The use of optical correlation algorithm to solve phase problem of complex optical fields

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Abstract

We propose an optical correlation algorithm for reconstructing the phase skeleton of complex optical fields from the measured two-dimensional intensity distribution. The essence of the algorithm consists in location of the saddle points of the intensity distribution and connecting such points into nets by the lines of intensity gradient that are closely associated with the equiphase lines of the field. This algorithm provides a new partial solution to the inverse problem in optics commonly referred to as the phase problem.

Keywords: Phase skeleton, saddle points, gradient lines

1. Introduction

Solving the phase problem in optics has attracted much attention, primarily in problems of diagnostics of an object structure within microscopy, pattern recognition, terrestrial telescope, and biomedical optics [1, 2]. In general, the phase problem consists in deriving the spatial phase distribution for complex fields, including speckle fields from a measured intensity distribution. On the whole, the inverse problem has no universal solution. A novel promising approach in solving the phase problem arises from the concept of singular optics [3]. To be precise, the following propositions are assumed as a basis for this approach: (i) amplitude zeroes (also named optical vortices, or wave front dislocations, or phase singularities) are the ‘reference’, structure-forming elements, whose set constitutes a singular skeleton of a field; (ii) spatially distributed amplitude zeroes obey the specific sign principle governing the characteristics (signs) of adjacent zeroes; (iii) the spatial distributions of intensity and phase in complex fields are interconnected. The aim of this paper is to substantiate – proceeding from simple intuitive suppositions – the algorithm for reconstruction of the spatial phase distribution from a measured spatial intensity distribution of a complex (speckle) field. We will represent the initial results of computer simulation carried out by us within the scalar approximation, viz. in the assumption of homogeneously polarized field. The proposed algorithm includes the following three actions: (i) bicubic spline interpolation [4]; (ii) location of the saddle points of intensity; (iii) connecting the saddle points of intensity by the gradient lines.

2. Location of the saddle points of intensity [5]

A saddle point is the point from the function domain that is stationary not being a local extremum. Derivative of the function equals zero at this point in the transverse directions. That is why, primary analysis consists in choosing all points where the two derivatives simultaneously equal zero. As maxima and minima of the function obey this condition, one must determine the criterion for selecting just the saddle points. The algorithm implemented by us is illustrated in Fig. 1. For pass tracing a saddle point of intensity (blue point) one goes by turns through the points of alternate minima and maxima shown in Fig. 1 by green and red points, respectively. Therefore, the magnitudes of maxima (minima) are always larger (less) than the magnitude of a function at the saddle point. Thus, one initially finds out the
stationary points of a field, where $dI/dx=0$ and $dI/dy=0$. Then, if passing a stationary point one meets alternating minima and maxima (with magnitudes larger and smaller than at the specified stationary point), this point is identified as a saddle point.

![Fig. 1. Saddle point of intensity (blue) with two maxima and two minima (red and green, respectively) in its vicinity. Green lines are the iso-intensity lines.](image)

3. Connecting saddle points by intensity gradient lines

As a rule, viz. with probability 95%-98%, the saddle points of intensity are located within areas of rapid changing phase [3]. In turn, the gradient lines of intensity going from the saddle points correspond to equiphase lines with the same probability. For this reason, the saddle points of intensity are just chosen as ‘structure-forming’, steady points from which the gradient lines of intensity are reconstructed for phase mapping of complex, spatially inhomogeneous optical fields.

From a saddle point of intensity, S, one draws the line connecting the points with the maximal numerical gradient directed towards the specified saddle point. At the phase map, this line corresponds to the area of the smoothest spatial change of the phase. Passing the saddle point of intensity, one always meets two minima, which means that the each saddle point is the origin of two gradient lines. These lines approach the spatial area with increasing intensity. As a rule, such gradient lines for the intensity in complex fields intersect and eventually form a spatial net.

We simulate a speckle field with specified phase distribution (within the far-field diffraction approximation). Reconstructing a pair of the intensity gradient lines (as an analogue to the equiphase lines for a field) originating from a saddle point, we compute the phase difference at the saddle point and at each point of the gradient line. Then, for the set of found magnitudes of phase difference, we establish histograms and estimate the confidence interval for deviation of a phase magnitude at the each point of the gradient line from the phase at the initial saddle point. Analysis has been performed for two different speckle field distributions. The histograms for the corresponding phase distributions are shown in Fig. 2.

For versatile proving the proposed approach, we have performed simulation for several groups of objects including both random and quasi-deterministic sets of point coherent sources.
Distribution of phase deviation (a) along the phase gradient line from a phase at the initial saddle point for the each line (b). Mean: $-10.31°$, confidence interval with probability 95%: $-12.03°$ -> $-8.60°$.

Distribution of phase deviation (c) along the phase gradient line from a phase at the initial saddle point for the each line (d). Mean: $-3.39°$, confidence interval with probability 95%: $-4.82°$ -> $-1.95°$.

Fig. 2. Histograms for phase distribution of the analyzed speckle field.

Analysis of the represented dependences shows that the width of the confidence interval does not exceed $4°$, i.e. the reconstructed gradient lines of intensity are really the equiphase lines of a field, within the accepted accuracy.
Let us shortly discuss the sources of errors in this simulation. At first, both in registering speckle pattern using a CCD camera (as in this study) and in attempts to compensate for the effect of atmospheric turbulence in terrestrial telescope by means of adaptive optics, one meets the serious (and universal) problem of digitization of ‘rough’ (intrinsically analogue) spatial distributions of intensity and phase. As a matter of fact, the edges and corners of cells in both cases inevitably produce additional phase singularities, whose number may be commensurable or even exceed the number of ‘true’ singularities intrinsic to the field per se. This circumstance can drastically distort the processed data and the result of processing. Thus, one must search for proper procedures for data smoothing, based either on bicubic spline interpolation [4] used by us, or on Gaussian smoothing [6]. Secondly, available means for registration of complex intensity distributions (such as CCD cameras) are characterized by strong nonlinearities, especially within the regions of vanishing intensity (amplitude zeroes). That is why, at least now, data proceedings based on CCD registration cannot be considered reliable enough.

4. Conclusion

We have introduced an optical correlation algorithm for reconstructing the phase skeleton of a complex optical field from a measured two-dimensional intensity distribution. The algorithm consists in location of the saddle points for the intensity distribution and subsequently connecting these points into nets by the lines of intensity gradient, which are closely related to the equiphase lines of the field. It has been demonstrated that the set of the saddle points of intensity and the intensity gradient lines facilitate the reconstruction of the phase skeleton of a complex scalar (homogeneously polarized) coherent optical field. The proposed algorithm provides a new partial solution to the phase problem in optics. Significance of this result follows from the possible use of complex two-dimensional optical fields within optical telecommunications. Actually, the proposed algorithm can be implemented for any polarization projection selected at the output of a detector even for inhomogeneously polarized field.

References