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DAYS ON DIFFRACTION 2021

ABSTRACTS

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FOREWORD

“Days on Diffraction” is an annual conference taking place in May–June in St. Petersburg since 1968. The present event is organized by St. Petersburg Department of the Steklov Mathematical Institute, St. Petersburg State University, and the Euler International Mathematical Institute.

The conference is supported by a grant from the Government of the Russian Federation, agreement № 075-15-2019-1620, and by Simons Foundation.

The abstracts of 108 talks, presented during 5 days of the conference, form the contents of this booklet. The author index is located on the last pages.

Full-length texts of selected talks will be published in the Conference Proceedings. Format file and instructions can be found at `http://www.pdmi.ras.ru/~dd/proceedings.php`. The final judgement on accepting the paper for the Proceedings will be made by editorial board after peer reviewing.

Organizing Committee

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In memoriam of
Alexander Yakovlevich Kazakov
1954–2020

Alexander Ya. Kazakov — a scientist of high international reputation — unexpectedly passed away on December 23, 2020, aged 66. Since 2015 he was a member of the Organizing Committee of the annual International conference 'Days on Diffraction' and a co-editor of its Proceedings. A. Ya. Kazakov, D.Sc., was a department head at the St. Petersburg State University of Industrial Technologies and Design and a Professor at the St. Petersburg State University of Aerospace Instrumentation.

He was a versatile researcher, whose contribution to various areas of mathematics and theoretical physics was significant, in particular, to plasma physics, optical signal processing, studies of fields near non-stationary caustics and wave propagation near boundaries with inflection points. His level of expertise was invaluable in discussions, reviewing countless publications and theses — it would be just a slight exaggeration to say that he was an opponent and referee everywhere. Special attention should be paid to his research concerning the Heun equation — a memorial session on this topic is included in the programme of this conference.

A. Ya. Kazakov will be remembered as a good friend, a gentleman with a keen sense of humor, a scientist with broad erudition and lively curiosity. It was a unique privilege to be colleagues and friends of such a wonderful person as Alexander Yakovlevich. Be remembered forever!

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Wave propagation in and by thin plates and waveguides containing inhomogeneities

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This talk is motivated by wave propagation in a duct containing an inhomogeneity or flexural wave scattering by a crack in a thin elastic plate. The former problem can be cast into a matrix Wiener–Hopf equation of a particular class, and for illustration we present two other model problems which give rise to equations of this same class. In the general case, we have not been able to obtain an exact factorization of the original Wiener–Hopf kernel, so instead reduce the equation, via pole removal, to an infinite linear algebraic system of equations. This generically has very slow convergence, but here we offer a novel approach to obtaining an accurate and very rapid solution via input of direct physical information on corner or other singular behaviour.

Vortex-generation in the system of multihelical and twisted anisotropic optical fibers

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To date, optical vortices (OVs) [1] carring orbital angular momentum (OAM) [2] due to their unique properties are widely used in different fields. In particular, one can example tweezers [3], information technologies [4], quantum optics [5], etc. The range of OVs application requires efficient methods of their generation. It is often convenient to use the methods of excitation of OVs directly in waveguides, since such a fiber vortex can be further coupled to another fiber more efficiently than a one created in free space.

Previous studies of the light propagation in twisted anisotropic and multihelical optical fibers have shown that they can be used as a medium for the transmission and controlling of OVs [6, 7]. This is the reason of their great potential in the field of information, communications and quantum technologies in which OVs are associated as carriers of information.

Since twisted anisotropic and multihelical fibers are happened to be possessed of the properties that make them friendly medium for the vortex-used technologies, we assume that the systems formed by these types of fibers may also have promising capabilities. In this work we study the light propagation in the system of connected one after the other multihelical and twisted anisotropic optical fibers. We demonstrate the generation of linearly polarised OV with nonzero OAM from an input EH-mode in such a system. At that, the even EH-mode is transformed to the x-polarised OV, and from the odd EH-mode the y-polarised OV is generated. We show the corresponding evolution of OAM and spin angular momentum of an input field. Parameters of twisted anisotropic and multihelical fibers are obtained for a numerical example of the established field transformation in the system.

This work was financially supported by the Russian Foundation for Basic Research (Project No. 20-47-910001).

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Asymptotic solutions of a system of gas dynamics with low viscosity that describe smoothed discontinuities

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We construct formal asymptotic solutions describing smoothed shock waves and tangential and weak discontinuities for the nonlinear system of gas dynamics of a fluid with small viscosity. We show that the profile of the smoothed shock wave is described by the ordinary differential equation while for th tengential discontinuity the profile is described by the evolutionary system of equations on a moving surface. The results are published in [1].

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Efficient asymptotic solution for linearized one-dimensional run-up problem with dispersion

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We consider a Cauchy problem with localized initial data for a one-dimensional pseudo-differential equation describing a run-up on a shore of water waves with dispersion. We present asymptotic formulas for the solution both before and after the moment in time, when the wavefront collides with the shore. The formulas are efficient from the practical viewpoint, and appeal only to trajectories of the corresponding Hamiltonian system. In the regime before the collision, the solution is described with help of Airy function. After the collision, the solution is a sum of incoming and reflected waves, where the former is given in terms of WKB asymptotics, and the latter in terms of Airy function. In a neighborhood of the shore, the sum of two waves admits an asymptotics in terms of Bessel functions.

The work is supported by the Russian Science Foundation (Project No. 16-11-10282).

Parametrization of phase space of Painlevé V equation

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All Painlevé equations can be considered as Hamiltonian systems. Their phase spaces are some algebraic symplectic manifolds. We consider the simplest Painlevé equation corresponding of the isomonodromic deformation of the differential system with irregular singularity. The presented theory explains the presence of the symplectic structure and gives a method of the canonical parametrization of the phase space.

Töplitz matrices in the BC-method

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The dynamical system of the form

$$\begin{aligned} & u_{tt} - \Delta u - \langle \nabla \ln \rho, \nabla u \rangle = 0 && \text{in} \quad \mathbb{R}^2_+ \times (0, T), \\ & u|_{t=0} = u_t|_{t=0} = 0 && \text{in} \quad \overline{\mathbb{R}^2_+}, \\ & u_y|_{y=0} = f && \text{for} \quad 0 \leqslant t \leqslant T, \end{aligned}$$

is considered in $\mathbb{R}^2_+ := \{(x, y) \in \mathbb{R}^2 \mid y > 0\}$, where $\rho = \rho(x, y) > 0$ is smooth, $f = f(x, t)$ is a Neumann boundary control, $u = u^f(x, y, t)$ is a solution (wave). The control operator of the system acts from $L_2(\mathbb{R} \times [0, T])$ to $L_2(\mathbb{R}^2_+)$ by $f \overset{W^T}{\mapsto} u^f(\cdot, \cdot, T)$. The response operator acts in $L_2(\mathbb{R} \times [0, T])$ by $f \overset{R^T}{\mapsto} u^f|_{y=0}$. The connecting operator $C^T = (W^T)^* W^T$ acts in $L_2(\mathbb{R} \times [0, T])$ and is expressed via R^{2T} in a simple explicit form well known in the Boundary Control Method.

Let R^{2T}_σ be the response operator given on controls supported in $\sigma \times [0, T]$, where $\sigma \subset \mathbb{R}$ is a finite interval. The *inverse problem* is to determine $\rho\big|_{x\in\sigma,\, 0\leqslant y\leqslant T}$ from the given R^{2T}_σ. To solve it numerically, one needs to invert the Gram matrix $G_{ij} = (C^T f_i, f_j)_{L_2(\mathbb{R}\times[0,T])}$ for a rich enough set of controls $f_1, \dots, f_N$ (with $N \sim 100$). This matrix is positive but strongly ill posed.

In the talk, we propose a way to reduce the inversion of G_{ij} to inversion of a matrix $\tilde{G}_{ij}$, which is simply determined by G_{ij} and has a block-Töplitz structure. This significantly reduces the amount of computation.

Some results on Electric Impedance Tomography of surfaces

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Let (M, g) be a smooth compact two-dimensional Riemannian manifold (*surface*) with a smooth metric tensor g and smooth boundary Γ. Its *DN-map* $\Lambda : C^\infty(\Gamma) \to C^\infty(\Gamma)$ is associated with the (forward) elliptic problem $\Delta_g u = 0$ in $M \setminus \Gamma$, $u = f$ on Γ, and acts by $\Lambda f := \partial_\nu u^f$ on Γ, where Δ_g is the Beltrami–Laplace operator, $u = u^f(x)$ the solution, ν the outward normal to Γ. The corresponding *inverse problem* (EIT-problem) is to determine the surface (M, g) from its DN-map Λ.

An algebraic version of the Boundary Control method (Belishev, 2003) is developed:

1) the version is extended to the case of *nonorientable* surfaces and a criterion of orientability (in terms of Λ) is obtained;

2) a *characteristic description* of Λ that provides the necessary and sufficient conditions for solvability of the inverse problem for orientable surfaces, is given;

3) a procedure that determines surfaces with (unknown) *internal holes*, is proposed.

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Rayleigh–Bloch waves above the first cut-off and higher-order resonant loads on straight-line cylinder arrays

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Linear theory predicts incident waves impose resonant loads on regularly spaced, straight-line arrays of surface piercing circular cylinders [1]. The lowest-order resonance has been shown to be caused by resonant excitation of so-called Rayleigh–Bloch waves (array bound modes) [2]. We use a transfer-operator method to investigate Rayleigh–Bloch waves above the first cut-off (i.e. above the lowest-order resonance), and show the connection with higher-order resonances.

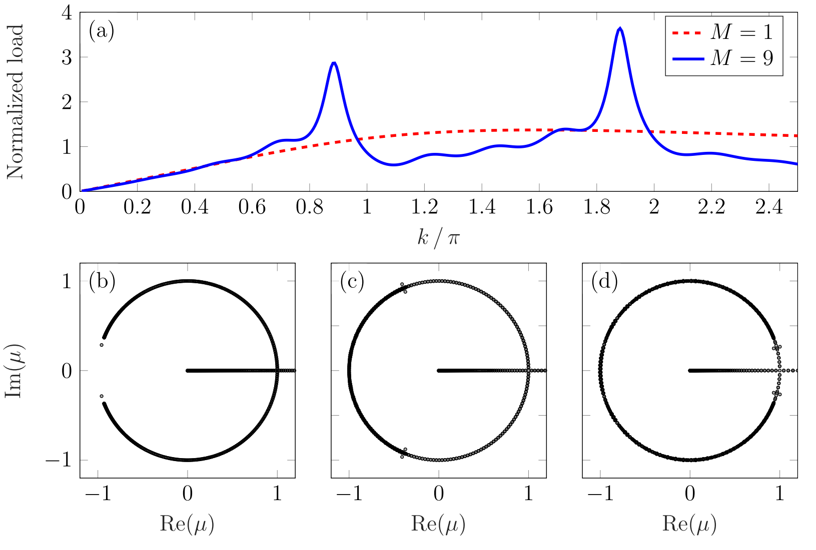

Fig. 1: (a) Normalized load at middle of M-cylinder array vs. wavenumber k, for cylinder radius $a = 0.2$ and centre-to-centre spacing $W = 1$, showing primary and secondary resonances around $k\,/\,\pi = 0.88$ and 1.88, respectively. (b) Transfer-operator eigenvalues, μ, in complex plane for $k\,/\,\pi = 0.88$, showing continuous spectrum (closely spaced dots) and discrete spectrum (connected with Rayleigh–Bloch waves). (c) Eigenvalues above cut-off at $k\,/\,\pi = 1.38$, where discrete spectrum has four elements and moved off unit circle. (d) Eigenvalues at $k\,/\,\pi = 1.88$.

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The brilliant Raman stumbles

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Raman argued that in a continuously varying layered medium, such as air above a hot road, a ray that bends so as to become horizontal must remain so, implying that the reflection familiar in the mirage cannot be explained by geometrical optics. This is a mistake (repeatedly made and corrected since 1799), as standard ray curvature arguments demonstrate. But a simple limiting process, in which the smoothly varying refractive index is approximated by a stack of thin discrete layers, is not quite straightforward because it involves a curious singularity, related to the level ray envisaged by Raman. In contrast to individual rays, families of rays possess caustic (focal) singularities. These can be calculated explicitly for two families of rays that are relevant to the mirage. Only exceptionally does the locus of reflection (lowest points on the rays) coincide with the caustics. Caustics correspond to the 'vanishing line', representing the limiting height of objects that can be seen by reflection. For these two families, the waves that decorating mirage caustics are described by the universal Airy functions Ai and Bi, and can be calculated exactly.

A Python implementation of the general Heun function using elementary integral series

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The Heun equation is known since the end of the 19th century [1, 2] but its importance in physics was recognised only recently, especially after the early 2000s when Heun functions started to be used as solutions of many physics problems [3]. As a consequence, the commercial computer algebra systems Maple and Mathematica have produced in-built functions to evaluate Heun-type functions, while freely available codes working on Matlab/GNU Octave are now also available. These codes enable the numerical evaluation of general and singly confluent Heun functions [4, 5] and all ultimately rely on series expansions and analytic continuation.

Using the Python language and its common modules NumPy and SciPy for numerical analysis, we present a computational implementation of the general Heun function which relies instead on the unconditionally convergent elementary integral series representation of Heun functions presented in [6]. The code can not only be used as a standalone Python program, but it can also be used in a Python-based system such as SageMath in order to serve as the numerical calculator of symbolic results such as those produced using the package from [7]. We will show that the code evaluates a general Heun function successfully in any region that does not cross over a singularity, is everywhere convergent up to infinity, is faster than existing alternatives and can easily be extended to singly, bi-, doubly- and tri-confluent Heun functions. Its current limitation, namely crossing over singularities, will be discussed as well and prospects for overcoming it will be presented.

