Multipartite entangled quantum states in coupled optical parametric interactions: density matrix and entropy characterization

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Abstract—We focus on the so called normal ordering method (NOM) for constructing the density matrix of multipartite entangled CV states generated via coupled parametric interactions in running wave regime. The method allows us to construct a simple way of getting the state vector and density matrix for multipartite field in parametric optical interactions. We demonstrate application of the NOM by calculation of the density matrix for three coupled parametric processes in which the four partite entangled state is formed. We analyze von Neuman entropy, conditional entropy and mutual information for this interaction.

Keywords—quantum optics; coupled parametric interactions; normal ordering method; entropy

Coupled optical parametric interactions are of interest as method of obtaining multipartite quantum entanglement. Application of multipartite entangled states in quantum information and quantum computations have a number of advantages compared with two-partite entangled states. Therefore, multipartite entangled quantum states are recently given considerable attention.

We consider formation of frequency and spatial multimode entangled light fields in inhomogeneous nonlinear photonic structures. Interactions under study include coupled optical parametric down- and up-conversions. When three or more coupled parametric processes are involved in the interaction of entanglement formation, one can produce four-partite entangled continuous-variable states and entangled continuous-variable states of higher order [1].

We focus on the process including one down-conversion followed by two up-conversions. Thus four-partite entangled states are formed. In order to find the density matrix for this multipartite state the so called normal ordering method is applied. The normal ordering method allows us to construct a simple way of getting the state vectors for multipartite field in parametric optical interactions. Moreover, for special classes of the method can be used for any number of the coupled parametric processes [2].

For the interaction under study we have obtained the following explicit formula for the system state:

$$|\psi> = C \sum_{m,n,k,l=0} Q_{12}^{m} Q_{34}^{n} Q_{13}^{k} Q_{24}^{l} \sqrt{C_{m+n}^{i} C_{n+k}^{j} C_{n+l}^{k}} |m + k>_1 |m + l>_2 |n + k>_3 |n + k>_4$$

The vector $|\psi>$ corresponds to the initial vacuum state, and $Q_{pq}$ is the matrix element determined by elements of the canonical transformation matrix. Subscript refers to the number of mode, integers represent the number of photons. The vector obtained allows one to calculate any statistical characteristics of the quantum state.

We analyse von Neuman entropy, conditional entropy and mutual information of fourpartite field. The sign of conditional entropy can serve as an indicator of mode entanglement. The state generated in our coupled parametric processes is Gaussian one. Since the reduced density matrix of pure Gaussian state is also Gaussian, we apply the formula of entropy for Gaussian states which is defined by the symplectic spectrum of the covariance matrix of the Gaussian state. The covariance matrix is explicitly expressed in terms of canonical transformation one.

This work was partially supported by the Russian Foundation for Basic Research under grant No. 14-02-00458.

REFERENCES