekamp, Conway and Guy conjectured that sequence $G(n)$ is periodic. Flammenkamp calculated the first 2^{38} values of $G(n)$ and found no periodicity. We present a new approach for calculating nim-values of Grundy's game using GPU. Currently we calculated 2^{43} num-values and results for the first 2^{38} of them fully match results of Flammenkamp. Our approach can be applied to other octal games.

Aleksei Rodin. *Finding bifurcation points of a piecewise smooth minimax solution of the Hamilton-Jacobi-Bellman equation.*
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Branch of RAS

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The paper considers the Cauchy boundary value problem of the Hamilton-Jacobi-Bellman equation, in the case where the Hamiltonian depends on the impulse variable. Particular attention is paid to the description and study of the features of the solution: non-smooth points and bifurcation points. The necessary conditions for finding different points of bifurcation will be found.

Olga Samsonuyk, Stepan Sorokin, Maxim Staritsyn. *Feedback Optimality Conditions with Weakly Invariant Functions for Nonlinear Problems of Impulsive Control.*
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We consider a broad class of optimal control problems for nonlinear measure-driven equations. For such problems, we propose necessary optimality conditions, which are based on a specific procedure of “feedback variation” of a given, reference impulsive control. The approach is based on using impulsive feedback controls designed by means of “weakly invariant functions”. The concept of weakly invariant function generalizes the notion of weakly monotone function; these are functions such that their 0-sublevel sets are weakly invariant with respect to the impulsive system. In the talk, we discuss the advantages of this approach and some perspectives of designing, on its base, nonlocal numeric algorithms for optimal impulsive control.

Alexander Shaburov. *Asymptotic Expansion of a Solution for One Singularly Perturbed Optimal Control Problem with a convex Integral Quality Index Depends on Slow Variables and Smooth Control Constraints.*

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The paper deals with the problem of optimal control with a convex integral quality index depends on slow variables for a linear steady-state control system with a fast and slow variables in the class of piecewise continuous controls with a smooth control constraints

$$\begin{cases} \dot{x}_\varepsilon = A_{11}x_\varepsilon + A_{12}y_\varepsilon + B_1u, & t \in [0, T], \quad \|u\| \leq 1, \\ \varepsilon \dot{y}_\varepsilon = A_{21}x_\varepsilon + A_{22}y_\varepsilon + B_2u, & x_\varepsilon(0) = x^0, \quad y_\varepsilon(0) = y^0, \\ J(u) := \varphi(x_\varepsilon(T)) + \int_0^T \|u(t)\|^2 dt \rightarrow \min, \end{cases}$$

where $x_\varepsilon \in R^n$, $y_\varepsilon \in R^m$, $u \in R^r$; A_{ij} , B_i , $i, j = 1, 2$ — are constant matrices of the corresponding dimension, and $\varphi(\cdot)$ — is the strictly convex and cofinite function that is continuously differentiable in R^n in the sense of convex analysis. In the general case, Pontryagin’s maximum principle is a necessary and sufficient optimum condition for the optimization of a such a problem. The initial vector of the conjugate state l_ε is the unique vector, thus determining the optimal control. It is proven that in the case of a finite number of control switching points, the asymptotics of the vector l_ε has the character of a power series.

Mark Sigalovsky¹, Anvar Azimov². *Numerical solution of geometrical inverse problem of gravimetry by genetic algorithm.*

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In this paper, the 2D inverse geometric problem of gravimetry is considered. Here, given a gradiometric data and conditions on the part of boundary, we have to find the center coordinates of geological heterogeneity, which causes the gravity anomaly. From the study of the target functional's derivative, it's non-differentiability by Gâteaux was stated. Due to this fact, it's impossible to use gradient method for numerical solution. Here, for a given problem statement, the genetic algorithm — based solution is presented, in that it doesn't require the property of functional's differentiability.

Nina Subbotina, Natalia Novoselova. *On Viability Set to Problems of Chemotherapy of Malignant Tumors.*

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A chemotherapy model for a malignant tumor is considered and the optimal control (therapy) problem is investigated, the goal of which is to minimize the number of tumor cells at the fixed final instant. The mathematical model is assumed to have piece-wise monotone dynamics. We introduce the viability set of initial states such that optimal trajectories started from the set stay inside restrictions on the states which compatible with life, up to the fixed final instant. We investigate construction of the viability set and influence of the parameters of the model to the structure the set.

Viktor Ukhobotov¹, Konstantin Kudryavtsev², Irina Stabulit³.

On the problem of comparing fuzzy numbers.

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In game problems, when every time the players implement their strategies, their payoffs are given by fuzzy numbers, the problem of comparing fuzzy numbers arises. The paper presents an approach when, according to the accepted procedure of comparing a real number with a fuzzy number, a comparison of two fuzzy numbers is carried out. Details are considered comparison of trapezoidal functions. A specific example of an antagonistic game is considered.

Vladimir Ushakov¹, Aleksandr Ershov¹, Maksim Pershakov².

Counterexamples in the Theory of α -Sets.

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In the early 2000s, V.N. Ushakov and his colleagues introduced the concept of so-called α -sets for classifying sets by the degree of their non-convexity. The parameter α denotes the least upper bound of the angle, under which the projections from the point onto the α -set from this point are seen. The basic properties of α -sets are considered, and we gave two counterexamples that refute two natural hypotheses.

Dmitriy Yaparov, Liudmila Prokudina. *Numerical modeling of the mass of the flowing liquid at transverse oscillations of the straight tube.*

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A numerical method for determining the oscillations of a straight tube section rigidly fixed at both ends is proposed. The tube to an external impulse effect, taking into account the flow and mass of the liquid is subjected. For this method, the results of dynamic measurements are used. A mathematical model of transverse vibrations of a straight pipe section is presented. An algorithm for solving a fourth-order differential equation using the transition to a finite-difference scheme is developed. The results of computational experiments oscillations of elements of a straight pipe are presented. The amplitudes of oscillations, the phase difference of the direct element of the pipe are determined.

Tatyana Zavyalova. *On the question of optimal control of systems with a random structure.*

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A controlled system with a random structure is considered. The dynamics of this system experiences a parametric random effect of a purely discontinuous Markov process. At a random moment of time, due to a change in the structural state of the system, the phase vector of the process trajectory changes abruptly. And the initial conditions for continuing the random process depend on the centered random variable. For such systems with a random structure and a random jump condition for the phase vector, the optimal control is sought using the Lyapunov function method and the small parameter method. The results are clearly illustrated by examples of the movement of a control object with a random environment, as well as by the example of the development of a biological population.

10 . Optimization in Approximation

Roman Akopyan. *Optimal recovery of a function analytic from its approximately given values on a part of the boundary.*

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We study the problem of optimal recovery of a function analytic in a multiply connected domain from its approximately given values on a measurable part of the boundary. The problem of the best approximation of the functional of analytic continuation into point of domain from a part of the boundary by bounded linear functionals is investigated.

Destiny Anyaiwe¹, Chika Moore². *Iterative Solutions for Variational Inclusion Problems in Banach Spaces.*

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Variational inclusion problems have become the apparatus generally used to constrain sundry mathematical equations in order to guarantee the uniqueness and existence of their solutions. The existence of these solutions was earlier studied and proven for uniform Banach Spaces using accretive operators. In this study, we extend the conditions to hold for arbitrary Banach Spaces using uniform accretive operators.