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Two-dimensional stationary massless Dirac equation: singularities of phases of semi-classical asymptotics

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Quasiparticles in graphene are described by the two-dimensional massless Dirac equation. We consider its stationary solution with a localized right side in an autonomic non-homogeneous electromagnetic field. The phase of its semi-classical asymptotics defines a singular Lagrangian submanifold. In the case of a generic electromagnetic field we present local normal forms of the projection of this Lagrangian submanifold onto the configuration plane.

Resolvents of elliptic operators on quantum graphs with small edges: holomorphy and Taylor series

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We consider an arbitrary metric graph, to which we glue a graph with edges of lengths proportional to ε, where ε is a small positive parameter. On such graph, we consider a general self-adjoint second order differential operator $\mathcal{H}_\varepsilon$ with varying coefficients subject to general vertex conditions; all coefficients in differential expression and vertex conditions are supposed to be holomorphic in ε. We introduce a special operator on a special graph obtained by rescaling the aforementioned small edges and assume that it has no embedded eigenvalues at the threshold of its essential spectrum. Under such assumption, we show that that certain parts of the resolvent of $\mathcal{H}_\varepsilon$ are holomorphic in ε and we show how to find effectively all coefficients in their Taylor series. This allows us to represent the resolvent of $\mathcal{H}_\varepsilon$ by an uniformly converging Taylor-like series and its partial sums can be used for approximating the resolvent up to an arbitrary power of $\mathcal{H}_\varepsilon$. In particular, the zero-order approximation reproduces recent convergence results by G. Berkolaiko, Yu. Latushkin, S. Sukhtaiev [1] and by C. Cacciapuoti [2], but we additionally show that next-to-leading terms in $\mathcal{H}_\varepsilon$-expansions of the coefficients in the differential expression and vertex conditions can contribute to the limiting operator producing the Robin part at the vertices, to which small edges are incident. We also discuss possible generalizations of our model including both the cases of a more general geometry of the small parts of the graph and a non-holomorphic $\mathcal{H}_\varepsilon$-dependence of the coefficients in the differential expression and vertex conditions.

The research is supported by the Russian Science Foundation (grant No. 20-11-19995).

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Optimal control with phase constraints for a quasilinear endovenous laser ablation model

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The optimal control problem for quasilinear equations of radiation-conductive heat transfer simulating the process of endovenous laser ablation in the bounded domain G with the boundary $\Gamma = \partial G$ consists in minimizing the functional

$$J(\theta) = \int_0^T \left(\int_{G_1} \theta(x,t)dx - \theta_d(t) \right)^2 dt \to \inf \tag{1}$$

on the solutions of the initial-boundary value problem:

$$\sigma\partial\theta/\partial t - \operatorname{div}(k(\theta)\nabla\theta) - \mu_a\varphi = u_1\chi, \quad -\operatorname{div}(\alpha\nabla\varphi) + \mu_a\varphi = u_2\chi, \quad x \in G, \;\; 0 < t < T, \tag{2}$$

$$\theta = 0, \quad 2\alpha\partial_n\varphi + \varphi = 0 \quad \text{on } \Gamma, \quad \theta|_{t=0} = \theta_0. \tag{3}$$

Moreover, the following restrictions take into account:

$$u_{1,2} \geq 0, \quad u_1 + u_2 \leq P, \quad \theta|_{G_2} \leq \theta_*. \tag{4}$$

Here, θ is the difference between the temperature in $\overline{G}$ and the constant boundary temperature, φ is the intensity of radiation averaged over all directions, α is the photon diffusion coefficient, μ_a is the absorption coefficient, $k(\theta)$ is the coefficient of thermal conductivity, $\sigma(x)$ is the product of the specific heat capacity and the volume density, u_1 describes the power of the source spending on heating the fiber tip, u_2 is the power of the source spending on radiation, P is the maximum power of the source, χ is the characteristic function of the part of the medium in which the fiber tip is located divided by the volume of the fiber tip. In the subdomain G_1, it is required to provide a given temperature profile θ_d, while the temperature in the subdomain G_2 cannot exceed the critical value θ_*.

Estimates for the solution of the controlled system (2), (3) are obtained. On the basis of these estimates the solvability of the control problem is proved. An algorithm for solving the optimal control problem is proposed, based on the approximation of the phase constraint in G_2 by a functional with a penalty. The efficiency of the algorithm is illustrated by numerical examples.

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Acoustic cloaking using thin resonant ligaments

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In this work, we consider the propagation of acoustic waves in a 2D waveguide Ω which is unbounded in the (Ox) direction and which contains an obstacle. This leads us to study the problem

$$\left| \begin{array}{rcll} \Delta u + \lambda^2 u & = & 0 & \text{in } \Omega \\ \partial_\nu u & = & 0 & \text{on } \partial\Omega \end{array} \right. \tag{1}$$

where ∂_ν corresponds to the derivative along the exterior normal. We fix the wavenumber $\lambda \in (0;\pi)$ so that only the modes $w^{\pm}(x,y) = e^{\pm i\lambda x}$ can propagate. We are interested in the solution to (1) generated by the incoming wave w^+ coming from $-\infty$. It admits the decomposition

$$u(x,y) = \left| \begin{array}{rl} w^+(x,y) + Rw^-(x,y) + \dots & \text{for } x \to -\infty \\ T\,w^+(x,y) + \dots & \text{for } x \to +\infty \end{array} \right. \tag{2}$$

where the ellipsis stand for evanescent terms and $R, T \in \mathbb{C}$ are reflection and transmission coefficients.

The goal of this presentation is to explain how to cloak the obstacle by adding thin ligaments of width $\varepsilon \ll 1$ as depicted in Figure 1-middle. More precisely, when adding ligaments, we create a new geometry Ω^ε and we denote by R^ε, T^ε the corresponding scattering coefficients. Our objective is to show how one can place the ligaments and tune their lengths to get, when ε tends to zero,

$$R^\varepsilon = o(1), \qquad\qquad T^\varepsilon = 1 + o(1)$$

as if, approximately, there were no obstacle. The difficulty in this task is that in general the dependence of the scattering coefficients with respect to the geometry is implicit and not linear. However, working with these thin ligaments which are almost 1D objects, we can get relatively explicit formula. The approach is based on asymptotic analysis for thin ligaments. A key ingredient in the approach is that we work around the resonance lengths of the ligaments. This allows us to get effects of order one with geometrical perturbations of size ε. This work will be published in the article [1].

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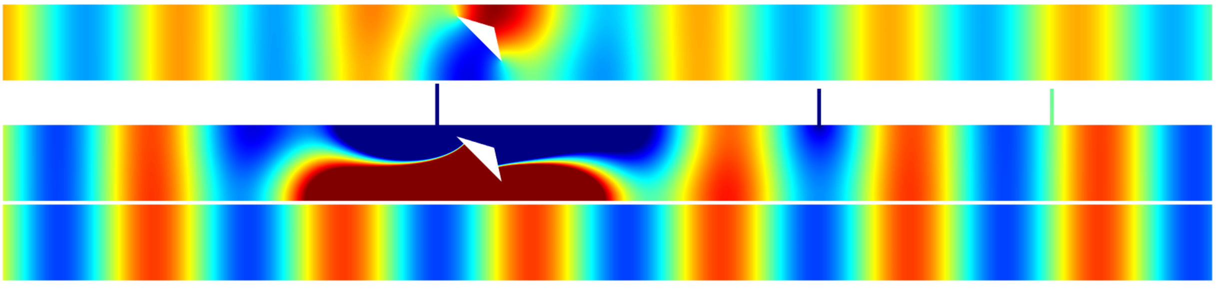

Fig. 1: Top: waveguide with an obstacle. Middle: the lengths of the three ligaments have been tuned to cloak the obstacle. Bottom: reference situation. A bit far from the obstacle, the fields in the waveguide with the cloaking device and in the reference situation are approximately the same.

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Design of a mode converter using thin resonant slits

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In this work, we are interested in the propagation of acoustic waves in 2D in a guide Ω infinite in the direction (Ox), that we assume to be governed by the problem:

$$\left\{ \begin{array}{rcll} \Delta u + \omega^2 u &=& 0, & \text{in } \Omega, \\ \partial_n u &=& 0, & \text{on } \partial\Omega, \end{array} \right. \tag{1}$$

where n is the unit outward normal to Ω. We choose the wave number ω so that only two propagative modes $w_1^\pm$ and $w_2^\pm$ exist. The study of the diffraction of the waves w_1^+ and w_2^+ leads to consider the matrices $R = (r_{jk})_{1\leq j,k\leq 2}$ and $T = (t_{jk})_{1\leq j,k\leq 2}$, where r_{jk} (respectively t_{jk}) is the coefficient of reflection (respectively transmission) of the mode k for the wave w_j^+. Our goal is to design a mode converter, that is to find a geometry such that the mode 1 is converted into the mode 2 and conversely, with the energy completely transmitted. In general, it is a difficult problem because the dependence of the diffraction coefficients with respect to the geometry is non linear and non explicit.

In our study, we choose to work with domains Ω made of two half-guides connected by thin ligaments of width $\varepsilon \ll 1$ (see Figure 1). This can seem paradoxical because in general, due to the features of the geometry, energy is almost completely reflected and we have, when ε goes to zero:

$$R^\varepsilon = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + o(1) \quad \text{and} \quad T^\varepsilon = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + o(1).$$

However, by carefully tuning the position of the ligaments and their lengths around the resonant lengths, we were able to show that we can obtain, when ε goes to zero:

$$R^\varepsilon = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + o(1) \quad \text{and} \quad T^\varepsilon = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + o(1),$$

that is the desired effect of mode conversion. The method lies on an asymptotic analysis with respect to the width of the ligaments which allows us to explicit the dependence of R^ε and T^ε with respect to the geometric parameters. In particular, working around the resonant lengths allows us to obtain an effect of order 1 with a ligament of width ε. We also crucially use the symmetry with respect to the (Oy) axis. Figure 1 shows one of the geometries obtained by this method. More details can be found in the article [1].

The work of S. A. Nazarov has been supported by the RNF, project 17-11-01003.

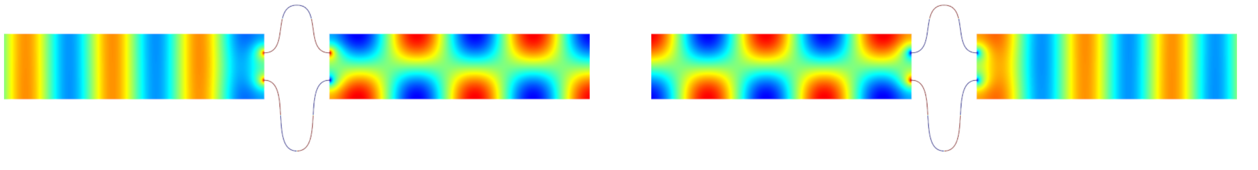

Fig. 1: Numerical illustration of the mode conversion: the geometry has been designed to allow one to convert the first mode into the second one (left), and the second mode into the first one (right).

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Exact asymptotics for the exit problem at infinity for diffusion processes

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We construct the exact asymptotic solution (nonoscillating WKB-type solution) to the problem

$$\frac{\partial u}{\partial t} = h\frac{a(x)}{2}\frac{\partial^2 u}{\partial x^2} + b(x)\frac{\partial u}{\partial x}, \; x \in (0,\infty), \; t > 0, \quad u\big|_{t=0} = 0, \; u\big|_{x=0} = 0, \; u\big|_{x\to\infty} = \mu(t), \qquad (1)$$

where $u = u(x, t, h)$, $a(x)$, $b(x) > 0$, $a(x), b(x) \in C^\infty$ with a described asymptotic behavior as $x \to \infty$, $\mu(t) \in C^\infty$, $h \to +0$ is a small parameter, and $\int_0^\infty b^{-1}(x)\,dx$ converges.

The solution u of problem (1) is probabilistic in nature [1].

To solve (1), we consider the fundamental solution G to the above initial boundary value problem:

$$G = 2(2\pi h)^{-1/2}\hat{V}(x, t, \tau + a^+, h)\exp\{-\zeta^2/h\}\big|_{\zeta=0},$$

where $V(x, t, \tau + y, h)$ is the symbol of initial boundary value problem which solves the problem

$$\frac{\partial V}{\partial t} = h\frac{a(x)}{2}\frac{\partial^2 V}{\partial x^2} + b(x)\frac{\partial V}{\partial x}, \quad V\big|_{t=0} = 0,\ V\big|_{x=0} = 0,\ V\big|_{x\to\infty} = \exp\left\{-\frac{(t-\tau-y)^2}{2h}\right\}$$

and a^+ is the creation operator [2, 3]. We justify the asymptotic solutions constructed here.

The study of V. G. Danilov was implemented in the framework of the Basic Research Program at the National Research University Higher School of Economics (HSE University) in 2021.

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On the analogue of Kirchhoff's formula for the wave equation in a two-dimensional domain

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Let $u(x, t)$ be a solution to the wave equation $\partial_t^2 u - \Delta u = 0$ in $\Omega \times \mathbb{R}$, Ω being a bounded domain in $\mathbb{R}^2$. We discuss the problem of determining u from the values of u, $\partial_\nu u$ on the space-time boundary $\partial\Omega \times \mathbb{R}$ (ν is the outward unit normal to $\partial\Omega$). In the three-dimensional case, the simplest way to solve the problem in consideration is to apply Kirchhoff's formula. The latter is derived by applying Green's formula to the solution u and the fundamental solution of the wave equation. However, in the two-dimensional problem, which is considered here, this approach requires Cauchy data on an unbounded time interval. The reason is that the corresponding fundamental solution (unlike that in the three-dimensional case) is non-zero for arbitrarily large t for any fixed x. Here we provide an algorithm of determining the solution u from data on $\partial\Omega \times I$, where I is some bounded time interval, in the two-dimensional case. To obtain this algorithm, we replace the fundamental solution in Kirchhoff's formula with a certain function, which vanishes for large t.

