SOLITON REFLECTION AND TRANSMISSION IN RANDOMIZED PHOTONIC LATTICES

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Soliton propagation in quasi-periodic and random refractive index landscapes and formation of stable stationary excitations has been extensively discussed (see [1] for review). The disorder may completely change the evolution of light; it results in the formation of spatially localized states even in linear randomized lattices. In nonlinear regime, the randomness may result in trapping of strongly localized walking solitons or vice versa, it may facilitate the transport of such states in quasi-periodic lattice. The disorder leads to such intriguing effects as Brownian soliton motion and percolation. While most of previous studies address the effect of randomness on light evolution in infinite periodic systems, the effects that may become available in finite disordered systems are not studied yet.

We consider soliton transmission and reflection by disordered nonlinear photonic lattices of finite width embedded into uniform cubic medium. Transmission of such lattices can be handled by “sewing” solutions in the uniform medium and lattice, using continuity conditions. In linear case, the waves in uniform medium should be matched with eigenfunctions of finite lattice, which in the absence of randomness have much in common with truncated Bloch waves. Randomization drastically affects the shape and localization of lattice eigenfunctions, consequently one should expect considerable modifications in transmission and reflection induced by randomization. Notice that backscattering experiments were already applied to detect Anderson localization in strongly scattering samples such as semiconductor powders or macroporous semiconductor networks (see [1] for review).

We use the nonlinear Schrödinger equation describing the interactions of one-dimensional light beams with randomized finite photonic lattice embedded into uniform cubic medium:

\[
\frac{\partial q}{\partial \xi} = \frac{1}{2} \frac{\partial^2 q}{\partial \eta^2} - p Q(\eta) q - |q|^2 q,
\]

where $\eta$ and $\xi$ are the normalized transverse and longitudinal coordinates, respectively; the parameter $p$ characterizes linear refractive index modulation depth; the lattice shape is described by the function:

\[
Q(\eta) = \sum_{m=-(N-1)/2}^{(N-1)/2} G(\eta - \eta_m), \quad G(\eta) = \exp(-\eta^6 / a^6),
\]

with waveguides having super-Gaussian profiles $G(\eta) = \exp(-\eta^6 / a^6)$ of the width $a$. The coordinates $\eta_m$ of waveguide centers are randomized such that $\eta_m = md + \epsilon_m$, where $d$ is the spacing in regular case, while $\epsilon_m$ stands for the random shift, that is uniformly distributed within the segment $[-S_d, S_d]$. This array with number of guides $N$ is embedded into uniform cubic medium. Further we set $p=11$, $a=0.3$, and $d=1.6$ that corresponds to refractive index contrast $\delta n \approx 10^{-3}$ at the wavelength $\lambda = 0.8 \mu m$, waveguide width of 3 $\mu m$ and mean spacing of 16 $\mu m$.

We start our consideration from the preliminary qualitative analysis where waveguides may be considered as scatterers characterized by complex amplitude reflection coefficients $\rho_l$. Then using Born approximation of weak scatterers one can show that the net amplitude reflection coefficient for the plane wave $\exp(i\alpha \eta)$ interacting with an array of $N$ guides can be estimated as $\rho_N = \rho_l \sum_{m=1}^{N} \exp(-i \varphi_m)$, where $\varphi_m$ is the phase difference for waves backscattered by first and $m$-th waveguides. In regular array where $\varphi_m = 2md\alpha$ the energy reflection coefficient of the plane wave is therefore given by $r_n = |\rho_l|^2 = \eta \sin^2(Nmd) / \sin^2(\alpha d)$, where $\eta = |\rho_l|^2$. If the Bragg condition $\alpha \cdot d = \pi M$ ($M$ is a positive integer) is accomplished, one gets considerable reflection with $r_N \sim N^2$. In contrast, in a disordered array the statistical mean value of energy reflection coefficient can be estimated as:

\[
\langle r_N \rangle = nN + n^2 \sum_{m=1}^{N} \langle \exp[i(\varphi_l - \varphi_m)] \rangle,
\]

where the angular brackets stand for statistical averaging and $\varphi_l$, $\varphi_m$ are random phases. In particular, in strong disorder limit, when $\varphi_l - \varphi_m$ fluctuates between $-\pi$ and $\pi$ for different array realizations, the last term in the above expression vanishes and mean reflection coefficient $\langle r_N \rangle \approx nN$ decreases substantially. This simple approach, which explains the effect of disorder on reflection of plane waves, is not so accurate in the case of finite-width beams walking across the array, since only scatterers covered by the beam are involved into the reflection process, multiple reflections (ignored in the Born approximation) and boundary effects enter the scene.
Focusing nonlinearity further complicates the picture, since it causes beam reshaping upon propagation. Therefore, further we use direct integration of Schrödinger equation to study beam reflection from disordered lattices.

