Spontaneous and Stimulated Transitions in the Atoms Embedded into Hyperbolic Metamaterials

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Abstract—Atom-field interaction models including density matrix description and polarization issues are presented. The peculiarities of eigenmodes of different metamaterials with hyperbolic or hyperbolic-like dispersion are discussed. Numerical results for populations and medium polarization dynamics are presented.

Keywords—density matrix; hyperbolic dispersion; transitions; polarization; dynamics

I. HYPERBOLIC METAMATERIALS

Recently, the investigation of atom-field interaction in so-called hyperbolic metamaterials (HM), which have the hyperbolic dispersion [1], where one of the principal values of susceptibility tensor have opposite sign with respect to others, become very popular (see e.g. [2-6]) due to peculiarities of spontaneous and thermal emission, wave propagation phenomena and other phenomena. These features originate from indefinite number of photonic density of eigenstates (DOS) along definite direction in these media. Most common examples of HM at optical wavelength are metamaterials made from ordered metal nanowires and 1D periodic graphene-dielectric layered medium. We are discussing the origin of hyperbolicity in these materials, utilizing the isofrequency surfaces and periodicity in k-space as the first approximation for calculated isofrequencies. We also discuss the approaches for HM design, the dispersion characteristics of HM containing ordinary anisotropic medium, and calculation of amplitudes of reflected and transmitted plane waves at HM interface. Besides, the effects of gain in embedding medium and losses in wires or graphene sheets are discussed.

II. SPONTANEOUS AND STIMULATED TRANSITIONS IN HYPERBOLIC MEDIUM

Due to high DOS in HM for extraordinary waves spontaneous emission from excited atomic levels are strongly enhanced. We have demonstrated the dynamics of spontaneous transition in two-level system using Wigner-Weisskopf approach [7] and numerical solution of the equations for probability amplitudes. Assuming that the laser field exists simultaneously with vacuum field corresponding equations for density matrix elements are derived and solved. Two cases are considered: laser excitation occurs between two exited levels and one of the levels is ground.

III. POLARIZATION AND POPULATION DYNAMICS

To treat more realistic atomic model it was assumed that the levels are degenerate according to magnetic quantum numbers. Corresponding equations for density matrix elements are written in so-called κ-μ representation [8], where κ is the rank of the polarization moment and μ is the index, which numbered its components. The relaxation terms and terms describing interaction with polarized laser field are derived. Due to strong anisotropy of spontaneous emission to extraordinary waves of HM, there exists a coupling of different polarization moments of the levels. This gives an alignment even in the absence of external field, which characterize the direction of emitting dipole or quadruple. This alignment can strongly change the character of interaction of the atom embedded in HM with polarized external field. The results of numerical solutions for different cases of transitions: j=1/2 ↔ j=3/2 (H), j=1 ↔ j=2 (Ne), j=11/2 ↔ j=13/2, (Er3+) etc. are presented