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The problem on the rotation of a two-phase drop

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The stability of an axially symmetric liquid drop $\mathcal{F}$ consisting of two viscous incompressible capillary fluids $\mathcal{F}^{\pm}$ and rotating about the x_3-axis with the angular velocity ω is considered. We study the free boundary problem for the perturbation of velocity vector field $\boldsymbol{v} = (v_1, v_2, v_3)$ and pressure function p of the two-phase fluid written in the coordinate system rotating with ω:

$$
\begin{gathered}
\rho^{\pm}(\boldsymbol{v}_t + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} + 2\omega(\boldsymbol{e}_3\times\boldsymbol{v})) - \mu^{\pm}\nabla^2\boldsymbol{v} + \nabla p = 0, \qquad \nabla\cdot\boldsymbol{v} = 0 \quad \text{in } \Omega_t \equiv \Omega_t^+ \cup \Omega_t^-,\ t>0,\\
\boldsymbol{v}(x,0) = \boldsymbol{v}_0(x), \quad x\in\Omega_0 \equiv \Omega_0^+\cup\Omega_0^- \in \mathbb{R}^3,\\
\mathbb{T}(\boldsymbol{v},p)\boldsymbol{n} = \Big(\sigma^- H^-(x) + \rho^-\frac{\omega^2}{2}|x'|^2 + p_0^-\Big)\boldsymbol{n}, \quad x\in\Gamma_t^-,\\
[\boldsymbol{v}]|_{\Gamma_t^+} \equiv \lim_{\substack{x\to x_0\in\Gamma_t^+,\\ x\in\Omega_t^+}} \boldsymbol{v}(x,t) - \lim_{\substack{x\to x_0\in\Gamma_t^+,\\ x\in\Omega_t^-}} \boldsymbol{v}(x,t) = 0, \qquad V_n = \boldsymbol{v}\cdot\boldsymbol{n},\ x\in\Gamma_t\equiv\Gamma_t^-\cup\Gamma_t^+,\\
[\mathbb{T}(\boldsymbol{v},p)\boldsymbol{n}]|_{\Gamma_t^+} = \Big(\sigma^+ H^+(x) + [\rho^{\pm}]|_{\Gamma_t^+}\frac{\omega^2}{2}|x'|^2 + p_0^+\Big)\boldsymbol{n},\ x\in\Gamma_t^+, \quad x' = (x_1,x_2,0),
\end{gathered}
$$

where V_n is boundary speed of the free outside surface Γ_t^- and the interface Γ_t^+, $\boldsymbol{n}$ is the outward normal to Γ_t, $\nu^{\pm}$, $\rho^{\pm}$ are the step-functions of viscosity and density, respectively, $\boldsymbol{v_o}$ is the initial velocity distribution, $\mathbb{T}$ is the stress tensor with the elements $T_{ik} = -\delta_i^k p + \mu^{\pm}(\partial v_i/\partial x_k + \partial v_k/\partial x_i),\ i,k = 1,2,3$; $\mu^{\pm} = \nu^{\pm}\rho^{\pm}$, δ_i^k is the Kronecker symbol, $\sigma^{\pm}\geq 0$ are surface tension coefficients; $p_0^{\pm}$ are given constants on $\Gamma_t^{\pm}$, $H^{\pm}$ are doubled mean curvatures of these surfaces, $\boldsymbol{e}_3$ is the unit vector in the direction of x_3. The dot means the Cartesian scalar product, “ $\times$ ” does the vector one. We suppose to comply with the conservation laws of volume: $|\Omega_t| = |\mathcal{F}|$, $|\Omega_t^+| = |\mathcal{F}^+|$, of the barycenter of both fluids, momentum and angular momentum, $\mathcal{F}$, $\mathcal{F}^+$ being axially symmetric equilibrium figures.

We obtain the global unique solvability of the problem provided that the initial data and the rotation speed are small, as well as the proximity of unknown surfaces to certain axially symmetric equilibrium figures. It is proved that if the second variation of energy functional is positive, the perturbation of an axially symmetric equilibrium figure tends to zero exponentially as $t\to\infty$, the motion of the drop going over to the rotation of the liquid mass as a rigid body. We develop the technique used by V. A. Solonnikov to prove the stability of a solution near the slow rotation of an axisymmetric equilibrium figure for a single fluid of finite volume [1]. Some preliminary considerations for the problem were given in [2]. In particular, the existence of axisymmetric equilibrium figures close to balls was demonstrated there.

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From Heun class equations to Painlevé equations

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In the first part of our paper we discuss linear 2nd order differential equations in the complex domain, especially Heun class equations, that is, the Heun equation and its confluent cases. The second part of our paper is devoted to Painlevé I–VI equations. Our philosophy is to treat these families of equations in a unified way. This philosophy works especially well for Heun class equations. We discuss its classification into 5 supertypes, subdivided into 10 types (not counting trivial cases). We also introduce in a unified way deformed Heun class equations, which contain an additional nonlogarithmic singularity. We show that there is a direct relationship between deformed Heun class equations and all Painlevé equations. In particular, Painlevé equations can be also divided into 5 supertypes, and subdivided into 10 types. This relationship is not so easy to describe in a completely unified way, because the choice of the "time variable" may depend on the type. We describe unified treatments for several possible "time variables".

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Constructive uniform semiclassical asymptotics of polynomials defined by second-order recurrent equations

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A lot of orthogonal polynomials $u_n(z)$ (n is the number of a polynomial, z is its argument), for example, Chebyshev, Hermite, Laguerre, Legendre polynomials, etc, are determined by second-order recurrence relations

$$a(z,n)u_{n+1}(z) + b(z,n)u_n(z) + c(z,n)u_{n-1}(z), \quad n = 0, 1, ..., N, ..., \quad z \in \mathbf{R}$$

with initial conditions in the form of polynomials $u_0(z) = v^0(z)$, $u_1(z) = v^1(z)$. Here $v^0(z)$, $v^1(z)$ are polynomials of z and $a(z,n)$, $b(z,n)$, $c(z,n)$ are polynomials of z and n.

We develop an uniform approach to describe the asymptotics of solutions of equations of this type as $n \to \infty$ that are uniform in the variable z. The idea of the method lies in the transition from a discrete equation to a continuous pseudodifferential one and application of the semiclassical approximation. A feature of the problem under consideration is that the symbol of the corresponding pseudodifferential operator is complex-valued. We suggest a method that reduces the problem to two equations with real-valued symbols and constructs the Plancherel–Rotach-type uniform asymptotics in the form of Airy or Bessel functions of a complex argument.

The talk is based on the joint work with A. V. Tsvetkova.

Bessel functions and semiclassical asymptotics

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The construction and application of the Maslov canonical operator are illustrated by the example of Bessel functions. Representations of these functions for real values of the argument via the canonical operator are derived, and well-known asymptotics are obtained as a corollary.

The study was supported by the Government program (contract no. AAAA-A20-120011690131-7).

Homogenization of nonstationary periodic Maxwell system in the case of constant permeability

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We study the Cauchy problem for the nonstationary Maxwell system in the case where the magnetic permeability is given by a constant positive matrix μ, and the dielectric permittivity is given by the rapidly oscillating (as $\varepsilon \to 0$) matrix $\eta^\varepsilon(\mathbf{x}) := \eta(\mathbf{x}/\varepsilon)$:

$$\begin{cases} \partial_t \mathbf{E}_\varepsilon(\mathbf{x},t) = (\eta^\varepsilon(\mathbf{x}))^{-1}\operatorname{curl}\mathbf{H}_\varepsilon(\mathbf{x},t), & \operatorname{div}\eta^\varepsilon(\mathbf{x})\mathbf{E}_\varepsilon(\mathbf{x},t) = 0, \quad \mathbf{x}\in\mathbb{R}^3,\ t\in\mathbb{R}; \\ \partial_t \mathbf{H}_\varepsilon(\mathbf{x},t) = -\mu^{-1}\operatorname{curl}\mathbf{E}_\varepsilon(\mathbf{x},t), & \operatorname{div}\mu\mathbf{H}_\varepsilon(\mathbf{x},t) = 0, \quad \mathbf{x}\in\mathbb{R}^3,\ t\in\mathbb{R}; \\ \mathbf{E}_\varepsilon(\mathbf{x},0) = (P_\varepsilon\mathbf{f})(\mathbf{x}),\ \mathbf{H}_\varepsilon(\mathbf{x},0) = \boldsymbol{\phi}(\mathbf{x}), & \quad \mathbf{x}\in\mathbb{R}^3. \end{cases}$$

Here a symmetric matrix-valued function $\eta(\mathbf{x})$ is periodic with respect to some lattice, positive definite, and bounded. Next, $\boldsymbol{\phi}\in L_2(\mathbb{R}^3;\mathbb{C}^3)$, $\operatorname{div}\mu\boldsymbol{\phi}(\mathbf{x}) = 0$, $\mathbf{f}\in L_2(\mathbb{R}^3;\mathbb{C}^3)$ and P_ε is the orthogonal projection of the space $L_2(\mathbb{R}^3;\mathbb{C}^3;\eta^\varepsilon)$ onto the subspace $\{\mathbf{u}\in L_2(\mathbb{R}^3;\mathbb{C}^3)\colon \operatorname{div}\eta^\varepsilon(\mathbf{x})\mathbf{u}(\mathbf{x}) = 0\}$.

It is well known that the electric and magnetic fields $\mathbf{E}_\varepsilon$ and $\mathbf{H}_\varepsilon$ weakly converge to the fields $\mathbf{E}_0$ and $\mathbf{H}_0$, which are the solutions of the homogenized Maxwell system with a *constant effective* dielectric permittivity η^0.

Denote by $\mathbf{D}_\varepsilon = \eta^\varepsilon\mathbf{E}_\varepsilon$, $\mathbf{D}_0 = \eta^0\mathbf{E}_0$, $\mathbf{B}_\varepsilon = \mu\mathbf{H}_\varepsilon$, $\mathbf{B}_0 = \mu\mathbf{H}_0$ the corresponding electric displacement vectors and the magnetic inductions. Our main results (see [1, 2]) are as follows.

- Let $\boldsymbol{\phi},\mathbf{f}\in H^2(\mathbb{R}^3;\mathbb{C}^3)$, and $\operatorname{div}\mu\boldsymbol{\phi} = 0$. Then for $t\in\mathbb{R}$ and $\varepsilon > 0$ we have

$$\|\mathbf{H}_\varepsilon(\cdot,t) - \mathbf{H}_0(\cdot,t)\|_{L_2(\mathbb{R}^3)} \leqslant C(1+|t|)\varepsilon(\|\boldsymbol{\phi}\|_{H^2(\mathbb{R}^3)} + \|\mathbf{f}\|_{H^2(\mathbb{R}^3)}),$$
$$\|\mathbf{B}_\varepsilon(\cdot,t) - \mathbf{B}_0(\cdot,t)\|_{L_2(\mathbb{R}^3)} \leqslant C(1+|t|)\varepsilon(\|\boldsymbol{\phi}\|_{H^2(\mathbb{R}^3)} + \|\mathbf{f}\|_{H^2(\mathbb{R}^3)}).$$

- Let $\mathbf{f}\in H^3(\mathbb{R}^3;\mathbb{C}^3)$, and $\boldsymbol{\phi} = 0$. Then for $t\in\mathbb{R}$ and $\varepsilon > 0$ we have

$$\|(\mathbf{E}_\varepsilon(\cdot,t) - \mathbf{E}_\varepsilon(\cdot,0)) - (\mathbf{1}+\Sigma^\varepsilon)(\mathbf{E}_0(\cdot,t) - \mathbf{E}_0(\cdot,0))\|_{L_2(\mathbb{R}^3)} \leqslant C|t|(1+|t|)\varepsilon\|\mathbf{f}\|_{H^3(\mathbb{R}^3)},$$
$$\|(\mathbf{D}_\varepsilon(\cdot,t) - \mathbf{D}_\varepsilon(\cdot,0)) - (\mathbf{1}+\widetilde{\Sigma}^\varepsilon)(\mathbf{D}_0(\cdot,t) - \mathbf{D}_0(\cdot,0))\|_{L_2(\mathbb{R}^3)} \leqslant C|t|(1+|t|)\varepsilon\|\mathbf{f}\|_{H^3(\mathbb{R}^3)}.$$

Here Σ^ε and $\widetilde{\Sigma}^\varepsilon$ are the so-called correctors of zero order. These results are sharp with respect to the norm type as well as with respect to the dependence on t. However, these estimates can be improved under some additional assumptions.

- Let $\boldsymbol{\phi},\mathbf{f}\in H^{3/2}(\mathbb{R}^3;\mathbb{C}^3)$, and $\operatorname{div}\mu\boldsymbol{\phi} = 0$. Then for $t\in\mathbb{R}$ and $\varepsilon > 0$ we have

$$\|\mathbf{H}_\varepsilon(\cdot,t) - \mathbf{H}_0(\cdot,t)\|_{L_2(\mathbb{R}^3)} \leqslant C(1+|t|)^{1/2}\varepsilon(\|\boldsymbol{\phi}\|_{H^{3/2}(\mathbb{R}^3)} + \|\mathbf{f}\|_{H^{3/2}(\mathbb{R}^3)}),$$
$$\|\mathbf{B}_\varepsilon(\cdot,t) - \mathbf{B}_0(\cdot,t)\|_{L_2(\mathbb{R}^3)} \leqslant C(1+|t|)^{1/2}\varepsilon(\|\boldsymbol{\phi}\|_{H^{3/2}(\mathbb{R}^3)} + \|\mathbf{f}\|_{H^{3/2}(\mathbb{R}^3)}),$$

- Let $\mathbf{f} \in H^{5/2}(\mathbb{R}^3;\mathbb{C}^3)$, and $\boldsymbol{\phi}=0$. Then for $t\in\mathbb{R}$ and $\varepsilon>0$ we have

$$\|(\mathbf{E}_\varepsilon(\cdot,t)-\mathbf{E}_\varepsilon(\cdot,0))-(\mathbf{1}+\Sigma^\varepsilon)(\mathbf{E}_0(\cdot,t)-\mathbf{E}_0(\cdot,0))\|_{L_2(\mathbb{R}^3)} \leqslant C|t|(1+|t|)^{1/2}\varepsilon\|\mathbf{f}\|_{H^{5/2}(\mathbb{R}^3)},$$
$$\|(\mathbf{D}_\varepsilon(\cdot,t)-\mathbf{D}_\varepsilon(\cdot,0))-(\mathbf{1}+\widetilde{\Sigma}^\varepsilon)(\mathbf{D}_0(\cdot,t)-\mathbf{D}_0(\cdot,0))\|_{L_2(\mathbb{R}^3)} \leqslant C|t|(1+|t|)^{1/2}\varepsilon\|\mathbf{f}\|_{H^{5/2}(\mathbb{R}^3)}.$$

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Prandtl equation in Kondratjev spaces

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We consider the Prandtl equation

$$\frac{a(t)}{1-t^2}u(t)-\frac{1}{2\pi}\int_{-1}^{1}\frac{u'(\tau)d\tau}{\tau-t}=f(t), \qquad t\in\mathcal{J}:=[-1,1]. \tag{1}$$

Here the known function in the right-hand side f belongs to the Sobolev–Kondratjev $\mathbb{KW}^{m-1}_p(\mathcal{J})$ or Hölder–Zygmund–Kondratjev $\mathbb{KZ}^0_{\mu-1}(\mathcal{J})$ space, while the solution u we look in the space $\mathbb{KW}^m_p(\mathcal{J})$ or $\mathbb{KZ}^0_\mu(\mathcal{J})$. The space $\mathbb{KW}^m_p(\mathcal{J})$ (the space $\mathbb{KZ}^0_\mu(\mathcal{J})$, $\mu=m+\nu$, $m=0,1,2,\ldots$, $1<p<\infty$, $0<\mu\leqslant 1$) consists of functions $\varphi(t)$ for which the function u itself and the weak Fuchs derivatives $\mathfrak{D}^k u:=(1-t^2)\dfrac{d^k}{dt^k}u$ belong to the Lebesgue space $\mathbb{L}_p(\mathcal{J})$ (to the Hölder–Zygmund $\mathbb{KZ}^0_\nu(\mathcal{J})$ space) for $k=1,\ldots,m$ (for more details see [2] where these spaces are introduced). The coefficient a is sufficiently smooth $a\in\mathbb{C}^\ell(\mathcal{J})$, where $\ell\geqslant m$.

Importance of the Prandtl equation (1) is well known (cf., e.g., [1]).

For the equation (1) we derive the criteria to be Fredholm in the Sobolev–Kondratjev and Hölder–Zygmund–Kondratjev spaces and find out that the Fredholm properties are independent of the integer parameter of these spaces m. These results imply, for example, that all Fuchs derivatives of a solution $\mathfrak{D}^k u(t)$ are smooth provided the right-hand side is infinitely smooth.

Conditions for the unique solvability of equation (1) are also indicated.

The same result is valid for more general equation with Fuchs derivatives

$$\sum_{k=0}^{n}\left[a_k(t)\mathfrak{D}^k u(t)-b_k(t)\int_{-1}^{1}\left(\frac{1-\tau^2}{1-t^2}\right)^{d_k}\frac{\mathfrak{D}^k u(\tau)d\tau}{\tau-t}\right]=f(t), \qquad t\in\mathcal{J},$$

where $d_k\in\mathbb{C}$ are complex numbers and coefficients are sufficiently smooth.

Similar results hold for equation on piecewise-smooth curves with corners, provided Fuchs derivatives and kernels of Mellin convolution type are defined properly.

For the investigation of the equations (1)–(2) and their analogues on curves with corners we apply localization and the resulting local representatives turn out to be systems of Mellin pseudodifferential equatins on semi-axes. Unique solvability of some equations of type (1) result from their connection with the boundary value problems for some partial differential equations.

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Parametrix for the inverse source problem of thermoacoustic tomography with reduced data

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Our goal is to solve the inverse source problem of thermo- and photoacoustic tomography, with data registered on an open surface partially surrounding the source of acoustic waves. The proposed modified time reversal algorithm recovers the source term up to an infinitely smooth error term. Numerical simulations show that the error term is quite small in practical terms. The present technique is applicable in the presence of a known variable speed of sound. We illustrate our results with numerical simulations in 2 and 3 spatial dimensions.

The influence of cortical bone thickness, soft tissue, and porosity on ultrasonic guided wave propagation

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At present, osteoporosis is one of the most common non-communicable diseases. It is associated with reducing bone mass and microarchitectural bone deterioration, leading to reduced bone quality and an increase in fracture risk. One of the promising methods for assessing the quality of bone tissue (cortical bone) is Quantitative Ultrasound (QUS), which is non-radiative, non-invasive, and relatively cheap [1]. Cortical bones support the propagation of ultrasonic traveling waves. Uncovering its dependence on the factors related to the state of the bone, such as the thickness of the cortical layer, porosity, radial gradient, and degree of mineralization, as well as properties of surrounding soft tissues, can provide hidden signs of osteoporosis to improve its diagnostics [2].

The purpose of this work is the study of the dispersion curves, frequency response, and time-frequency characteristics of ultrasonic guided waves (GWs) piezoelectrically generated in samples mimicking real bones (phantoms) to identify signs predicting osteoporosis. The analytically based computer models have been adapted to simulating the GW excitation and propagation in multilayered phantoms mimicking waveguide properties of tubular bones. The models are based on the explicit integral representations in the form of inverse Fourier transform path integrals of the waveguide's Green matrix and the source load vector, from which the GWs are extracted using the residue technique [3]. The influence of thickness, elastic properties, and porosity on the GW frequency response and time-frequency characteristics are analyzed and discussed.

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Hybrid numerical-analytical scheme for 3D scattering problems

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The study of guided wave (GW) diffraction by local inhomogeneities (obstacles) is a classical wave problem in various practical applications. For example, ultrasonic structural health monitoring (SHM) of thin-walled constructions (aircraft skins, pipelines, chemical tanks, etc.) is based on the GW property to propagate over long distances and interact with local defects of any nature. The application of SHM technologies requires the development of mathematical and computer models describing the GW excitation, propagation, and diffraction in elastic waveguides with obstacles of various nature and shape (cracks, cavities, inclusions corrosion, and others). A standard finite element method (FEM) can provide a numerical solution for arbitrary scatterers. However, the FEM is intended for finite domains and not well-suited for wave problems in prolonged structures (waveguides). Besides, the FEM numerical results are not physically evident. They do not give the wave structure of the solution, e.g., the wave energy distribution among the excited GWs, without post-processing.

To obtain efficient solutions, hybrid numerical-analytical approaches are currently being developed, based on the coupling of numerical solutions in local domains containing the source or obstacles with explicit analytical representations in the external semi-infinite domain. Such hybrid methods allow reducing computational costs and obtaining explicit analytical representation for the scattered GWs. However, they are not widely spread because the possibility of such coupling with an external multimode wave field is generally not provided in standard (commercial) FEM software. We have proposed a scheme avoiding this difficulty using the FEM software as a black box [1]. The main idea is to use the FEM for obtaining a set of local numerical solutions that serve as a basis in the inner domain with the source or scatterers. The expansion coefficients of both the FE decomposition in the inner domain and the external modal expansion are determined from the continuity conditions at the artificial boundary between them. This scheme has already been implemented in the 2D case. We present its further development for 3D scattering problems, starting from examples of its implementation for acoustic waveguides with arbitrary obstacles based on COMSOL MultiPhysics software.

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Wormhole solitons in lasers with saturable absorption

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Topological one-, two-, and three-dimensional (1D-, 2D-, and 3D-) dissipative optical solitons forming in media or schemes with energy input and output, were studied in a large number of publications, see a review [1]. Here we analyze a type of topological solitons which is intermediate between 2D and 3D. Being generated by 2D-solitons, such 3D dissipative solitons in a laser medium or in a cavity of sufficiently large size, acquire nontrivial properties.

The model under consideration represents a matrix in which centers with nonlinear (saturable) gain and absorption are embedded. The matrix is linear, with linear gain and frequency dispersion.

The centers are characterized by effectively two-level scheme of energy levels with fast response to the field. The medium can be placed into a cavity of large size. Radiation propagation is described by the generalized complex Ginzburg–Landau equation for a slowly varying envelope which is scalar in approximation of fixed radiation polarization and two-component vector when polarization variations are taken into account.

For scalar structures, the generating structure is one of known stable 2D-vortex laser solitons or a complex of such solitons with point-wise vortices (zeros of the complex envelope). A 3D- initial field distribution results by multiplication of 2D (transverse) distribution by an exponential multiplier describing purely phase variation in the third (longitudinal) dimension. Then vortices form one or several vortex lines, and 3D solitons acquire wormhole shape. More exactly, the exponential multiplier can include phase variation in transverse dimensions, and all these distributions represent exact solutions of the governing equation (assuming that 2D-solution is known). Therefore, the 3D wormhole solitons form a family with continuous spectrum of spectral parameter.

The linear stability analysis shows the instability of such "ideal" wormhole solitons in a medium with infinite size. It is confirmed by direct numerical solution of the governing equation. However, situation changes for a cavity with finite length L. The wormhole solitons maintain symmetry and straight vortex lines for short lengths L. For larger L, the instability leads to soliton symmetry breaking accompanied by bends or even breaks of vortex lines. The critical value of length separating these two scenarios, can be given by the linear stability analysis. There are two ways of stabilization of vortex lines and solitons bending in cavities with finite length. First, stabilization can result finally to some fixed bending of soliton; interesting, that even more stable are structures with "double-helix" vortex lines. Second, stable symmetric solitons are possible with an additional closed vortex line. Such structures are demonstrated both for bright, with intensity vanishing at periphery, and dark, where instead of vanishing, stabilization of intensity takes place, topological scalar solitons.

For vector case, a large number of new topological polarization phenomena arise [2]. We discuss some types of additional polarization singularities appearing in the wormhole laser solitons.

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Solutions to quasiperiodic equations: selfsimilarity and quantization conditions

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Quasiperiodic Shrödinger equations arise in many problems of modern physics. The behaviour of their solutions at infinity was studied by many authors, see review [1]. Computer calculations lead to the conjecture that this behavior can be multiscale and selfsimilar. This conjecture looks very natural in view of papers of Sinai, Helffer and Sjöstrand, Buslaev and Fedotov, and Fedotov and Klopp, see [2–5]. The first satisfactory result (and the first rigorous result) was obtained by Jitomirskaya and Liu [6] in the so called *localization* regime.

In this talk, we discuss the Maryland equation that is one of the most popular models in the theory of almost periodic equations and was introduced by specialists in solid state physics Grempel, Fishman and Prange [7]. We describe solutions to the Maryland equation in the semiclassical approximation. In the case when our approach can be applied, there are no well localized solutions.

Roughly, we show that, after some smoothing, the graphs of a solution on certain large intervals look like the graphs of solutions to the Maryland equations with new parameters on an interval of the length of the order of one. Moreover, studying solutions of such a new *renormalized* equation, one can reconstruct the behavior of solutions of the input Maryland equation. One can say that the semiclassical behavior of a solution of the input equation is determined by a series of quantization conditions, and that that is the behavior of a solution to the renormalized equation that determines which of these conditions are satisfied. In this talk, we focus on this effect. The talk is based on a joint work with Frederic Klopp [8].

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On self-similar solutions to some difference equations

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We study a function σ satisfying the difference equation

$$\sigma(z+h) = (1+e^{-iz})\,\sigma(z-h), \tag{1}$$

where z is a complex variable, and $h \in (0,\pi)$ is a parameter. This function and kindred ones arise in many problems of mathematical physics, e.g., in the scattering theory [2], in the theory of almost periodic operators [3], in the theory of integrable systems [4], etc.

The function σ can be considered as a trigonometric analogue of the Euler Γ-function: it satisfies a homogeneous first order linear difference equation with a coefficient being a first order trigonometric polynomial.

The σ satisfies a series beautiful functional relations, one knows the location of its zeros and poles, and its asymptotics as $\operatorname{Im} z \to \pm\infty$. But, the behavior of σ along the lines parallel to the real axis is not well understood. In this talk, assuming that $\operatorname{Im} z$ is bounded away from zero, we study $\sigma(z)$ as $\operatorname{Re} z \to \infty$. We show that σ has a multiscale behaviour that depends on number theoretical properties of the parameter h. The theoretical results are illustrated by results of computer calculations.

Let us note that the asymptotic analysis of $\sigma(z)$ as $\operatorname{Re} z \to \infty$ is equivalent to the one of the *logarithmic* sum

$$S(z,h,N) = \sum_{k=1}^{N} \ln\left(1+e^{-i(z+h-2hk)}\right)$$

as $N \to \infty$. To study the latter, we use a method similar to one developed in [1] to study the Gaussian exponential sums. In particular, we obtain a renormalization formula that expresses the input logarithmic sum via the logarithmic sum with a smaller number of terms and new constant parameters instead of h and z.

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On adiabatic evolution generated by a one-dimensional Schrödinger operator

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Consider the Schrödinger equation

$$i\frac{\partial\Psi}{\partial t} = -\frac{\partial^2\Psi}{\partial x^2} + v(x,\varepsilon t)\Psi, \quad t \in \mathbb{R}, \quad x \geq 0, \quad \varepsilon \to 0, \tag{1}$$

with the boundary condition $\Psi|_{x=0} = 0$. This equation describes adiabatic evolution in $L_2[0,+\infty)$ generated by the time-dependent stationary operator $H(\tau) = -\partial^2/\partial x^2 + v(x,\tau)$ with the Dirichlet boundary condition at zero, $\tau = \varepsilon t$ being the rescaled time.

We consider the model case

$$v(x,\tau) = \begin{cases} -1 & \text{if } 0 \leq x \leq 1-\tau, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

In this case the spectrum of $H(\tau)$ consists of the (absolutely) continuous spectrum filling $[0,+\infty)$ and a finite number of negative eigenvalues. $H(\tau)$ has exactly n eigenvalues if $\tau_{n+1} \leq \tau < \tau_n$, where

$$\tau_n = 1 - \pi(n - 1/2), \quad n \in \mathbb{N}. \tag{3}$$

As τ approaches the critical value τ_n, the n-th eigenvalue approaches the edge of the continuous spectrum and, having reached it, disappears.

We study a particular solution Ψ_n of (1) constructed by Fedotov [1], which has asymptotics of the form

$$e^{-\frac{i}{\varepsilon}\int\limits_{\tau_n}^{\tau} E_n(s)\,\mathrm{d}s} \sum_{m=0}^{\infty} \varepsilon^m \psi_{n,m}(x,\tau), \quad \varepsilon \to 0, \tag{4}$$

as long as the n-th eigenvalue exists. Here $E_n(\tau)$ is the n-th eigenvalue and $\psi_{n,0}(\cdot,\tau)$ is the corresponding eigenfunction of $H(\tau)$. During and after the disappearance of $E_n(\tau)$ the asymptotic behaviour of Ψ_n is different. Asymptotics inside the potential well have been described by Fedotov [1, 4]. We describe asymptotics outside of the potential well.

This problem in large part mirrors the problem of wave propagation in variable-depth shallow water, which was studied heuristically by physicists, e.g. Pierce [3], and then rigorously by Fedotov [2].

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Lorentzian Calderón problem under curvature bounds

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We introduce a method of solving inverse boundary value problems for wave equations on Lorentzian manifolds, and show that zeroth order coefficients can be recovered under certain curvature bounds. The set of Lorentzian metrics satisfying the curvature bounds has a non-empty interior in the sense of smooth, compactly supported perturbations of the metric, whereas all previous results on this problem impose conditions on the metric that force it to be real analytic with respect to a suitably defined time variable. The analogous problem on Riemannian manifolds is called the Calderón problem, and in this case the known results require the metric to be independent of one of the variables. Our approach is based on a new unique continuation result in the exterior of the double null cone emanating from a point. The approach shares features with the classical Boundary Control method, and can be viewed as a generalization of this method to cases where no real analyticity is assumed. The talk is based on joint work with Spyros Alexakis and Lauri Oksanen.

Detection of small-scale inhomogeneities in an elastic-acoustic medium

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The paper is devoted to the mathematical modeling of the plaques detection in the vein. Mathematically, we consider the problem of elastic-acoustic medium visualization. The visualization is conducted using Reverse time migration. Results of numerical experiments are presented.

This work was supported by the Volkswagen Foundation.

The monotonicity method for the inverse crack scattering problem

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Let $\Gamma \subset \mathbb{R}^2$ be a smooth non-intersecting open arc (crack), and we assume that Γ can be extended to an arbitrary smooth, simply connected, closed curve $\partial\Omega$ enclosing a bounded domain Ω in $\mathbb{R}^2$. Let $k > 0$ be the wave number, and let $\theta \in \mathbb{S}^1$ be incident direction. We consider the following direct scattering problem: For $\theta \in \mathbb{S}^1$ determine u^s such that

$$\Delta u^s + k^2 u^s = 0 \text{ in } \mathbb{R}^2 \setminus \Gamma, \tag{1}$$

$$u^s = -\mathrm{e}^{ik\theta\cdot x} \text{ on } \Gamma \tag{2}$$

$$\lim_{r\to\infty} \sqrt{r}\left(\frac{\partial u^s}{\partial r} - iku^s\right) = 0, \tag{3}$$

where $r = |x|$, and (3) is the *Sommerfeld radiation condition.* It is well known that there exists a unique solution u^s and it has the following asymptotic behaviour:

$$u^s(x) = \frac{\mathrm{e}^{ikr}}{\sqrt{r}}\left\{u^\infty(\hat{x},\theta) + O\big(1/r\big)\right\},\ r\to\infty,\ \ \hat{x} := \frac{x}{|x|}. \tag{4}$$

The function u^∞ is called the *far field pattern* of u^s. With the far field pattern u^∞, we define the *far field operator* $F : L^2(\mathbb{S}^1) \to L^2(\mathbb{S}^1)$ by

$$Fg(\hat{x}) := \int_{\mathbb{S}^1} u^\infty(\hat{x},\theta)g(\theta)ds(\theta),\ \hat{x}\in\mathbb{S}^1. \tag{5}$$

The inverse scattering problem we consider is to reconstruct the unknown arc Γ from the far field pattern $u^\infty(\hat{x},\theta)$ for all $\hat{x}\in\mathbb{S}^1$, all $\hat{x}\in\mathbb{S}^1$ with one $k>0$. In other words, given the far field operator F, reconstruct Γ.

In order to solve such a problem, we use the monotonicity method. The feature of this method is to understand the inclusion relation of an unknown target and artificial object by comparing the data operator with some operator corresponding to an artificial one. For recent developments of the monotonicity method, we refer to [2–4]. The following theorems are our main results for solving the inverse crack scattering problem.

Theorem (Theorem 1.1 in [1])
Let $\sigma\subset\mathbb{R}^2$ be a smooth non-intersecting open arc. Then,

$$\sigma\subset\Gamma \quad\Longleftrightarrow\quad H_\sigma^* H_\sigma \leq_{\mathrm{fin}} -\operatorname{Re} F, \tag{6}$$

where the Herglotz operator $H_\sigma : L^2(\mathbb{S}^1)\to L^2(\sigma)$ is given by

$$H_\sigma g(x) := \int_{\mathbb{S}^1} \mathrm{e}^{ik\theta\cdot x} g(\theta)ds(\theta),\ x\in\sigma, \tag{7}$$

and the inequality on the right-hand side in (6) *denotes that $-\operatorname{Re} F - H_\sigma^* H_\sigma$ has only finitely many negative eigenvalues, and the real part of an operator F is self-adjoint operators given by* $\operatorname{Re} F := \dfrac{1}{2}(F+F^*)$.

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Diffraction of a waveguide mode at the open-end of a dielectric-loaded circular waveguide

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Contemporary beam and THz technologies are tightly interlaced during last years. Strong THz fields allow realization of THz driven electron guns, THz bunch compression, streaking [1] and THz driven wakefield acceleration [2]. Inversely, dielectric capillaries similar to those used for THz bunch manipulation can be in turn utilized for development of high-power narrow-band THz sources [3]. Mentioned cases involve interaction of THz waves and particle bunches with an open end of certain dielectric loaded waveguide structure, most frequently a circular capillary. For further development of the discussed prospective topics a rigorous approach allowing analytical investigation of both radiation from open-ended capillaries and their excitation by external source would be extremely useful.

We present an elegant and efficient rigorous method for solving circular open-ended dielectric-loaded waveguide diffraction problems based on Wiener–Hopf technique. We deal with the case of uniform dielectric loading and internal excitation by a waveguide mode. Corresponding S-parameters, near-field and far-field distributions are presented.

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Formation of solitary strain waves in viscoelastic waveguides

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Polymeric materials play an essential role nowadays as they are used in a huge number of structures; therefore, the study of strain waves propagation in polymers is an important problem. In polymeric materials, for example, in polymethylmethacrylate (PMMA), viscoelastic relaxation processes significantly affect the dynamics of strain waves. The viscosity is characterized by the loss tangent, which is often weakly dependent on the wave frequency and remains almost constant in the range from fractions of a hertz to hundreds of kilohertz [1].

During physical experiments, a short longitudinal strain wave was generated at the end of the polymeric bar and the resulting deformations were recorded at different positions along the bar axis. It was shown that a short wave rapidly decays and a long solitary wave forms behind it, which has a much longer path and can propagate over long distances with almost the same amplitude [2].

The existence of the solitary waves was predicted by the nonlinear theory of elasticity. The Korteweg – de Vries and Boussinesq-type equations were asymptotically derived for the long waves in nonlinearly elastic waveguides [3]. However, this theory does not take into account viscoelastic effects, which play a crucial role in the formation of the long solitary wave.

In the present work we develop a model for the longitudinal strain waves in a viscoelastic bar, which is valid for both long and short waves. We consider the full three-dimensional problem and reduce it to a two coupled one-dimensional integro-differential equations. These equations can be further reduced to the single Boussinesq-type equation if viscosity is neglected. The predictions of our model are in good agreement with the experimental results.

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General Heun functions in free shear layer acoustics

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In this talk, we present our recent research on acoustics and stability of free shear layers based on the Euler equations for compressible flows. Our approach is based on finding an exact solution to the underlying linearized problem. We further show the challenge offered by the exact solution given in terms of the generalized Heun function.

In the first part of our work, the linearized compressible Euler equations were reduced to an ordinary differential equation, the well-known Pridmore–Brown equation [1], using a normal mode approach. We show that this equation can be reduced to the standard known form of the generalized Heun's differential equation [2] for the case of a hyperbolic tangent base flow profile we consider. Furthermore, we show how the distinction between the acoustic problem and the stability problem is expressed in the formulation of the eigenvalue problem.

In the second part, we focused on the solution of the eigenvalue problem and on the challenges posed by Heun's differential equation. In particular, the so far unsolved two point connection problem of the generalized Heun function prevents a direct solution of the eigenvalue problem. Accordingly, approaches to overcome this challenge are presented: due to the boundary conditions, we were able to formulate integral criteria for the stability problem which restrict the amount of possible eigenvalues. To answer the question of reflection and transmission of acoustic waves through the shear layer, an approach is presented which uses a further transformation of the generalized Heun equation to a symmetric form [3] in order to be able to link the solution branches at the different singularities of Heun's ODE.

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The boundary integral equation method for the simulation guided waves scattering by a distribution of cracks in a laminate

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Laminates are now widely employed in various areas since they can improve the reliability and durability of elastic structures due to high fatigue resistance or reduction in weight. However, the adhesive joints under in-service conditions give rise to some defects as micro-cracks, voids or delaminations. Therefore, an early-stage detection of degradation is an important issue for many industrial applications. Guided waves showed their efficiency in the field of nondestructive evaluation for crack identification. In this study, guided waves scattering by a distribution of strip-like micro-cracks situated in a multi-layered waveguide is considered. A semi-analytical hybrid approach for the simulation of the guided waves excitation by a piezoelectric transducer presented in [1] is extended here. Both the problems are investigated of a laminate with a distribution of open micro-cracks without face contacts, and a laminate with partially closed delaminations modelled via the effective spring boundary conditions. Random variables of centres and widths of the micro-cracks are numerically simulated by a function of the probability density, for example, the uniform distribution. The wave scattering by distributional of cracks is modelled according to the boundary integral equation method [2]. Guided waves transmission through the damaged zone is compared for the two considered cases. The possibility of the employment of the effective boundary conditions to substitute a distribution of micro-cracks at higher frequencies is analysed and discussed.

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A combined optimization technique for the engineering of optical devices

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A combined optimization technique for advanced device engineering is suggested. The proposed method is a fusion of the Bayesian and SGD (Stochastic gradient descent) optimization schemes dedicated to obtaining structure designs with improved performance. The developed technique was applied specifically to improve the characteristics of the diffraction gratings and QC (Quantum Cascade) amplifiers/detectors.

For the diffraction grating optimization task, the oblique-incident (off-plane) radiation scattering from one-periodical gratings (2D structures) with arbitrary conductivity and varying border profile is considered. The solution of the direct problem includes heavy computation of the numerical solution of the 2D Helmholtz equation using the boundary integral equation method ([1], Ch. 12).

For the QC structures, AlGaAs/GaAs active region materials are considered. The solution of the direct problem consists of Schrödinger–Poisson's system equation self-consistent solution.

Several numerical experiments were performed to test the proposed technique. The applied technique turns out to be at least 2 times faster, compared to the earlier genetic algorithm results [2]. The results also show that the SGD algorithm applied on the latest stages (on a plateau) of optimization contributes to the improvement of the results (Fig. 1). The obtained optimization improvements can lead to the development of more efficient device generation strategies and the creation of new devices: spectral instruments, beam splitters, detectors, amplifiers, etc.

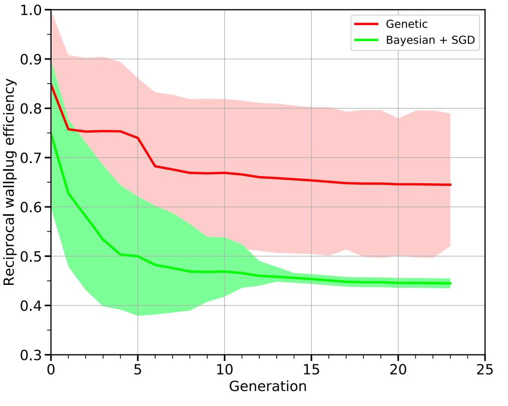

Fig. 1: The optimization curves for genetic and proposed algorithms. Shaded regions indicate standard deviation from the mean reciprocal wallplug efficiency.

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Geoacoustic inversion of vertical line array data in shallow water with an ice cover

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A technique for solving the inverse problem of estimating the effective acoustic parameters of the bottom has been developed for shallow water with an ice cover. The initial data for the inversion are the experimental dependence of the sound field amplitude on depth recorded at a vertical line array, and a set of precalculated dependences simulated for different effective bottom parameters. The range between a sound source and the array is an order of magnitude higher that the water depth. In the calculations, the bottom is considered as a liquid homogeneous medium with different values of the sound speed, density, and attenuation coefficient. Numerical modeling is carried out within the framework of normal-mode theory. The obtained values of bottom parameters are considered effective if they provide a maximum agreement between experimental and simulated data. The technique has been tested in a field experiment on Lake Baikal with a solid ice cover. The work was supported by the Russian Foundation for Basic Research (project no. 19-02-00127)

Horizontal refraction of acoustic waves in a shallow water waveguide with inhomogeneous bottom

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Mode parabolic equations and modal ray theory are applied to study horizontal refraction of acoustic waves in a planar waveguide with an inhomogeneous lower boundary, typical for shallow water arctic seas. Water depth is equal to $H \sim \lambda - 3\lambda$, where λ is the acoustic wavelength. In the model, depth and sound speed in the waveguide are constant. Inhomogeneities in the bottom are associated only with the spatial variability of the sound speed. Numerical simulations show that the horizontal refraction in such a waveguide is significant in an area where bottom sound speed is approaching the water sound speed. Modal ray trajectories demonstrate that the angle of refraction is up to 10 degrees. The work was supported by the Russian Foundation for Basic Research (project no. 19-02-00127).

Asymptotic Bergman kernels in the analytic case

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In this talk, we shall be concerned with the semiclassical asymptotics for Bergman projections in exponentially weighted spaces of holomorphic functions, with real analytic strictly plurisubharmonic weights. Here a result due to O. Rouby, J. Sjöstrand, S. Vũ Ngoc, and to A. Deleporte, establishes that one can describe the Bergman projection up to an exponentially small error. We shall discuss a direct approach to the construction of asymptotic Bergman projections in the analytic case, developed recently with A. Deleporte and J. Sjöstrand, which allows us to give a simplified proof of this result.

On semi-classical spectral series for an atom in a periodic polarized electric field

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As in [1] we consider the 3-D time-dependent Schrödinger operator $H(t) = -h^2\Delta + V + E(t)\cdot x$ where V is a radial potential and $E(t)$ a circularly polarized field with uniform frequency ω. Floquet operator that takes the system through a complete period $T = 2\pi/\omega$, turns out to be unitarily equivalent to $e^{iTP(x,hD_x)/h}$. Up to a linear gauge transformation, $P(x, hD_x))$ identifies with a magnetic Schrodinger operator with symbol $p(x,\xi) = (\xi_1 + \frac{1}{2}\omega x_2)^2 + (\xi_2 - \frac{1}{2}\omega x_1)^2 + \xi_3^3 + W(x)$, where $W = V+$ a quadratic term. Thus the semi-classical spectrum of Floquet operator is implied by this of P. If $V(r)$ has a non degenerate critical point at $r = r_0$, then $p(x,\xi)$ has itself a critical point at some $\rho_0 = (x_0,\xi_0) \in T^*\mathbf{R}^3$, and for some values of the parameters $V''(r_0),\omega$ and energy E, Floquet exponents are purely imaginary. Then $\{\rho_0\}$ is indeed a microlocal well for $p(x,\xi)$ and near ρ_0 we

can construct germs of quasi-modes that resemble those associated with the celebrated Landau levels (the degeneracy being lifted), i.e. Hermite functions. The Magnetic Birkhoff Normal Form [2] allows also for semi-excited states.

To consider semi-classical "shape resonances" for P, we assume a "profile" for V, with a Coulomb like singularity at $r = 0$, featuring the "inner sea", and a long range decay near $r = \infty$, featuring the "outer sea". This is typically the situation where resonances are implied by tunneling from $\{\rho_0\}$ to the inner and outer seas through the "phase-space forbidden region" $\Omega \subset T^*\mathbf{R}^3$. The quasi-modes constructed already near ρ_0 are extended in Ω along some extremal (complex) trajectories as follows: (1) Considering the Hamiltonian $q(x, \xi) = |\xi|^2 + W(x)$ obtained from $p(x, \xi)$ by removing the magnetic field, we are in the usual situation of shape-resonances ([3, 4]). In particular there are WKB solutions of $(Q(x, hD_x) - E)u \sim 0$ of the form $w(x; h) \sim e^{i\phi(x,E)/h} a(x, E; h)$ where ϕ (solution of the eikonal equation) is purely imaginary in the classically forbidden region; (2) Assuming the strength ω of the magnetic field to be small, we seek for a solution of $(P(x, hD_x) - E)u \sim 0$ of the form $u(x; h) = e^{i\psi(x,E)/h} b(x, E; h) w(x, E; h)$. We use a quantization deformation with respect to ω modifying the symplectic canonical 2-form to the magnetic canonical 2-form (Kostant–Souriau connexion), and show that ψ verifies indeed a first order PDE (new "eikonal equation") whose characteristics, as $\omega \to 0$, are close to those of q. Thus we get a good approximation of the rate of decay of quasi-modes that gives in turn an estimate on the imaginary part of the resonances.

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The time domain enclosure method for an inverse obstacle problem governed by the Maxwell system

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The *time domain enclosure method* with *dynamical back-scattering data* for various inverse obstacle problems governed by the scalar wave equations in three dimensions was initiated in the article [1]. We have already applied the idea to inverse obstacle problems governed by the Maxwell system in [2] for a perfectory conductive obstacle, and an obstacle with the Leontovich boundary condition in [3] and [4]. Those obstacles are impenetrable ones. In this talk we present a recent result [5] on finding a *penetrable obstacle* via the time domain enclosure method.

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On linear cocycles over irrational rotations with secondary collisions

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We consider a skew-product map

$$F_A : \mathbb{T}^1 \times \mathbb{R}^2 \to \mathbb{T}^1 \times \mathbb{R}^2, \tag{1}$$

defined for any $(x, v) \in \mathbb{T}^1 \times \mathbb{R}^2$ by

$$(x, v) \mapsto (\sigma_\omega(x), A(x)v),$$

where $\sigma_\omega(x) = x + \omega$ is a rotation of a circle $\mathbb{T}^1$ with irrational rotation number ω and

$$A : \mathbb{T}^1 \to SL(2, \mathbb{R})$$

is a differentiable function. We suppose the transformation A has a special form

$$A(x) = R(\varphi(x)) \cdot Z(\lambda(x)),$$

where

$$R(\varphi) = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}, \quad Z(\lambda) = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix}$$

and $\varphi : \mathbb{T}^1 \to \mathbb{T}^1$, $\lambda : \mathbb{T}^1 \to [\lambda_0, \infty)$ are C^1-functions such that φ has index 0 and λ_0 is sufficiently large positive constant.

It is also assumed that the critical set $\mathcal{C}_0 = \{x \in \mathbb{T}^1 : \cos\varphi(x) = 0\}$ consists of two points $\{x_0, x_1\}$ and the function φ satisfies the following conditions

$$|\varphi'(x)| \geq C_1\varepsilon^{-1}, \quad x \in \mathcal{C}_0; \quad |\cos\varphi(x)| \geq C_2, \quad \forall\, x \in \mathbb{T}^1 \setminus U_\varepsilon(\mathcal{C}_0),$$

where $U_\varepsilon(\mathcal{M})$ denotes a ε-neighborhood of a set $\mathcal{M}$, C_1, C_2 are some positive constants and $\varepsilon \ll 1$.

One may note that as a trajectory of a point $x \in \mathbb{T}^1$ under the rotation σ_ω hits $U_\varepsilon(\mathcal{C}_0)$, the hyperbolic properties of the corresponding cocycle generated by F_A become weaker. Due to irrationality of ω every trajectory intersects $U_\varepsilon(\mathcal{C}_0)$ and, particularly, each point of $\mathcal{C}_0$ interacts both with $U_\varepsilon(x_0)$ and $U_\varepsilon(x_1)$. The hyperbolicity of the system (1) strictly depends on whether the trajectory of x_0 hits $U_\varepsilon(x_0)$ (primary collision) before or after its intersection with $U_\varepsilon(x_1)$ (secondary collision) [1].

In the present work we study the case when

$$\sigma_\omega(U_\varepsilon(x_0)) \cap U_\varepsilon(x_1) \neq \emptyset.$$

Using approach developed in [1, 2] we show that for sufficiently small values of the parameter ε and under some additional requirements on the function λ the secondary collisions compensate weakening of the hyperbolicity due to primary collisions and the cocycle generated by F_A becomes hyperbolic in contrary to the case when secondary collisions can be partially eliminated [1].

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Linear combinations of Seeley–DeWitt coefficients and their properties

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The asymptotic expansion of the heat kernel at small values of proper time can be written as a series with Seeley–DeWitt coefficients [1, 2]. Using these coefficients, we construct functions of a special type and study their properties. Some applications of these functions are considered. In particular, we show their relation to the fundamental solution of the Laplace operator in d-dimensional space at $x \sim y$.

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Homoclinic transversal trajectories of singularly perturbed periodic Hamiltonian systems with disappearing separatrix

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We study a singularly perturbed Hamiltonian system with a Hamiltonian

$$H(q,p,\tau,\varepsilon) = \frac{\varepsilon^2}{2}p^2 - \frac{1}{2}\varphi(\tau)q^2 + \frac{1}{2}q^4, \quad \varepsilon \ll 1. \tag{1}$$

It is assumed the function φ to be a periodic C^2-function with period 1, which satisfy the following condition

(H_1): there exist $\tau_k \in [0,1), k = 1,\dots,2m$, such that $\varphi(\tau_k) = 0$, $\varphi'(\tau_k) \neq 0$.

Systems of type (1) appear in many branches of physics, for example, they describe dynamics of charged particles in the Earth's magnetospheric tail, where the Larmor radius of the particles is larger than the minimum radius of curvature of the magnetic field lines.

Introducing the fast time $t = \tau/\varepsilon$, we consider equations of motion corresponding to (1) as the simplest example of the slow-fast systems:

$$\dot q = p, \quad \dot p = \varphi(\tau)q - 2q^3, \quad \dot\tau = \varepsilon.$$

Thus, (q,p) can be treated as fast variables and τ as a slow one. In the limit $\varepsilon = 0$ one gets the so-called "frozen" system. The origin is an equilibrium of this system. Moreover, for those values of τ for which $\varphi(\tau) > 0$ the "frozen" system possesses a figure-eight separatrix. However, each time the parameter τ passes through the point τ_k the pitchfork bifurcation occurs and the separatrix disappears or appears again. Such scenario leads to existence of very chaotic dynamics in the domain correpsonding to oscillatory motion of the "frozen" system. One of key ingredients responsible for such chaotic behaviour are homoclinic transversal trajectories, i.e. trajectories which approach the origin at $\pm\infty$.

Using various asymptotic techniques [1, 2] in different regions of the extended phase space we construct a set of such doubly-asymptotic trajectories. Particularly, we prove that there exists $\varepsilon_0 > 0$ and a subset $\mathcal{E}_h \subset (0, \varepsilon_0)$ such that

1. for any $\varepsilon_1 < \varepsilon_0$ the Lebesgue measure $leb\left((0,\varepsilon_1) \setminus \mathcal{E}_h\right) = O\left(\mathrm{e}^{-c/\varepsilon_1}\right)$ with some positive constant c;
2. for any $\varepsilon \in \mathcal{E}_h$ the origin is a hyperbolic equilibrium of the system (1);
3. there exist positive constants $C_1, C_2, \nu < 1$ such that for any natural k, corresponding $\varphi(\tau_k) > 0$ and natural $N \in [C_1\varepsilon^{-1} - C_2\varepsilon^{-(1-\nu)}, C_1\varepsilon^{-1}] \cap \mathbb{N}$ there exists a homoclinic transversal trajectory with the following behaviour. It stays exponentially close to the origin for $\tau \in \mathbb{R} \setminus [\tau_k, \tau_{k+1}]$. In contrary, passing the point τ_k it detaches from the origin and comes inside the separatrix loop of the "frozen" system, where it oscillates exactly N times around new-born equilibrium of the "frozen" system.

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Modeling of electrodynamic processes by means of a micropolar continuum

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It is known that, before the beginning of the 20th century, all physical processes were described by means of mechanical models. These models were based on the concept of the ether as a substance occupying the whole free space, being able to penetrate into the material bodies endowing them with additional physical properties. The concept implies that the ether implements the interactions of the material bodies located at a distance from each other. In particular, when describing electromagnetic processes both in vacuum and in matter, scientists of the 19th century believed that they were describing processes in the ether. In the 19th century, the mechanical models played an important role for constructing the physical theories, namely, they were an intermediary between scientist's understanding of the nature of physical processes and mathematical description of these processes, see [1]. Using the ether concept, 19th-century scientists (Volta, Ampere, Poisson, Oersted, Young, Fresnel, Stokes, Navier, Cauchy, Green, Strutt, Neumann, Weber, Gauss, Riemann, Thomson, Maxwell, Helmholtz, Kirchhoff, FitzGerald, et al.), in fact, separated the process of constructing a new theory into two stages. At the first stage, they created a mechanical model corresponding to their intuitive understanding of the nature of the physical process. At the second stage, they derived differential equations describing this mechanical model. At the turn of the 20th/21st centuries, the interest in mechanical models of physical processes began to revive, see [2]. A large number of review papers have been published over the past 30 years. In addition, studies aimed at creating mechanical models of physical processes resumed in the second half of the 20th century and in the 21st century.