In simulations, we use as an input the soliton beam \( \phi(\xi) = \chi \text{sech}(\chi(n - n_0)) \exp[i \alpha(n - n_0)] \), having form-factor \( \chi \) and launched into uniform nonlinear medium, \( \alpha \) stands for the incidence angle. We focus on calculation of statistically averaged (over \( 10^3 \) realizations) reflection \( R = \langle U_{\text{red}} \rangle / U_{\text{in}} \) and transmission \( T = \langle U_{\text{tr}} \rangle / U_{\text{in}} \) coefficients where \( U_{\text{in}} = 2\chi \) in the input soliton energy flow. The level of disorder was controlled by the \( S_d \) parameter. In regular lattice, the transmission and reflection coefficients for broad low-power solitons depend dramatically on the incidence angle \( \alpha \). While the total reflection from regular lattice occurs at certain angle \( \alpha = 5.26 \) [Fig. 1(a)], moderate detuning of \( \alpha \) from this value results in almost complete soliton transmission. The addition of even weak disorder \( (S_d = 0.3) \) drastically increases mean transmission coefficient \( T \) of the reflecting lattice. Interestingly, stronger randomization does not necessarily mean larger transmission: Fig. 1(c) illustrates that further growth of disorder up to \( S_d = 0.42 \) unexpectedly increases average reflection on the lattice. With increase of the number of waveguides \( n \), the reflection becomes stronger and the width of statistically averaged reflected beam grows. Reflected beam is asymmetric, and it may have long exponentially decaying tail due to multiple reflections. Randomization can also results in an opposite effect: it suppresses complete soliton transmission, for instance, at \( \alpha = 4.80 \).

![Fig. 1](image1.png)

Averaged reflection dynamics of a broad soliton \( (\chi = 0.05) \) from disordered waveguide arrays at \( \alpha = 5.26, n = 14 \) and \( S_d = 0.00 \) (a); 0.30 (b); 0.42 (c).

Statistical aspects of soliton transmission are addressed in Fig. 2. For incident angles corresponding to reflection (curves 1), the statistically averaged reflection coefficient \( R \) first diminishes remarkably, indicating on disorder-induced transparency, but then starts performing decaying oscillations, gradually approaching certain limiting value. In contrast, for \( \alpha \) values detuned from the resonant angle (curves 2), one observes almost monotonic growth of reflection coefficient with increase of \( S_d \) that indicates on disorder-induced reflection.Remarkably, it appears that irrespectively of \( \alpha \) value, with growth of \( S_d \) the reflection coefficient \( R \) approaches almost the same limiting value, which increases with the number of waveguides \( n \) in finite lattice (thus, for \( N = 15 \) this value is close to 0.3 , while for \( N = 31 \) it is close to 0.5 ). This suggests that largest relative modifications of reflection are available in arrays with small \( N \), while largest relative modifications of transmission are possible in arrays with large \( N \). The dependencies of \( R,T \) coefficients on \( N \) in disordered lattice are presented in Fig. 2(c). Summarizing, disorder-induced soliton transmission or reflection reported here provides an example of nontrivial interplay of randomness and nonlinearity. Our results may find applications in diagnostics of disordered periodic lattices and implementation of energy-selective soliton mirrors.

![Fig. 2](image2.png)

(a) Reflection coefficient versus disorder level for \( n = 15 \) at \( \alpha = 5.26 \) (curve 1) and \( \alpha = 4.80 \) (2); (b) the same for \( n = 31 \); (c) reflection and transmission versus \( n \) at \( \alpha = 5.26, S_d = 0.30 \). In all cases \( \chi = 0.05 \).

References: