Vortex Mode Soliton Propagation in Graded-Index Optical Fiber with Longitudinal Inhomogeneity

Vortex vs azimuthal modes comparative analysis

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Abstract—Propagation of a vortex mode of a short pulse in the graded-index optical fiber is studied with an analytical technique taking into account either nonlinearity of the propagation process and longitudinal inhomogeneity of the fiber. Quantitative estimates are obtained for differences in the soliton envelope dynamics of vortex and azimuthal modes.

Keywords—optical soliton; graded-index optical fiber; vortex mode; longitudinal inhomogeneity

I. INTRODUCTION

Self-focusing and self-defocusing of large-power beams of continuous radiation appear to be a well-known manifestation of nonlinearity in stationary wave processes. At first this phenomenon had been attributed to the beams in which the wave field distribution was smooth and had no singularity, however discovered afterwards were self-guided gyratory optical beams [1]. These have temporal analogues – the vortex modes – arising in dielectric waveguides and being distinguished from ordinary azimuthal modes [2]. At now a technique of effective excitation of vortex modes in optical fibers has been elaborated [3] and applications to fiber optical communication line mode multiplexing were discussed [4].

The present paper is aimed at analytical description of the nonlinear regime of propagation of a short pulse in the form of the first-order vortex mode, and comparison of its envelope dynamics with that [5] for other mode types.

II. AZIMUTHAL AND VORTEX MODES IN GRADED-INDEX OPTICAL FIBERS

The nonlinear wave equation modeling the pulse propagation is solved by means of an asymptotic approach using a small parameter of the order of magnitude of the wave field. The principal approximation to the complex amplitude \( F \), as well as the propagation constant \( r(s) \), result from equation

\[
\frac{\partial^2 F}{\partial s^2} + \frac{1}{\rho} \frac{\partial F}{\partial \rho} + (\beta^2(\rho, s) - r^2(s))F + \frac{1}{\rho^2} \frac{\partial^2 F}{\partial \rho^2} = 0
\]  

(1)

\( \rho \) radial, \( \varphi \) azimuthal coordinates in the fiber cross-section, \( s \) longitudinal coordinate, \( \beta^2(\rho, s) \) the fiber refractive index. The solution of (1) must be bounded for \( \rho \to 0 \), vanish for \( \rho \to \infty \), and be \( 2\pi \)-periodic with respect to \( \varphi \).

The azimuthal dependence can be described with either \((\cos m \varphi)\) or \((\exp \pm im \varphi)\) functions. The real functions imply a stationary distribution of the wave field in the fiber cross-section, just as the complex exponentials correspond to a vortex mode with a screwed wave front. The envelope \( U(\theta, s, \rho) \) the phase of envelope, obeys a nonlinear Schrödinger equation

\[
2i r(s) \frac{\partial U}{\partial s} + g(s) \frac{\partial^2 U}{\partial \rho^2} + i r'(s) + h(s) |U|^2 U = 0
\]  

(2)

with coefficients \( g(s) \) and \( h(s) \) depending on type and index of the mode. They can be expressed explicitly in a special case of quadratic dependence of \( \beta \) on the radial coordinate \( \rho \), thus providing analytical evaluations of differences in dynamics of azimuthal and vortex modes.

III. MAIN RESULTS

The asymptotic approach presented in this paper enables an analytical description of soliton envelopes of either radial and vortex modes. Equation (2) characterizing the envelope shape has a soliton solution of the \( \exp(i \varphi) \) \( \cosh(\theta - \Delta \theta) \) type, despite of the variable coefficients [5]. Proceeding from the expression for the envelope phase via longitudinal coordinate and time, one obtains that the relative discrepancy in speed of propagation of basic and first-order vortex mode turns out a magnitude on the order of \( 10^{-1} \). This deviation should be compensated in mode multiplexing in long-trunk fiber optical communication lines.

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REFERENCES