Our model is based on rotational degrees of freedom only. This is the fundamental difference between our model and the models of the 19th century. The basic steps we take to construct our model are: to formulate equations of a special type micropolar continuum, and then to suggest analogies between quantities characterizing the stress–strain state of the continuum and quantities characterizing electrodynamic processes. We introduce mechanical analogies of the electric field vector, the electric induction vector, the magnetic field vector, the magnetic induction vector, the electric current density and the electric charge density, see [3]. In the framework of the suggested model, we obtain a set of differential equations that coincide with Maxwell's first equation, the Gauss law for electric field and the charge conservation law. As a debatable question, we discuss the possibility of modifying the Maxwell–Faraday equation and the Gauss law for magnetic field.

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Boundary control for transport equations

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We consider 2 types of control for the linear stationary Boltzmann equation in a bounded and smooth domain X: first, control of a transport solution on a subdomain of X from the incoming boundary condition, and then control of the outgoing solution from incoming condition. In the first case we prove exact control under appropriate convexity assumption of the domain. Behavior of the control in a diffusive regime is also considered. In the second case we show that control is not feasible for well chosen optical parameters (absorption and scattering) of the domain X.

This a joint work with G. Bal (Chicago University).

Canonical representation of C^*-algebra associated with metric graph

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The eikonal algebra $\mathfrak{E}(\Omega)$ is a C^*-algebra associated with the metric graph Ω. This algebra was introduced in [1] and later studied in [2]. Eikonals are bounded self-adjoint operators determined by the dynamical system that describes the propagation of waves from the boundary into the graph with finite velocity. Eikonals and algebra $\mathfrak{E}(\Omega)$ can be determined (up to an isometric isomorphism) from dynamical and/or spectral inverse boundary data of the graph. It is shown that for arbitrary graph algebra $\mathfrak{E}(\Omega)$ has a canonical block structure:

$$\mathfrak{E}(\Omega) \cong \oplus \sum_{j=1}^{N} \left\{ \phi \in C([0,\delta_j], \mathbb{M}^{\kappa_j}) |\ \phi(0) \in \mathfrak{M}_0,\ \phi(\delta_j) \in \mathfrak{M}_{\delta_j},\ \mathfrak{M}_0, \mathfrak{M}_{\delta_j} \subseteq \mathbb{M}^{\kappa_j} \right\}.$$

The structure of its spectrum (the set of irreducible representations) is studied by analysis of the canonical block representation and is rather simple due to homeomorphism:

$$\hat{\mathfrak{C}} \cong [0, \delta_j],$$

where $\mathfrak{C} = C([0, \delta_j], \mathbb{M}^{\kappa_j})$ and $\hat{\mathfrak{C}}$ is its spectrum.

Coordinatization of the spectrum, which uses the eikonals as coordinates, enables one to construct a new graph deeply connected with the original Ω. This result is the next step towards solving the inverse problem that is reconstruction of the metric graph via its inverse boundary data.

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Optimization of resonances in photonic crystals and related Hamilton–Jacobi–Bellman equations

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Optical resonators with small decay rate (high-Q cavities) are important components in the contemporary optical engineering because they enhance intrinsically small light-matter interactions. The related mathematical problem is to design a photonic crystal structure that, under certain constraints, generates a resonance as close as possible to the real line.

The engineering and computational aspects of the problem have drawn great attention since the first fabrication success almost two decades ago. However, the analytic background of spectral optimization for non-Hermitian eigenproblems is still in the stage of development.

While the existence of optimizers was handled by the Pareto optimization settings of [2] and the associated Euler–Lagrange eigenproblems for 1-D optimizers were derived [2], the structure of optimal resonators is not completely understood (cf. [3–6]).

In the talk, it is planned to outline the recently developed minimum-time control approach to optimization of resonances in layered optical cavities [3]. In particular, this includes the derivation of the associated Hamilton–Jacobi–Bellman equations. On the other hand, the shooting methods [3, 4] for the effective computation of optimal symmetric structures and their connection with Maximum Principle will be described. We also plan to discuss briefly the analytical optimization of multidimensional resonances [1].

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The study of acoustic modes back-scattering by bottom relief inhomogeneities using the invariant imbedding method

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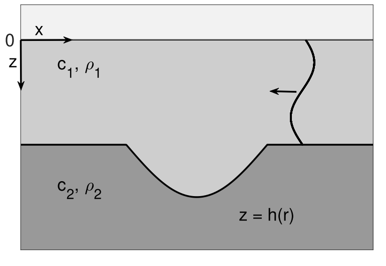
A problem of sound propagation in the 2-D waveguide of a shallow sea $\Omega = \{(x,z)|z \geq 0\}$ shown in Fig. 1 is considered. The waveguide consists of the water layer $0 \leq z \leq h(x)$ separated from the liquid penetrable bottom by the interface $z = h(x)$. It is supposed that the sea has the constant depth $h = h_0$ outside the interval $x \in [L_0, L]$. Thus inhomogeneity of bottom relief is localized on this interval.



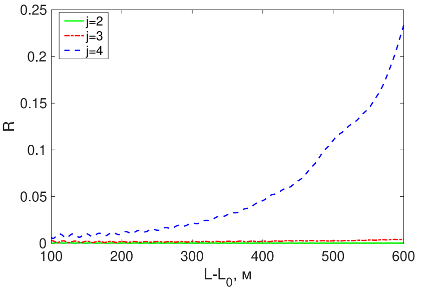


Fig. 1: (a) shallow-water waveguide with bottom relief inhomogeneity; (b) the relationship between the reflection coefficient and the width of bottom relief inhomogeneity.

Sound propagation in such waveguide is described by the 2-D Helmholtz equation

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} + \frac{\omega^2}{c^2} P = 0, \tag{1}$$

where $P(x,z)$ is the acoustic pressure, and c is the sound speed, ω is the angular frequency. It is supposed that the normal mode $\exp(-i\, k_{0,j} x)\varphi_{0,j}(z)$ is incident on the relief inhomogeneity on the right where $\varphi_{0,j}$ is an eigenfunction of an acoustic spectral problem, and $k_{0,j}$ are corresponding horizontal wavenumbers. It is assumed that water depth perturbation is small and sound propagation can be considered adiabatic, i.e. a solution in the waveguide can be represented by the formula $P(x,z) = A_j(x)\varphi_j(x,z)$.

The invariant imbedding method is used to obtain the following imbedding equations for the mode amplitude from the Helmholtz equation (1)

$$\begin{cases} \frac{\partial}{\partial L} A(x,L) = ikA(x,L) + \frac{1}{2} ik\varepsilon(L)(1+R(L))A(x,L), \; A(x,L)|_{L\to x} = 1 + R(x); \\ \frac{d}{dL} R(L) = 2ikR(L) + \frac{1}{2} ik\varepsilon(L)(1+R(L))^2, \qquad R(L_0) = 0. \end{cases} \tag{2}$$

The aim of the research is estimation of the reflection coefficient R_L of a normal mode from bottom relief inhomogeneity. Some parameters of bottom inhomogeneity are varied in order to study and analyse the reflection coefficient.

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Inverse problem for non-linear parabolic equations

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We consider the inverse problem of determining some general class of non-linear terms appearing in nonlinear parabolic equations. Our goal is to determine these non-linear terms from lateral boundary measurements of solutions of the equation. In order to prove these results we apply some arguments based on the new idea of multiple linearizations that we adapt to the framework of parabolic equations. This talk is based on some joint work with Ali Feizmohammadi and Gunther Uhlmann.

Matching creeping waves with lit area diffraction field

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This paper continues the series of classical publications [1–4] on the shortwave diffraction of the plane wave and authors' papers [5–9] on diffraction on the prolate bodies of revolution with axial symmetry. Shadow creeping waves for both Dirichlet and Neumann boundary conditions using Fock's asymptotics as the initial data were constructed. Numerical comparison of the Dirichlet and Neumann currents showed that the wave field in the Fock's boundary layer transforms continuously into the ray field in the lit zone and decays exponentially in the shadow zone.

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Evolution of basins of attraction in perturbed Painlevé-2 equation

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Solutions of the perturbed Painlevé-2 equation are typical for describing a dynamic bifurcation of soft loss of stability in wide the phenomena. The perturbed Painlevé-2 can be derived as primary term in narrow layer when one use the matching of asymptotic expansions before and after the dynamical bifurcation. Dynamics in the narrow layer is crucial important to the behavior after this layer.

Far from the bifurcation as $t \to -\infty$ the typical trajectories oscillates around $u = 0$. After the bifurcation there are two basins of attractions for the trajectories as $t \to \infty$. First one is a basin near $\sqrt{t/2}$ another one is near $-\sqrt{t/2}$. The boundary for there basins for $t \to -\infty$ was described by A. R. Its and A. A. Kapaev in 1987.

Perturbations of the Painlevé equations make deformations of this boundary and more of them the perturbations can break it. The properties of modulated the boundary depending on the perturbation are obtained. Both analytic and numerical results are given.

Reconstruction of unknown sources in cerebral oxygen transport model

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A continuum model of cerebral oxygen transport is considered. It is assumed that the blood and tissue compartments occupy the same spatial region $\Omega \subset R^3$ and have volume fractions, σ and $1-\sigma$, respectively. Following [1], the oxygen transport can be described by the following coupled equations:

$$\partial\varphi/\partial t - \alpha\Delta\varphi + \mathbf{v}\cdot\nabla\varphi = G + \sum_{j=1}^{m} q_j(t) f_j, \quad \partial\theta/\partial t - \beta\Delta\theta = -\kappa G - \mu, \quad x \in \Omega, \quad t \in (0,T). \tag{1}$$

Here, φ and θ are the blood and tissue oxygen concentrations, respectively; μ describes the tissue oxygen consumption; $G = c(\theta - \psi)$ is the intensity of oxygen exchange between the blood and tissue fractions, where ψ is the plasma oxygen concentration; $\kappa = \sigma(1-\sigma)^{-1}$, where σ is the volumetric fraction of vessels; $\mathbf{v}$ is a prescribed continuous velocity field in the entire domain G; α and β are diffusivity parameters of the corresponding phases; f_j are the characteristic functions of the disjoint subdomains $\Omega_j \subset \Omega$, $j = 1, ..., m$, which are some neighborhoods of the ends of arterioles and venules. That is the contribution from arterioles and venules are described by the source functions of equations (1) with unknown intensities q_j, $j = 1, ..., m$. Notice that, in [1], this contribution is described by the appropriate boundary conditions.

There are nonlinear monotonic dependencies of the tissue oxygen metabolic rate, μ, on the tissue oxygen concentration, θ, and of the plasma oxygen concentration, ψ, on the blood oxygen concentration, φ [1].

Equations (1) are supplemented by the boundary conditions imposed on $\Gamma = \partial\Omega$ and initial conditions,

$$\alpha\partial_n\varphi + \gamma(\varphi - \varphi_b)|_\Gamma = 0, \quad \beta\partial_n\theta + \delta(\theta - \psi_b)|_\Gamma = 0, \quad \varphi(x,0) = \varphi_0(x), \quad \theta(x,0) = \theta_0(x). \tag{2}$$

Here, ∂_n denotes the outward normal derivative at points of the domain boundary. Nonnegative functions φ_b, ψ_b, γ, and δ are given.

The *Inverse Problem* consists in finding intensities $q_1(t), ..., q_m(t)$ and the corresponding solution $y = \{\varphi, \theta\}$ of the boundary-value problem (1), (2) with the following integral overdetermination:

$$\int_{\Omega_j} \varphi dx = Q_j(t), \quad t \in (0, T), \quad j = 1, ..., m.$$

Here, Q_j are the prescribed avereged values of the functions φ with respect to subdomains Ω_j.

The inverse problem is reduced to an integro-differential equation for φ, a priori estimates for its solution are derived, from which unique solvability follows. An algorithm to find solutions is proposed and implemented.

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On the numerical solution of the iterative parabolic equations by the ETDRK pseudospectral methods

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We consider the Helmholtz equation (NHE) in a nonlinear Kerr-type medium:

$$E_{zz} + E_{xx} + k_0^2(1 + \epsilon|E|^2)E = 0\,,$$

where $E = E(x, z)$ denotes the wave (acoustic, electromagnetic) field. Following the standard multi-scale technique we can find [1], that solutions of NHE in the case of one-way wave propagation can be approximated by the series of solutions of iterative parabolic equations (IPEs):

$$E(x,z) \approx \sqrt{\frac{2}{\epsilon}}\exp(\mathrm{i}\tilde{z})\sum_{s=0}^{N} A_s(\tilde{x},\tilde{z})\,, \qquad 2\mathrm{i}A_{s,z} + A_{s,xx} + 2\sum_{j=0}^{s} A_j \sum_{i=0}^{s-j} A_i A^*_{s-i-j} + A_{s-1,zz} = 0\,,$$

where $\tilde{x} = k_0 x$, $\tilde{z} = k_0 z$, $s = 0 \ldots N$, in the equations for A_s tildes are omitted, and $A_{-1} \equiv 0$.