Vitalii Arestov. *Best approximation of a differentiation operator on the set of smooth functions with exactly or approximately given Fourier transform.*

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Let Y^n , $n \geq 2$, be the set of continuous bounded functions on the numerical axis with the following two properties: (1) the Fourier transform of a function is a function of bounded variation on the axis (in particular, a summable function); (2) a function is $n - 1$ times continuously differentiable, its derivative of order $n - 1$ is locally absolutely continuous, and the n th order derivative is bounded, more exactly, belongs to the space L_∞ . In the space Y^n , consider the class $\mathcal{Q}^n$ of functions, for which the L_∞ -norm of the n th order derivative is bounded by a constant, for example, by 1. The following two approximation problems are discussed: the best approximation of the differentiation operator D^k of order k , $1 \leq k < n$, by bounded operators on the class $\mathcal{Q}^n$ and the optimal calculation of the differentiation operator D^k on functions from the class $\mathcal{Q}^n$ under the assumption that their Fourier transform is given with a known error in the space of functions of bounded variation, in particular, in the space L . In interrelation with these two problems, we discuss the exact Kolmogorov type inequality in the space Y^n between the uniform norm of the

k th order derivative of a function, the variation of the Fourier transform of the function, and the L_∞ -norm of its derivative of order n .

Evgeny Derevtsov. *Some properties of generalized attenuated 3D ray transform and its angular moments.*

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Properties of generalized operator of attenuated 3D ray transform (ART), which is connected tightly with a problem of emission computer tomography, are investigated. New operators of ART of order m contain a complex-valued absorption, the weights assume to be of more general form, and the sources may depend on time. Connections between ART of various orders are established and their differential equations are obtained. Uniqueness theorems of boundary-value and initial-boundary value problems for the derived equations are proved. Angular moments of ART of order m are defined and connections between the moments of different orders are established. Close connections of generalized ART with integral geometry and tomography are marked.

Anastasia Drushlyak. *Coincidence points and fixed points in the category of uniform spaces.*

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The paper shows that a number of classical results on fixed points and points of coincidence of maps in metric spaces admits a generalization to the case of a much more extensive class of uniform spaces. It is shown that the classical Andre Weil theorem implies the possibility of representation any separable uniform space as a projective limit of metric spaces. On this basis, a theorem on the representation of any uniformly continuous mapping between uniform spaces in the form of a continual matrix, whose elements are uniformly continuous mappings between metric spaces, is proved. This allows applying the known principles of metric regularity to the elements of a given matrix to obtain the corresponding principles of uniform regularity. Examples and applications are considered.

Ganesh Perumal M, Srinivasa Prasanna Gn. *Projections on polytopes in Euclidean space.*

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In applications based on linear decision support system feasible region is represented as polytope and any infeasible point with respect to this polytope

need to be corrected so that the point becomes feasible. This correction is done such that the distance between original infeasible point and new feasible point is minimum. If p is infeasible point this problem can be solved as convex quadratic optimization problem, $\min \|x - p\|_2$ subject to $Ax \geq b$, $x \geq 0$ which is solvable in polynomial time. In this work, we discuss an Alternate Projection Method based solution to solve this problem in a much simpler manner by exploiting properties of polytopes and extend to project higher dimensional objects on Polytopes.

Ekaterina Kolpakova. *Open-loop Strategies in Nonzero-sum Differential Game with Multilevel Hierarchy.*

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The paper is concerned with the construction of open-loop strategies for the n -person nonzero-sum differential game with multilevel hierarchy. The dynamics of the first player (leader) is defined by its own position and control. The player of further levels of hierarchy knows the position and control of the players of the upper hierarchical levels. At the same time the dynamics and payoff functional of the player do not depend on the position and control of lower hierarchical levels. We solve this problem with the help of consequent solutions of optimal control problems for each player. Using the solution of Hamilton - Jacobi equation and the results of optimal control theory we construct the open-loop controls of the players. The specifics of this problem is the construction of solution for the Hamilton - Jacobi equation with the Hamiltonian discontinuous w.r.t. phase variable. In this case we use the notion of multivalued solution proposed by Subbotin. We show that the open-loop controls provide a Nash equilibrium in the differential game with multilevel hierarchy and the set of payoffs of the players is described by the multivalued solution of the corresponding Hamilton - Jacobi equation.

Oxana Matviychuk. *On ellipsoidal estimates for reachable sets of the control system.*

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The problem of the ellipsoidal estimation of reachable set for control system under uncertainties. The matrix included in the differential equations of the system dynamics is uncertain and only bounds on admissible values of this matrix coefficients are known. It is assumed that the initial states of the system

are unknown but belong to a given star-shaped symmetric nondegenerate polytope. This polytope may be non-convex set. Under such conditions the dynamical system is nonlinear and reachable set loses convexity property. A Minkowski function is used in the investigation to describe the trajectory tubes and their set-valued estimates. The step by step algorithm for constructing external and internal ellipsoidal estimates of reachable sets for such bilinear control systems is proposed. Numerical experiments were performed. The results of these numerical experiments are included.

Robert Namm, Georgiy Tsoy. *A modified duality scheme for solving 3D elastic problem with a crack.*

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We consider the equilibrium problem for 3D elastic body with a crack. Inequality type boundary conditions are considered at the crack faces to prevent a mutual penetration between the faces. This leads to the formulation of the problem with unknown contact area, which admits a variational formulation in the form of minimization of the energy functional on the set of admissible displacements. To solve the problem, we consider the Uzawa algorithm based on the modified Lagrange functional and compare it with classical analogue. Numerical results illustrating the efficiency of the proposed algorithm are presented.

Vladimir Noghin. *Optimizing a Numerical Function over the Fuzzy Set.*

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The main idea of the paper is to consider the optimization problem with a numerical function over the fuzzy set as a multicriteria optimization problem. The proposed method consists of two stages. At the first stage, a membership function of the fuzzy feasible set is added to the objective numerical function and an axiomatic approach that has been developed by the author earlier is applied to the formed bicriteria optimization problem. According to this approach some fuzzy information has to reveal. This information provides reducing the Pareto set of the bicriteria problem and forming a new membership function which is associated with the result of this reduction. Then the new membership function is involved in bicriteria objective function and thereby a tricriteria problem is formed. At the second stage of the proposed method an appropriate scalarization is used to solve this tricriteria problem. An illustrative example is considered.

Alexandr Seliverstov. *Elliptic Points on the Graph of a Third Degree Polynomial.*

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The existence of an elliptic point on a real cubic hypersurface allows to prove the absence of any straight line of singular points of the hypersurface. There is a large set of real hypersurfaces having an elliptic point. In particular, for almost all inhomogeneous third degree polynomials in two or three variables, the graph of the polynomial contains an elliptic point. The recognition of elliptic points on a cubic surface can be used in computer-aided geometric design because fragments of rational cubic surfaces are widely used to model complicated surfaces. On the other hand, an elliptic point can be an obstacle to finding the global extremum of a multivariate polynomial in a bounded domain.

Elena Tabarintseva. *The accuracy of approximate solutions for a boundary value inverse problem with final overdetermination.*

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We study a boundary value inverse problem with final overdetermination (the problem of the most accurate heating of a rod). Namely, we should recover the boundary condition in a mixed boundary value problem for the heat transfer equation from knowledge of the solution at the final time moment.

For linear ill-posed problems classical spectral technique is widely used to obtain estimates of the error for the approximate solutions on compact sets (correctness classes) and the error of the optimal approximate methods.

For linear ill-posed problems, the technique of calculating the error of the optimal method of the is based on the relation between the error of the optimal method and the module of continuity of the inverse operator, which can be calculated for specific operators and correctness classes

In the classical spectral technique the commuting of the operator of the problem with the operator defining the correctness class (reflecting a priori information about the exact solution of the inverse problem) plays the main role but for some important inverse problems in the classical formulation these operators do not commute.

We use the technique of the continuation to the complex domain and the expansion of the unknown function into a Dirichlet series (exponential series) to formulate the inverse problem as an operator equation of the first kind in the space isometric to the space of the initial data and the space of the solutions. This allow us to calculate the module of continuity and investigate the order-optimal approximate methods for the inverse problem under study.

Dmitry Yamkovoï. *Harmonic interpolating wavelets in Neumann boundary value problem in a circle.*

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The Neumann boundary value problem (BVP) in a unit circle is discussed. For the solution of the Neumann BVP, we built a method employing series representation of given 2π -periodic continuous boundary function by interpolating wavelets consisting of trigonometric polynomials. It is convenient to use the method due to the fact that such series is easy to extend to harmonic polynomials inside a circle. Moreover, coefficients of the series have an easy-to-calculate form. The representation by the interpolating wavelets is constructed by using an interpolation projection to subspaces of a multiresolution analysis with basis 2π -periodic scaling functions (more exactly, their binary rational compressions and shifts). Such functions were developed by Subbotin and Chernykh on the basis of Meyer-type wavelets. We will use three kinds of 2π -periodic scaling functions, where two out of the three generates systems, which are orthogonal and simultaneous interpolating on uniform grids of the corresponding scale and the last one generates only interpolating on the same uniform grids system. As a result, using the interpolation property of wavelets mentioned above, we obtain the exact representation of the solution for the Neumann BVP by series of that wavelets.

11 . Operations Research

Natalia Aizenberg, Nikolai Voropai. *The interaction of consumers and load serving entity to manage electricity consumption.*

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The paper is concerned with the coordination of interaction between various types of consumers and a load serving entity to manage electricity consumption by using a several models: the model of pricetaking, the Nash equilibrium pricing, the adverse selection model based on contract theory. A method is proposed to form tariff options for load curve optimization for different types of consumers and a power supply company for different market configurations. The utility functions describe sufficiently well the real situation and allow the implementation of a system of incentives for load curve optimization (load shifting from a peak time of the day), and the rates providing a separating equilibrium are determined. We compared the effectiveness of different retail market models for demand side management.

Ivan Davydov¹, Daniil Tolstykh². *An evolution based approach for the traffic lights optimization problem.*

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We consider the traffic lights optimization problem which arises in city management due to continuously growing traffic. Given a road network and predictions (or statistical data) about the traffic flows through the arcs of this network the problem is to define the offsets and phase length for each traffic light in order to improve the overall quality of the service. The latter can be defined through a number of criteria, such as average speed, average trip duration, total waiting time etc. For this problem, we present an evolution based heuristic approach. We use a simulation model on the basis of the SUMO modeling system to evaluate the quality of obtained solutions. The results of numerical experiments on real data confirm the efficiency of the proposed approach.

Nadezhda Dresvyanskaya, Oleg Khamisov. *Bilevel model of long-term power system development under network capacity constraints.*

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In our talk we consider a long term equilibrium model for power system capacity and network expansion. The main difficulty is caused by the network constraints. These constraints seriously affect on existence of the equilibrium, which is understood in the sense of Nash. A bilevel approach is suggested.

Network development represents the first level and interaction between generating companies under given network capacity stands for the second level. Different reduction to one level nonconvex optimization problem are considered and results of some numerical experiments are presented.

Elina Dyaminova¹, Anna Filippova¹, Lidia Vasilyeva², Yulia Valiahmetova². *The matrix technology of harnessing the data for solving the complex problem of geometrical placement.*

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The matrix technology of harnessing the data for designing solutions of geometrical placement problems is featured in the article. Performance evaluation of the algorithms designed on the above technology is being tested on the example of solving a double-criteria complex problem of geometrical placement. The mathematical model is offered for the given problem. The stages of solving it are described. There are given the computing experiment results based on the specially generated non-waste examples.

Oleg Khamisov, Natalia Mikhakhanova. *Numerical methods for finding equilibrium on heat energy markets.*

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We consider an equilibrium model describing compromise behaviour of heat generating companies and analyze conditions under which such equilibrium exists. Each company corresponds to a parametric optimization problem. A numerical process of solving the equilibrium problem is presented as well as preliminarily numerical results.

Anton Kolosnitsyn. *Stochastic Analogue of Long-term Development Model of Power Energy Systems.*

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We consider a special long-term development and operation model of power energy systems. This model is called the CANOE model that means CAPacity and Network Optimization and Expansion model. It represents a linear programming problem in which we minimize the total annual system cost subject to the balance equations and other system-specific constraints. In initial statement CANOE model takes into account expert valued demand in consumption nodes. We assume that demand in consumption nodes is a random variable with

specified distribution law. To provide the necessary level of energy load with certain probability the chance-constrained stochastic optimization problem was formulated. In our study we apply a special technique to transform chance-constrained optimization problem into the equivalent deterministic convex optimization problem. The testing results of stochastic and deterministic CANOE models will be given in this paper.

Alexandr Krivonogov¹, Galina Zakharova². *Using of language R for solving optimization problems in the educational program of the university.*

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The effectiveness of the programming language R for solving optimization problems is shown on the example of a linear programming task using specialized built-in packages. These opportunities are implemented in the master's educational program "Artificial Intelligence in assets management" in the Ural Federal University.

Sergey V. Kruglikov. *Operation Research Presentation of the State Support for the Regional Industry.*

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The dynamic model for the forming of common information images in the process of exchanging structured data and coordinating the information spaces of participants is implemented to the problem of annual budget planning. The paper discusses the analytical tools for assessing the effectiveness of state support for the timber industry for two neighboring regions of Russian federation. The study is based on the previously proposed technique of forming of industrial policy in the sectoral and territorial aspect. The multi-agent approach is applied to describe the local industry as the combination of several major enterprises and the SME population. The mathematical model of cost-benefit analysis presents the economic forecast. The total cost of ownership presents the state utility function estimate. Optimization problem in guaranteed statement allows to examine comparatively the effectiveness of current regional support for the timber industries in sample regions.

Sergey I. Kumkov¹, Zinaida V. Kataeva², Oleg V. Shilovskikh².

Procession of post-operation ophthalmic data under conditions of uncertainty.

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The paper describes decision making in procession of ophthalmic data of the post-operation status criteria between two groups of patients. The groups differ by the type (technology) of fixing the intraocular lenses (IOL). Validity of each type of technology was estimated by computation of distinction of corresponding samples between groups. The samples on each criterion are very short; in each criterion, as a rule, the samples of groups overlap each other; any probabilistic characteristics of the measuring criteria are unknown; any probabilistic characteristics of the measuring errors are also unknown. So, the standard methods of mathematical statistics can be applied only in the formal way and have shown to be inefficient. In contrast, the Hausdorff distance (from the set theory) as the index of distinction between two samples (both for one- and, especially, for two-dimensional criteria) demonstrated to be reliable to distinct results of two groups. Computations of the Hausdorff distance are valid for any relative location of point sets under comparison.

Ilya Kurochkin, Yakov Grinberg, Alexandra Prun. *Heuristic composite algorithm for sequential routing on a telecommunication network graph.*

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An mathematical simulation model of the functioning of the telecommunications network was created. Within the simulation model, a sequential laying of paths on the network graph between different pairs of nodes is carried out according to various criteria. The article presents several approaches to the formation of criteria for laying paths on the network graph, as well as criteria and algorithms. The creation of a set of test networks of certain network topologies, such as stochastic, dragonfly and fattree, is discussed. A composite algorithm is proposed that combines a linear programming approach and heuristic sequential routing algorithms.

Sergey Lavlinskii, Artem Panin, Alexander Plyasunov. *Stackelberg Model and Public-Private Partnerships in the Natural Resources Sector of Russia.*

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A comparative analysis is conducted of the efficiency of different partnership models in the natural resources sector of Russia. The first one is a classic public-private partnership (PPP) model used in developed countries, whereby a private company builds an object of public property and transfers it to the

government either immediately after the construction or after a certain period of operation of the object. The second model represents for the government a costly alternative of the former and is used in Russia in underdeveloped regions. This model assumes that the government supports the investor in infrastructure development and, in part, in the implementation of mandatory environmental protection measures and can also provide tax incentives. This work aims to look into possible ways of transforming the current Russian PPP model towards the classic forms of partnership. To conduct the comparative analysis of the PPP models, Stackelberg models are formulated and original iterative algorithms are developed for solving the corresponding two-level Boolean programming problems based on probabilistic local search. The properties of the equilibrium solutions are studied using real data for the Transbaikal krai. Based on the modeling results, the different partnership models are compared to find out the conditions under which the private investor would choose to invest in publicly owned industrial infrastructure facilities in Russia. This work is supported by the Complex program of basic scientific research of the SB RAS no. II.1 (project no. 0325-2018-0004).

Anna Lempert¹, Alexander Kazakov¹, Quang Mung Le². *On the Thinnest Covering of Fixed Size Containers with Non-Euclidean Metric by Incongruent Circles.*

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The paper is devoted to the circle covering problem with unequal circles. The number of circles is given. Also, we know a function, which determines a relation between the radii of two neighbouring circles. The circle covering problem is usually studied in the case when the distance between points is Euclidean. We assume that the distance is determined by means of some special metric, which, generally speaking, is not Euclidean. The special numerical algorithm is suggested and implemented. It based on optical-geometric approach, which is developed by the authors in recent years and previously used only for circles of equal radius. The results of a computational experiment are presented and discussed.

Yakov Lvovich¹, Igor Lvovich¹, Andrey Preobrazhenskiy¹, Oleg Choporov², Yuriy Klimenko¹. *Management of distributed energy systems based on rating, optimization and expert approaches.*

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Currently many methods and approaches related to the management of distributed energy systems are associated with the collection of large amounts of information. The results of the rating assessment from the management point of view are limited. In many cases we need to involve modeling and optimization techniques in the management process. This paper shows how an integral assessment of the efficiency of distributed energy systems is formed. The optimization model is developed and the procedures of expert evaluation of management decisions are formed. On the basis of the methods used, the results demonstrating their efficiency are obtained.

Tatyana Makarovskikh¹, Egor Savitskiy². *Optimization of Pierce Points Number for Cutting Plan with Combined Cuts.*

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The technology of plasma cutting claims presence of some space to place a pierce point. The image of cutting plan is a plane graph G with specified list L of faces allowing piercing. The outer face $f_0 \in L$. We review the polynomial algorithms for all the possible restrictions: (1) part cut off a sheet does not require further cuts (constructing of OE-route); (2) there are some restrictions on placement of pierce points (constructing of PPOE-cover). In this paper we consider necessary conditions for existence of technologically realizable routes (PPOE-routes). Earlier we have proved the existence of OE-chain for a plane Eulerian graph and defined upper bound for the number of such chains with fixed transitions system. This bound may be used for easy definition of an OE-chain starting from a fixed vertex without running algorithm if one of OE-chains is defined. The route may start from any vertex incident to outer face if G is Eulerian. As for semi-Eulerian graph, the route may start only from vertex of odd degree. If both of these vertices are incident to outer face f_0 then the route must start from any of these vertices. Generally, we have the problem of constructing the PPOE-route consisting of chains starting from vertices incident to faces from L . In this paper we consider the problem of constructing the technologically realizable routes with minimal number of pierce points.

Yurij Mezentsev, Yuliya Korotkova. *Optimization Problems and Algorithms for Airline On-Time Performance Management.*

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The schedule is the basis of any airline. In this paper, we provide one of the problems of schedule theory. This is the problem of optimal management of flight schedule to minimize flight departure delays and to improve flight on-time performance. A conceptual and formal statement of this problem is given. Defined such the terms as “Delay propagation” and as “On-Time Performance”. The considered task of the optimal fleet management consists in such operation management of schedule, which minimizes airline’s losses from violations of the planned Flight schedule. The problem’s solution is to manage the assignments of specific aircraft on flights considering the operational information about an estimated time of departure and delays. The problem is NP-hard and cannot be solved accurately for any real-life number of dimensions. We propose the parametric effective algorithm for its approximate solution and provide the results of calculations for the test dataset.

Artem Novikov. *Bilevel stochastic model for resource region development program formation.*

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Raw-materials base is one of the largest industries for financial investments in Russia. There are various mathematical descriptions for the development of regions with resource-based economy. This presentation proposes a new approach to public-private partnership modelling, including a bilevel linear stochastic programming problem. This model assumes that budget restraints of state and investor can vary in a random manner with a specific probability distribution. We put forward two methods to solve this problem, namely problem’s reduction to the deterministic bilevel one and formulation of deterministic problems sequence with help of Monte Carlo methods. In order to solve the deterministic problems of integer programming, we use two approaches: direct enumeration and heuristic “Game” approach. The numerical experiments for proposed algorithms validation are conducted on the actual data of Zabaykalsky Krai development. Multiple input parameters of the model vary in these experiments. Finally, we present a brief analysis of the obtained solutions to the stochastic linear programming problems with Boolean variables.

Alexander Petunin, Alexander Chentsov, Alexander Sesekin, Pavel Chentsov. *Tool path design for the CNC Sheet Metal Cutting Machines. Optimization models and "dynamic" constraints.*

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The tool path problem for the CNC sheet metal cutting machines is considered. Known algorithms for solving this problem mainly use various variants of the mathematical model of the generalized traveling salesman problem (GTSP) with additional constraints such as precedence constraints. At the same time, when forming the allowable tool path of a CNC machine, it is necessary to take into account the thermal and mechanical deformations of sheet metal arising during cutting. This leads to the emergence of a new type of technological constraints (“dynamic” constraints) that arise in the process of the tool route design. The paper describes the mathematical formalization of this type of constraints and proposes an algorithm for solving the problem, based on the dynamic programming method.

Olga Sokolova. *Modeling message transmission in networks with mobile nodes.*

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We consider data transmission networks in which communication takes place between devices located in vehicles, as well as between these devices and roadside units (for example, Vehicle ad-hoc networks VANET). Consider the task messages transmission from some vehicle or stationary unit to other road users. To simulate the placement of nodes in the network, you can use the Poisson random network model. We assume that the message transmissions from the nodes are broadcasting, that is, the message is always transmitted in the range of the transmitting device – r . Those nodes that were able to receive the message transmit it further. At each stage, nodes must be chosen so as to avoid interference, which is unavoidable when several nodes transmit messages simultaneously. The proposed data transmission algorithms on a specific schedule to reduce the influence of interference. Simulation has confirmed that the influence of interference is reduced in this case.

Arseniy Spiridonov, Kumkov Sergey. *Non-Conflict Merging Aircraft Flows under Given Arrival Schedule.*

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Nowadays, aircraft move along routes consisting of horizontal tunnels and vertical flight levels. With that, the routes can split or join. At the point of route joining, a problem of aircraft flow merging appears. Such a problem is highly important near airports, where air traffic is very dense. The main demand for aircraft flow merge is the presence of the minimal safe time interval between arrival instances at the merge point. There are two main tools for

changing arrival instant of an aircraft to a checkpoint. The first of them is control of the aircraft velocity, which allows to obtain relatively small changes of the arrival instant both to earlier or later times. To get larger delays one uses the second tool, delay schemes. As a result of designing system of delay schemes for a certain airport, one has information about possible acceleration and deceleration of aircraft moving along each route. Further on the basis of this information, it is necessary to study capabilities of the constructed system for formation of safe aircraft flow merge. In the talk, a formalization is set forth for the problem of optimal formation of aircraft arrival schedule under the present delay scheme system as a finite-dimensional optimization problem. Also, the authors discuss applicability of different methods for search of multivariable function extrema to this problem. Results of numerical computations are demonstrated.

Andrey Takmazian. *Finite automata in the network flow application to the locomotive crews assignment problem.*

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TBC

Alexandr Tesselkin¹, Valeriy Khabarov². *Optimal Design of Observations on Traffic Flows for the Origin-Destination Matrix Estimation.*

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The problem of the optimal design of observations on flows in the transportation network is relevant for organizing traffic flow surveys, the estimation of origin-destination matrices, the calibration of network equilibrium models etc. Observations on traffic flows may be very expensive, so it is necessary for observations to be as informative as possible. The criterion of informativeness is Fisher information in the nodes of observations. For solving the problem the transportation network is represented as an aperiodic discrete Markov chain. The problem of the optimal design of observations is stated as the problem of allocating some resource to the nodes of the transportation network. For example, the resource may depend on the time spent on counting the traffic flows. The maximum of some functional based on the Fisher information matrix is used to solve the problem. These matrices are calculated using maximum likelihood estimates and Bayesian estimates. Observation designs are evaluated using the criterion of D-optimality. If maximum likelihood estimates are employed, an analytical solution to the problem was obtained. In the case of

Bayesian approach the nonlinear optimization problem with linear constraints has to be solved. Bayesian approach allows taking into account prior information on flow volumes on the network. The designs of observations obtained are non-degenerate and imply that some part of the resource is given to observers at all network nodes. In practice the “design truncation” procedure can be used due to the limited number of observers. Examples and recommendations for applying the proposed approach are given.

Galina Timofeeva, Alexander Martynenko. *Estimation of Distribution Parameters in Probabilistic Model of Passengers Preferences.*

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A mathematical model for describing passenger preferences is studied. The model was proposed earlier by the authors for predicting passenger traffic for high-speed links of the railway network.

Choosing a route by a randomly selected passenger is considered as the problem of minimizing of the “generalized cost of transportation”, which is a linear combination two criteria (cost and time of transportation) and depends on a random parameter of the passenger preferences.

The properties of the model are investigated. The estimation problem for the random parameter distribution via statistical data on the passengers’ preferences is studied.

12 . Economics

Sergey Antsyz. *On one approach to modeling developments economics.*
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A new approach to the construction of mathematical models of complex socio-economic systems is described. The period of economic growth consisting of intervals equal in duration is considered. In contrast to the classical Ramsey model, it is proposed to minimize the integral income for the period, provided that per capita consumption at each interval is limited from below. The neoclassical function of economic growth is approximated at each interval by a linear function. The choice of restrictions from below providing growth of welfare of individuals is offered. It is shown that the strategy of economic development, obtained with the help of a new approach, can reduce the damage to the environment.

Ivan Belyaev¹, Igor Bykadorov^{1,2}. *Equilibrium in Dixit-Stiglitz-Krugman Model: the Case of Nonlinear Production Costs.*

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We study the market equilibrium in the Dixit-Stiglitz-Krugman international trade model. The transport costs are of "iceberg types". It is known that, in procompetitive case and under linear production costs, the social welfares w.r.t. transport costs decrease near free trade and increase near total autarky. We generalize these results on the case of nonlinear production costs.

Igor Bykadorov. *Social Optimality in International Trade under Monopolistic Competition.*

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We study the homogeneous model of international trade under monopolistic competition of producers. The utility function assume additive separable. The transport costs are of "ice-berg types". It is known that, in the situation of market equilibrium, under linear production costs, the social welfare, as function of transport costs, decreases near free trade while (counter-intuitively!) increases near total autarky. Instead, we study the situation of social optimality. We show that the total welfare decreases. We restrict our study by the case of two countries, "big" and "small". Moreover, we study two important "limited" situations: near free trade and near total autarky. We show that near free trade, the welfare in small country decreases; as to the big country, we find examples

when (1) the welfare decreases and (2) the welfare (counter-intuitively!) increases. Besides, in the autarky case, we describe the situations of decreasing/increasing of welfare in each country.

Nikolai Chernavin¹, Pavel Chernavin², Fedor Chernavin³. *Application of the committee machine method to the analysis of stock market technical indicators.*

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Application of the committee machine method to the analysis of stock market technical indicators

In the article is studied problems of the committee machine method application to forecast stock prices, based on set of technical indicators. All calculations were done on a data from Moscow Stock Exchange for the period of February 2010 till December 2018.

**Christof Defryn¹, Julian Golak¹, Alexander Grigoriev¹,
Veerle Timmermans².** *Inland waterway efficiency through skipper collaboration and joint speed optimization.*

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We address the problem of minimizing the aggregated fuel consumption by the vessels in an inland waterway (a river) with a single lock. The fuel consumption of a vessel depends on its velocity and the slower it moves, the less fuel it consumes. Given entry times of the vessels into the waterway and the deadlines before which they need to leave the waterway, we decide on optimal velocities of the vessels, that minimizes their private fuel consumption. Presence of the lock and possible congestions on the waterway make the problem computationally challenging. First, we prove that in general Nash equilibria might not exist, i.e., if there is no supervision on the vessels velocities, there might not exist a strategy profile from which no vessel can unilaterally deviate to decrease its private fuel consumption. Next, we introduce simple supervision methods to guarantee existence of Nash equilibria. Unfortunately, though a Nash equilibrium can be computed, the aggregated fuel consumption of such a stable solution is high compared to the consumption in a social optimum, where the total fuel consumption is minimized. Therefore, we propose a mechanism

involving payments between vessels, guaranteeing Nash equilibria while minimizing the fuel consumption. This mechanism is studied for both the offline setting, where all information is known beforehand, and online setting, where we only know the entry time and deadline of a vessel when it enters the waterway.

Chiang Kao. *Measuring the most favorable Russell efficiency under the framework of data envelopment analysis.*

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Conventional radial efficiency measurement models in data envelopment analysis are unable to produce appropriate efficiency scores for production units lying outside the cone generated by the convex hull of the efficient production units. Moreover, in the case of production technologies with variable returns to scale, the efficiency scores measured from the input and output sides are usually inconsistent. To solve these problems, the Russell measure of efficiency that takes both the inputs and outputs into account has been proposed. However, the conventional Russell efficiency is measured under the least favorable conditions, rather than the general custom of measuring under the most favorable ones. This paper develops a linearly constrained nonlinear combinatorial optimization program to measure Russell efficiency under the most favorable conditions. A case of the Taiwanese commercial banks demonstrates that the Russell measures are more reliable and representative than the radial measures. The Russell measures obtained under the most favorable conditions are greater than or equal to those obtained under the least favorable conditions. More importantly, the corresponding rankings are different. It can be concluded that the most favorable Russell measures of efficiency are more reliable and are more persuasive to the production units being evaluated.

Eugeniia Markova, Inna Sidler. *Optimization problem in an integral model of developing system without prehistory.*

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We consider an integral model of the developing system consisting of elements of n age groups. The model described by means of the nonclassical Volterra equation of the first kind. The moment of system origin coincides with the beginning of the modeling, so there is no prehistory and for $t = 0$ all the age groups of the elements are empty. Based on this model, we set the problem of optimizing the system age structure and the moment when the elements are dismantling. The results of numerical calculations are presented.

Vladimir Mazurov^{1,2}, Ekaterina Polyakova². *Ural Problems and Committee Method.*

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1. Idea: when do the structural properties of system algebra represent the consequence of subsystem algebra?

2. Committee and factor analysis theory is connected with separation of sets. Examples: ore body cuts, thin rock sections. Mineral and rock sections. In addition, geological structures of space.

3. Group structural theory was the subject of research conducted by professor of the Ural State University P.G. Kontorovich. The question studied within this topic was whether the group inherits the structural properties of its subgroups. Far from always the subgroup properties are inherited. For example, the simple fact: the proper subgroups can be commutative while the group itself can not.

4. Factor nomination. The problem has not been solved yet. Linguistics has helped us.

Further on, the following issues are discussed: construction methods of linear inequalities committee systems. Committee existence criterion. Committees and collective agreements. This topic was discussed in the research by Vl. D. Mazurov, M. Y. Khachai. The beginning can be found in the research by Vl. D. Mazurov, the classic conclusion — in that by M. Y. Khachai. This research imposes the following MX-logic of conflicting data processing:

□ Problem reduction to algebraic inequality system.

□ Further on, application of MX-logic of deduction drawing from conflicting data. The place of MX-logic is among the non-classic logics. This is the logic of dead-end subsystem analysis.

Further on, connection with the hereditary theory of set properties. Examples:

1. Inheritance of starting set properties in mathematical economics.
2. Group properties conform to the structural properties of its subgroups.
3. In perceptron — the use of sensor layer by parts in the process of perceptron learning.
4. Optimization method including discriminative analysis to record non-formalized factors.
5. Optimization and recognition of generalized equilibrium.
6. Nature of non-formalizability.
7. Theory of virtual equilibrium bypass
8. Practical problems of the Urals: geology, mining and smelting industry, furnace charge property forecast.
9. Rock diagnostics. Ore body geometry. Computer geological map.

Let us refer to the origins: Imaginary geometry of N. I. Lobachevsky, imaginary logic of N. A. Vasiliev, non-Aristotelian, intuitionistic, many-valued, inconsistent (MX) logic. Paraconsistent logic, an ill-defined term, was carelessly used in Latin America.

Nicholas Olenov. *An endogenous production function and its identification for some countries.*

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An endogenous production function with the limited age of the vintage production capacities and its identification for some countries is presented.

Sergey V. Plotnikov. *Coordination of decisions by committee methods in game situations.*

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In the report we discuss the possibility of using committee structures [1] to reconcile conflicting requirements in information models that use the game-theoretic concept of the core.

Let $N = \{1, \dots, n\}$ be the set of agents (players) in the cooperative game Γ with side payments defined by the payment function $v : 2^N \rightarrow R_+$. Coordination of coalition interests through the formation of the core leads to a linear model

$$x_1 + \dots + x_n = v(N), \quad \sum_{i \in K} x_i \geq v(K) \quad (\forall K \in 2^N), \quad (1)$$

which may be inconsistent even if the payment function v is super additive. Nevertheless, it is well known that the core will be non-empty if, for example, v is super modular. There are generalizations and extensions of the core concept, which play an important role for construction of an externally stable set of divisions of payments.

If the core is empty, we propose a committee solution of the system of linear relationships (1) as its approximation. Moreover, such a solution, due to the simplicity of the relationships, can be expressed explicitly in the general case when the payment function is super additive. In applications of this problem the systemic requirement

$$x_1 + \dots + x_N = v(N)$$

is too important to ignore even for a part of the committee members. We are convinced that this requirement should be mandatory for all members of the committee, counting on the natural interpretation of the committee solution.

Without assuming any additional properties of the payment function, such as super additivity, for example, it will not be possible in the general case to build a majority committee with the requirement $x_1 + \dots + x_N = v(N)$ for all members of the committee. Suppose that the payment function is super additive:

$$v(K \cup S) \geq v(K) + v(S), \quad K \cap S = \emptyset.$$

In this case, it is possible to build a majority committee with the directive requirement for all its members.

The conditions for the existence of a committee in the presence of directive restrictions and non-strict linear inequalities are also discussed.

References. 1. Mazurov V.D. The method of committees in optimization and classification. Nauka, 1990. 324 p.

Nina Plyaskina. *Optimal Control of the Multilevel Economic System Based on Lagrange Multipliers.*

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Complication of hierarchical systems caused expediency of adoption of management decisions on the basis of problems of optimization for each level with the subsequent coordination of their solutions. The two-level system of models is considered. The top level is represented by the optimization model of the megaproject, the lower level - the optimization models of the development of the separate projects which are its part. The development of the system as a whole and its subsystems is described by network models in the form of oriented graphs. The method of coordination of the network models of the system as a whole and the subsystems of the lower level is proposed. Searching of a global optimum in a two-level system is presented as an iterative process. This process is based on the principle of comparison of the Lagrange multipliers and the condition for the existence of a saddle point.

Yulia Polozhishnikova. *Equity-linked notes. Pricing framework: mathematical models, software architecture and implementation tools.*

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This paper addresses pricing of equity-linked notes (ELN). ELN is a financial instrument that can be viewed as a bond with fixed and floating coupons. The floating coupon is represented in terms of an embedded option that depends on the behavior of a certain underlying asset or a basket of assets. A choice of the embedded option allows to construct different types of payoffs catering

needs of all kinds of investors. In this paper we set up a general framework highlighting subtleties that one can encounter when trying to estimate the value and risk-metrics of ELNs. Firstly we demonstrate some mathematical models and derive ELNs pricing principles. Secondly we develop the suitable software architecture, which based on classes' hierarchy. Then we provide numerical examples in Python which comprise pricing ELNs in different techniques with several implementation challenges. The main troubles in pricing ELNs with embedded path-dependent option are the rate of convergence of Monte-Carlo simulations, memory and time consumption. We suggest some solutions on how to accelerate a rate of convergence, collect memory garbage and optimize the code. The fundamental difficulty in pricing structured products is calibration of model parameters, especially in absence of some market data. We provide some tricks to circumvent this.

Vladimir Servakh, Svetlana Malakh. *The net present value maximization in inventory management system.*

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The paper researches the model of the profit maximization for a commercial company, taking into account the intensity of the sale of goods, the cost of purchase, the cost of delivery, the cost of storage and the cost of sale of goods. The alternative investments of available capital is also taken into account. It is shown that the profit function, depending on the period of delivery of goods, has a single maximum point. A model of the problem of the profit maximization in multi-product systems with limited working capital has been built and an algorithm for solving it has been developed.

Alexander Smirnov, Vladimir Mazurov. *Conditions under which any optimal control is preserved, in one formalization of optimal exploitation renewable resources problem.*

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Previously, the authors proposed a formalization of the problem of rational use of renewable resources based on the representation of the system being controlled as a discrete dynamic system. In the particular case of a structured ecosystem described by the non-linear binary Leslie model, conditions were obtained under which optimal controls preserving this system exist. This article examines the conditions under which any optimal control in this problem retains all the units of the system being controlled. It is shown that there are restrictions on the controlled system, under which for any objective function the optimal control retains all the units of the system being controlled. For

this, in particular, some generalization of the classical concept of irreducibility for nonlinear maps, the concept of local irreducibility was used. The results obtained make it possible to assess the applicability of the proposed formalization in the specific problems of the exploitation of ecological populations.

Olga Tilzo¹, Igor Bykadorov^{1,2}. *Retailing under Monopolistic Competition: a Comparative Analysis.*

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We study the Dixit-Stiglitz monopolistic competition model with retailing. Thus, two-level model in the structure "Producer – Retailer – Consumer" arises. We consider the various types of interaction: the situations of Nash equilibrium, Stackelberg equilibrium (in two cases: the leadership of Retailer and the leadership of Producer). A comparative analysis of the solutions obtained allows clarifying the effect of each of these interactions on consumers and society.

Eugene Vitvitsky¹, Saida Khairova¹, Okxana Kulikova¹, Elena Khoroshilova¹, Bari Khairov². *Application of the tabu search algorithm to the task of optimal scheduling of small freight shipping in the city.*

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The purpose of the study is the approbation of the tabu search algorithm for solving the problem of optimal scheduling of small trucking in the city. The article contains a mathematical formulation of the problem as a binary programming problem. When setting the task, restrictions are formulated that take into account the requirements for the quality of freight shipping, such as the timeliness of trucking, the number of failures in service, and the criteria determining the business reputation of the trucking company, defined by its competitiveness indicators. The classification of situations allowing to identify cities according to the level of requirements for small trucking companies has been developed. The algorithm is implemented in the MATLAB software package. The design experiment was carried out for each class of cities in accordance with the developed classification. The results of the study. 1) Application of the tabu search algorithm allowed us to solve the task of optimal scheduling of small trucking in the city, which satisfies the described limitations of the task in a reasonable time. 2) It has been found that the higher the requirements for the trucking companies for small trucking, the smaller the number of customers

that can be served within one turnover. 3) Tightening customer requirements contributes to the need to increase the trucking company's volume of time reserves and material reserves. 4) The lack of the possibility of attracting third-party motor vehicles forces trucking companies not to take into account the interests of drivers above the regulatory requirements.

Dmitry Zavalishchin¹, Artem Khazimullin². *Adaptive Management of the Loyalty Program.*

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Modern loyalty programs on the CRM (Customer Relationship Management) platform are investigated. The optimal strategies found depend on the market situation. There are encouraged to use mathematical modeling using the game theory. This will allow the seller and the buyer to find the best strategies. In addition, the possibility of finding the best strategies of the seller in case of a change in the matrix of profits due to certain circumstances is considered. In this case, the behavior of the customers flow is given by a highly likely mixed strategy.

13 . Machine Learning

Anastasiya Andrianova. *Some modifications of the SVM-method optimization problem to reduce errors in the classification problem.*

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For the binary classification problem - one of the important problems in big data analysis and machine learning, some modifications of the classical formulation of the optimization problem for support vector machine method (SVM) are considered. The main feature of the proposed modifications is the use of other types of the penalty function for an errors of a training set examples in order to improve the accuracy of the constructed classifier. Of particular interest in the formation of this penalty functions are those errors that lead to a confident erroneous classification of the object, and not to fall into the "strip" that separates the classes.

Vladimir Berikov. *Semi-Supervised Classification Using Multiple Clustering and Low-Rank Matrix Operations.*

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This paper proposes a semi-supervised classification method which combines machine learning regularization framework and cluster ensemble approach. We use low-rank decomposition of the co-association matrix of the ensemble to significantly speed up calculations and save memory. Numerical experiments using Monte Carlo approach demonstrate the efficiency of the proposed method.

**Marat Bogdanov¹, Aleksander Dumchikov², Aigul Akhmerova²,
Dajan Nasyrov¹, Ivan Dokuchaev¹.** *Secured Telemedicine Service based on Deep Learning.*

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The article discusses the possibility of using Deep Learning to improve the security of telemedicine services performing diagnosis of cardiovascular diseases using electrocardiograms. It's compared the accuracy of biometric identification and diagnosis of heart diseases with Multilayer Perceptron (MLP) neural network and Long Short Term Memory networks (LSTM) depending on the time of signal registration. The maximum accuracy of biometric identification using MLP is 0.972, LSTM - 0.051, diagnostics using MLP is 0.945, LSTM - 0.691.

Valentina Bykova, Choduraa Mongush. *On “safety” decomposition of a binary context in data analysis and combinatorial optimization.*

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The problem of finding all formal concepts of a given formal context is investigated. The problem occurs in data mining when information is represented in the form of a binary object-attribute matrix, i.e. a matrix the rows of which correspond to objects, and the columns correspond to attributes that take a value from the two-element set 0, 1. Herewith the value 1 of the element of a matrix is interpreted as the presence of corresponding attribute to the object, and 0 is as its absence. Such a representation of a data set allows to be used the algebraic approach of R. Wille and B. Ganter, known in the literature as Formal Concept Analysis. Within of this approach the initial object-attribute matrix is called the formal context, and any of its maximal full submatrix is a formal concept. In the problem of finding all formal concepts, it is required to find the set of all formal concepts for a given formal context. This problem belongs to combinatorial enumeration problems and is $\#P$ -complete. The high computational complexity of the problem is due to the fact that in the general case the number of formal concepts exponentially depends on the size of the initial formal context. Currently many algorithms have been developed to solve the problem, among them NextClosure, Close-by-One, Norris. The execution time of these algorithms in the worst case exponentially depends on the dimension of the initial context, and therefore they are unsuitable for practical analysis of contexts of large dimension. We propose a decomposition method for solving the considered problem. In this method the fragments of the initial context are boxes. It is proved that the division of context into boxes is “safety” relatively of the formal concepts. This means that at decomposition the formal concepts are not lost and new formal concepts do not arise. We proved that the number of boxes arising at each iteration of the decomposition is equal to the number of unit elements of the 0,1-matrix representing the initial formal context. We showed how on a set of boxes a partial order relation can be defined. We also showed that at each iteration of the decomposition process the number of boxes can be reduced using constructing mutually disjoint chains. The results of computational experiments are given, indicating that the application of the proposed decomposition method allow significantly to increase the performance of algorithms for finding all formal concepts of a given context. The considered problem is equivalent to the problem of determining all maximally complete submatrices of a 0,1-matrix. Other cognate problems exist. This is the problem of enumerating all maximal bicliques, problems of finding different covers of a bipartite graph by maximal bicliques and etc. These graph-theoretical problems belong to the class $\#P$ -complete or NP-complete. These problems are well

studied and have many applications. However, most of the known algorithms for solving these problems work unacceptably long on large graphs. Improve the performance of these algorithms can also be through the use of our proposed decomposition approach.

Valeriy Kalyagin. *Clustering in Random Variables Network.*

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Random variables network is a graph with n nodes, associated with a random vector of dimension n . Nodes of the graph are associated with random variables (components of the random vector) and weights of edges are given by some measure of association (dependency, similarity,...) between them. Data mining in random variables network is an important problem with numerous applications. In this paper we consider the clustering problem in random variables network. The main question is how to handle uncertainty of clustering algorithms generated by uncertainty of the data. We suggest an approach based on decision theory and construct a robust clustering algorithms in random variables network.

Vladimir Krutikov¹, Mikhail Zhalnin², Lev Kazakovtsev³,

Vladimir Kazakovtsev⁴. *New Methods of Teaching Two-Layer*

Sigmoidal Neural Networks with Regularization.

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Artificial Neural Networks (ANN) are able to approximate dependencies of any complexity, however, they also show interference. There are problems with creating an ANN due to the complexity of estimating its parameters. Normally, we select an initial approximation randomly [1]. Often the working areas of neurons exceed the approximation area or cover only a part of it, and some data areas are not covered by neurons, and the approximation cannot escape an area of a local minimum. In the proposed algorithm for teaching two-layer sigmoidal type ANN, we use the idea of bordering a given surface with the surfaces of an excess number of neurons distributed by some clustering method over the data area to select the initial ANN approximation. Further, we adjust free ANN parameters with fixed neuron parameters. Due to the use of regularization at this stage, it is possible to avoid significant retraining of

the neural network and to ensure the approximation of the training sample data which guarantees the accuracy of the data approximation with further training of the network without fixing neurons. The main stage of the neural network training includes the sequential steps of learning and the removal of insignificant weights of the network. At the stages of learning, we use non-smooth regularization [2], which eliminates redundant components of the ANN model and suppresses interference. A computational experiment with real data was performed to compare the approximation quality by the ANN models and logistic models with various types of regularization. The proposed algorithm in combination with nonsmooth regularization allows us to obtain efficient ANN models for classification problems which can solve a wide range of problems. In this research, we performed a series of calculations with various types of regularization to confirm the efficiency of the developed algorithms. The results of experiments were compared with known algorithms.

**Artyom Makovetskii¹, Sergei Voronin¹, Vitaly Kober²,
Aleksei Voronin¹.** *A generalized point-to-point approach for orthogonal transformations.*

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The known Iterative Closest Point (ICP) algorithm often utilize point-to-point or point-to-plane approaches. The point-to-plane ICP algorithm uses an information on points coordinates and normal vectors for aligning of 3D point clouds, whereas point-to-point approach uses an information on points only. This paper proposes a new algorithm for orthogonal registration of point clouds based on the generalized point-to-point ICP algorithm for orthogonal transformations. The algorithm uses the known Horn algorithm and combine information on points and normal vectors.

Igor Masich. *Increasing the informativeness of logical patterns through using pseudo-Boolean optimization algorithms.*

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The question of identifying logical patterns in the data with the aim of constructing logical decision rules to solve the classification problem is considered. The problem of identifying logical patterns is formulated as a conditional pseudo-Boolean optimization problem: search for patterns with the highest coverage, subject to the inadmissibility (or limited admissibility) of covering

training observations of another class. To solve this problem, the greedy algorithm is traditionally used, which, however, does not ensure the finding of optimal patterns. In this paper, it is proposed to use the algorithm developed by the author based on the scheme of the branch and bound method, the principle of searching for the boundary points of a feasible region and greedy heuristics. This algorithm improves the solution obtained by the greedy algorithm and ensures that more informative patterns are found.

Timur Merembayev¹, Yedilkhan Amirgaliyev¹, Shahriar Shamiluulu², Didar Yedilkhan¹. *Using machine learning algorithm for diagnosis of stomach disorders.*

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Medicine is one of the rich source of data, generating and storing massive data, begin from description of clinical symptoms and end by different types of biochemical data and images from devices. Manual search and detecting biomedical patterns is complicate task from massive data. Data mining can improve the process of detecting patterns. Stomach disorders are most common disorders that affect over 60% of human population. In this work, the classification performance of four non - linear supervised learning algorithms i.e. Logit, K - Nearest Neighbour, XGBoost and LightGBM for five types of stomach disorders are compared and discussed. Clinical dataset with over 1000 instances and 24 attributes was used with disease prevalence of 65% in the dataset. The objectives of this research is to find trends of using or improvements of machine learning algorithms for detecting symptoms of stomach disorders, to research problems of using machine learning algorithms for detecting stomach disorders. Results of the research shows algorithms that base on gradient boosting technique (XGBoost, LightGBM) get better accuracy >95% on test dataset. For diagnostic and confirmation of diseases need to improve accuracy, in the article we propose to use optimization methods for accuracy improvement with using machine learning algorithms.

Anastasiya Polyakova, Leonid Lipinskiy, Eugene Semenkin. *Reference Sample Reduction Methods Investigation for Ensemble Output with Fuzzy Logic-Based Systems.*

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One of the main methods in data reduction processes is the instance selection method. Reducing the dataset has two main objectives: reducing the requirements for computing resources, and the time for processing the learning task. Thus, the choice of the correct subset of examples allows reducing the sample size and increasing the efficiency of training. But with regards to regression datasets, this problem is not fully understood, taking into the complexity of this type of dataset. The paper studies the problem of reducing the size of a reference set of points (reference sample) in collective decision making. In the paper, the reference sample refers to the sample that is used in ensemble output using of fuzzy logic system. The fuzzy controller makes a decision about the agent should be used for each point from a test set. The nearest point from the reference sample is determined for any point from a test set. Depending on the distance to the object from a test set and the successfulness of the algorithm on this object, the confidence of the algorithm on this test point is determined. Also, it is proposed to apply the instance selection to select instances for the reference set from the training set when solving regression problems based on such methods as genetic algorithms (GA), the K-Means clustering algorithm, and the random instance selection (RIS). Computational experiments show that effective instance selection in the reference set can significantly reduce the computational costs while maintaining the accuracy of the result.

Guzel Shkaberina, Viktor Orlov, Elena Tovbis, Lev Kazakovtsev.

Identification of the Optimal Set of Informative Features for the Problem of Separating a Mixed Production Batch of Semiconductor Devices for the Space Industry.

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In this paper, we investigate the problem of separation of a mixed production batch of semiconductor devices of space application into homogeneous production batches. The results of the mandatory testing for each item contain a large number of parameters. Many optimization models and algorithms were developed for solving this clustering problem in the most efficient way. However, due to a rather high data dimensionality, such algorithms take significant computational resources. We analyzed methods of reducing the dimensionality of the data set with the use of factor analysis based on Pearson matrix in order to improve the accuracy of the separation. We investigated efficiency of the proposed method for separating a mixed lot of semiconductor devices which consists of two, three, four and seven homogeneous batches, with various methods of selection and rotation of factors. It was shown on real data that with any orthogonal rotation, with an increasing number of homogeneous batches in the sample, the clustering accuracy decreases. Moreover, it was impossible

to identify a universal clustering model with limited a number of factors for dividing a mixed lot composed from an arbitrary number of homogeneous batches. Thus, the use of the multidimensional data was shown to be inevitable.

Al'fiya Surina¹, Alexander Tyrsin². *Risk Management in Gaussian Stochastic Systems as an Optimization Problem.*

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In article the algorithm of risk management in Gaussian stochastic system is describes. The model of risk management represents an optimization problem. The algorithm of risk management is realized on the basis of a method of the penalty functions. The features of this nonlinear programming problem are not the convexity of the accessible solution region and the presence of stochastic restrictions on the required risk. The software implementation of the algorithm in the form of a separate module is performed. Using the Monte Carlo statistical test method, the algorithm was investigated. The algorithm showed stable control. Its efficiency is proved. Results of a research are presented in article. Recommendations on practical application of the algorithm are given.

Anton Ushakov, Igor Vasilyev. *A computational comparison of parallel and distributed k-median clustering algorithms on large-scale image data.*

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Algorithms for solving the well-known k-median problem are one of the most commonly used clustering algorithms. Their main advantage is that they are flexible in choosing dissimilarity measures, thus they can be used with semimetrics. They are also known to be more robust to noise and outliers in comparison with k-means algorithms. In spite of that, they have been of limited use for large-scale clustering due to their high computational and space complexity. This work aims at computational comparison of k-median clustering algorithms in a specific large-scale setting - clustering large image collections. We implement distributed versions of the most common k-median clustering algorithms and compared them with our parallel heuristic algorithm for solving large-scale k-median problem instances. We analyze clustering results with respect to external evaluation measures and run time.

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