Fermi-Ulam Problem for Classical and Quantum Particles in Dynamical Traps

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Abstract—We analyze and compare regular and chaotic dynamics of classical and quantum particles in traps with periodically oscillating walls. For classical particles, effect of collisions’ inelasticity is taken into account. For quantum particles, quasienergy states are found and studied; in the resonance (two-level) approximation the Rabi oscillations of levels’ populations are demonstrated.

Keywords—Fermi-Ulam problem; dynamical traps; Rabi oscillations

I. Introduction

The problem of Fermi [1] with stochastic acceleration of particles colliding with moving bodies and that of Ulam [2] with particles bouncing between motionless and oscillating walls are of great importance for various fields of physics [3]. As for current traps for single particles using for high-precision measurements, important is that in dynamical traps – that with oscillating walls – it is possible simultaneously to localize, excite and manipulate the particles. In this talk, we analyze and compare the dynamics of classical and quantum particles in one-dimensional traps with weak modulation of their walls’ position. For such light objects as atoms, the walls can be organized as optically induced movable mirrors [4].

II. Classical Particles

Let us consider a classical point particle moving along the axis z between motionless (z = 0) and oscillating (z = L0 (1 + μ cos Ωt)) walls, where L0 is the average length of the trap, μ and Ω are the modulation depth and frequency, and t is time; μ > 1. For non-elastic collision of a particle with moving wall at the moment t, the velocities of the incident v_i and reflected v_r particle are connected by the relation: v_r = w(t) = q (v_i − w(t)) where w = dz/dt and q < 1 is the inelasticity parameter; for ideal elastic collisions, q = 1.

Depending on the problem parameters and on the initial particle position and velocity, there are regular regimes with periodic or quasi-periodic variation of particle velocity, and regimes of dynamical chaos. Though there are energy losses during inelastic collisions, they can be compensated by particle accelerations due to the same collisions. Correspondingly, there are stable periodic regimes both for elastic and inelastic collisions. The effect of inelasticity is in the asymptotic type of regular regimes’ stability, whereas their stability is only neutral for the case of elastic collisions. Usually, the dynamics is more regular for inelastic case, but nevertheless even then there are chaotic regimes also.

III. Quantum Particles

The specific feature of quantum particles in a trap is the discreteness of their energetic spectrum and its strong non-equidistance in the considered case [5]. Therefore, additionally to oscillations of particle wavepacket similar to the classical variant, it is possible to excite some levels selectively for the proper choice of the modulation frequency Ω. In this case, the simplified resonance, or two-level model describes the system with high precision. The Rabi oscillations with deep modulation of the resonance levels population follow from this model. Beyond the framework of the resonance approximation, an additional higher-frequency modulation arises connected with higher-order resonances, similar to multi-photon transitions. It gives the possibility to excite selectively different levels of quantum objects.

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