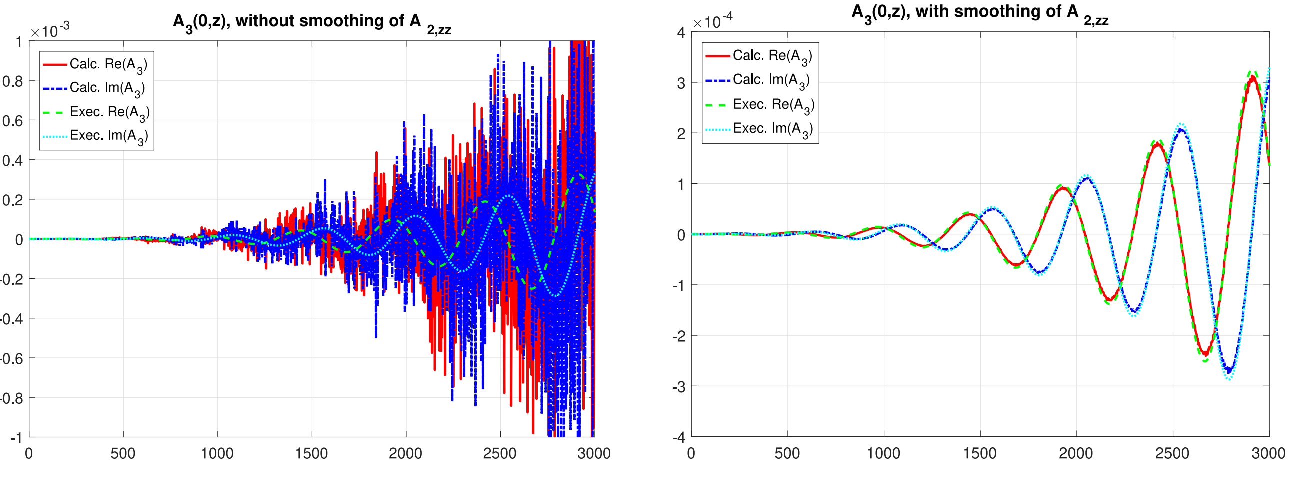

Fig. 1: Numerical solution for $A_3(0, z)$ by ETD4RK method without (left), or with (right) polynomial smoothing of $A_{2,zz}$, compared with the exact solution.

In this work we develop a few approaches of numerical integration of the IPEs based on exponential time differencing methods ETD2RK, ETD3RK, ETD4RK from [2]. The main difficulty in numerical solution of IPEs is in a proper approximation of the second derivative $A_{s-1,zz}$ which can dramatically spoil the accuracy of the solutions when $s \geq 3$. One of the approaches for this problem is in using of Chebyshev polynomials of the discrete variable. With their help we calculate the mean-square polynomial trend of the p-th order for a series of previously obtained $A_{s-1,z}$ at the n grid points immediately preceding the point of calculation. Then by differentiating the obtained polynomial we estimate the value of $A_{s-1,zz}$ at the point of calculation of A_s. In the most of our calculations $p = 4$ and $n = 120$. Comparison with the exact solutions from [1] reveals good results.

The developed approach can be easily extended to the case of 3D NHE, not saying of the usual cases of linear 2D and 3D Helmholtz equations. Solutions of such equations can be in demand in problems of acoustics and laser optics.

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Radiation resistance of a crossed-loop antenna in a magnetoplasma

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Sources that are capable of exciting electromagnetic waves with helical phase fronts in a magnetoplasma have recently been discussed in some detail, in particular with application to the whistler frequency range which is important for numerous applications (see, e.g., [1] and references therein). Such waves propagate both axially and azimuthally with respect to a static magnetic field superimposed on the plasma. As a result, they can carry orbital angular momentum along with the spin angular momentum. Although the excitation of whistler waves with helical phase fronts has already been studied experimentally in large plasma devices [2, 3], the radiation efficiency of the sources of such waves still needs separate consideration.

In the present work, it is our purpose to analyze the radiation resistance of an antenna in the form of two orthogonally crossed circular loops with quadrature-phased currents using an approach that is based on an eigenfunction expansion representation of the excited field [4, 5]. The antenna is assumed to be embedded in a cold collisionless magnetoplasma modeled upon the Earth's ionosphere and can excite waves with helical phase fronts, whose helicity type is determined by the sign of the phase shift, $\pi/2$ or $-\pi/2$, between the currents in the loops. The emphasis is placed on calculating both the total radiation resistance and the distribution of the radiated power over the spatial spectrum of the excited waves in the resonant part of the whistler frequency range. Analytical and numerical results will be reported for the radiation characteristics of such an antenna as functions of its size and the plasma parameters. The results obtained can be useful in understanding the basic features of exciting whistler waves with helical phase fronts in a magnetoplasma.

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Numerical simulation of time-fractional diffusion-wave processes applied to communication in bacterial populations

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In recent years, fractional calculus has provided promising scientific tools in the analysis of research problems in physics, engineering, biology, and economics. In many practical cases, fractional differential equations can be used to describe complex processes in more accurate way than the classical integer equations. In addition, the fractional differential theory is considered as a generalization of the integer analogue. Deterministic approaches (in particular, time-dependent PDEs equations) are widely used in mathematical biology and *in silico* researches. Here we consider a class of models of biological systems, namely, bacterial communities. In this concept, a bacterium is not considered to be a simple and primitive microorganism, the collective behavior of a bacterial community due to the "quorum sensing" regulation is realized. Bacterial resistance to antibiotics resulting from the quorum sensing causes the increasing challenges in medicine.

The various models of the bacterial quorum sensing formalized by "reaction-diffusion" PDEs have been proposed previously. For instance, the hybrid fractional-stochastic model is described in [1]. The experimental data suggest the appearance of time-dependent fluctuations of signal substances providing the quorum sensing during the bacterial population dynamics [2]. In this way, we can introduce the time-fractional diffusion-wave modification of the quorum sensing model as a generalization of the classical model in order to analyze different dynamical regimes of the biological system. The current study is aimed at developing numerical techniques to solve the time-fractional diffusion-wave problem with application to bacterial communication processes. The semilinear model is governed by a time-fractional diffusion-wave partial differential equation:

$$\begin{cases} \dfrac{\partial^\alpha u}{\partial t^\alpha} = D_u \dfrac{\partial^2 u}{\partial x^2} - \gamma_u u - \gamma_{L\to u} L u + F_1(x,u), \\ \dfrac{\partial^\alpha L}{\partial t^\alpha} = D_L \dfrac{\partial^2 L}{\partial x^2} - \gamma_L L + F_2(x,u),\ 0 < x < l,\ t > 0, \end{cases} \tag{1}$$

where $u(x,t)$ and $L(x,t)$ are the concentrations of special substances produced by bacteria; l is linear size of the domain solution; D_u, D_L, γ_u, γ_L, $\gamma_{L\to u}$ are the model parameters; the production terms are defined by the normal distribution and the Hill's law; α is the order of fractional derivative in Caputo sense, for $0 < \alpha < 1$ the equation (1) is referred to as the subdiffusion equation, $\alpha = 1$ corresponds to the ordinary diffusion; where as for $1 < \alpha < 2$ the equation (1) is the superdiffusion equation; $\alpha = 2$ characterizes the wave regime or "ballistic diffusion". The equation (1) is combined with the corresponding initial and boundary conditions. To solve the problem numerically, we derived an implicit computational scheme using the finite-difference approximation of Caputo derivative, and an iterative procedure in view of the nonlinear reaction term. Computational experiments were performed to estimate time-dependent characteristics of bacterial quorum process. The developed approach allows generalizing and essentially expanding diffusion-wave computational models applied for simulation of dynamical biological system.

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Frame-based Gaussian beam shooting analysis of monostatic scattering

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This work is aimed at taking advantage of paraxial Gaussian beam properties to analyse very large monostatic scattering problems.

In the first step of the proposed frame-based Gaussian beam shooting algorithm, the incident field is expanded into superpositions of frame windows [1] on the planar faces of a surface exterior to the scattering environment. Each frame window radiates in the form of a Gaussian beam, which propagates from the first to the last reflecting surface through multiple bounces, in as much as the initial frame window parameters guarantee sufficient paraxiality of the radiated beam. This first step yields a frame-based transfer function of the scattering environment.

In the second step, the scattering cross section (SCS) is computed on a grid of directions. Due to their spectral confinement (down to a given threshold), only a small number of Gaussian beams contribute to each incident plane wave and to the corresponding scattered one in the opposite direction. Hence the complexity of the proposed algorithm is expected be $O(BN)$, with B the maximum number of beam bounces and $N = (kR)^2$ the usual large parameter (k is the wavenumber and R is the radius of the circumscribing sphere).

Initial results of this research have been reported in [2], using "urban like" scenarios composed of rectangular parallelepipeds, as shown in Fig. 1. In Fig. 2, the SCS of such an environment is computed at the 0.01 m wavelength, for $S = 25$ m, $W = 20$ m, $H = 24$ m, and for various values of the frame window width parameter L.

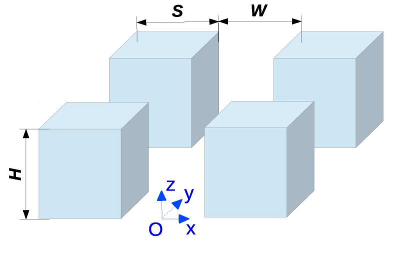

Fig. 1: "Urban like" scenario.

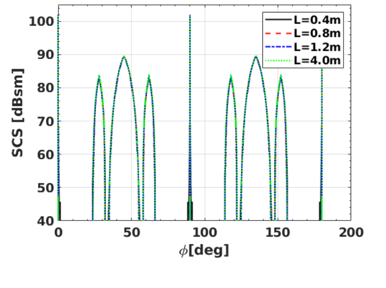

Fig. 2: Computed SCS for various values of L.

This communication will present further results of this ongoing work, exploring the conditions for given accuracy of the obtained results, by comparison with Physical Optics results, and discussing experimental computational efficiency of the method.

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A model problem of fluid current in a deep canal with rigid walls terminated by a sloping plane

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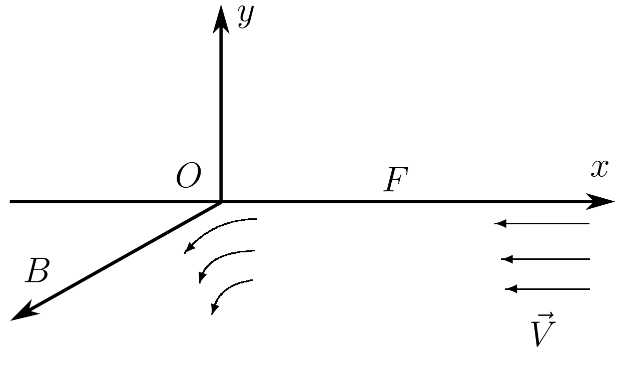
We study a problem of Neumann–Kelvin type that deals with current of a fluid in a canal of fixed width and of infinite depth. The fluid moves along vertical rigid plane walls of the canal having free surface F: $y = 0$, $x > 0$, $0 < z < b$. The fluid flux meets a sloping 'plane' B: $y = x \tan\alpha$, $0 < z < b$, $y < 0$, $0 < \alpha < \pi/2$, then goes under this surface B to infinity in depth. The potential U of the velocity satisfies the Laplace equation [1], the boundary condition of the second order on the free surface F, the inhomogeneous Neumann condition on B and a complementary condition at $x = y = 0$. A Meixner's type condition and conditions at infinity are also assumed.



The problem is actually two-dimensional because its solution is represented in the form $U(x, y, z) = \cos(kz)\, u(x, y; k)$ with $k = \pi/b$. The unknown function u fulfills the modified Helmholtz equation $\triangle u - k^2 u = 0$, the second order boundary condition on half-line $y = 0, x > 0$, the inhomogeneous Neumann condition on $y = x\tan\alpha$, $y < 0$, the complementary condition at the origin (at the edge), and other necessary conditions. The behaviour of the solution at infinity is actually clarified after getting an explicit solution of the problem at hand. To this end, the problem is solved by splitting of its solution into two separate modes i.e. in the form of linear combination of two solutions of simpler auxiliary problems with the first order boundary conditions on the free surface F. The unknown constants in the combination are then determined explicitely, in particular, by use of the complementary condition at the edge.

Each of the auxiliary problems is solved in a traditional way by means of the Sommerfeld integral representation and of reduction to a system of functional equations of Malyuzhinets type studied in a class of meromorphic functions [2]. The closed form solution given as the sum of two modes is then asymptotically evaluated at far distances from the edge. The questions of uniqueness of the solution obtained are aslo addressed.

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Hilbert spaces of functions related to dynamical system with discrete time associated with finite and semi-infinite Jacobi matrices.

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In [1, 2] the authors proposed a method of association of some special Hilbert spaces of analytic functions with one-dimensional dynamical systems. Among others there have been considered the dynamical system with discrete time associated with finite Jacobi matrices. Now we use this method to study the case of semi-infinite Jacobi matrices.

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Asymptotics of one-dimensional standing long waves on shallow water

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We consider one-dimensional shallow water equations (see [1, 2]) and look for periodical formal asymptotic solutions in the two case: (a) the basin with two shores, (b) the basin with one shore and vertical wall on the other side. The nonlinear problem is considered in the interval with variable boundary. We consider coordinate transform similar to linearized Carrier–Greenspan transform [3] that "fixes" the boundary. For resulting system periodical formal asymptotics can be constructed as asymptotics of linearized shallow water equations from [4]. The error of such formal asymptotics appears to be small in the case when nonlinear waves does not break.

Constructed asymptotics are compared with experimental results and fit well. Standing waves in experimental vessel (similar to [5]) are induced by vertical oscillations with parametric resonance. We consider two shapes of bottom (a) parabolic (asymptotics are defined using the Legendre polynomials) and (b) slopping with vertical wall. Considered approach provides effective analytical-numerical algorithm for finding approximate solutions.

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Elastic surface waves on a coated vertically inhomogeneous half-space

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This study is focussed on surface waves on an isotropic elastic half-space coated with a thin, vertically inhomogeneous coating layer, subject to prescribed vertical surface stress. The material properties of the coating, including the Lamé elastic parameters and the volume mass density, are assumed to be dependent on the vertical coordinate. The effective boundary conditions, replacing the effect of a thin inhomogeneous coating layer, are first discussed. Then, the explicit hyperbolic-elliptic model for surface waves is formulated, developing the methodology in [1]. The main feature of the model is a singularly perturbed wave equation for the elastic potential on the interface between the layer and the substrate, acting as a boundary condition for the elliptic equation governing decay over the interior. The developed asymptotic formulation is then implemented, allowing simplified analysis of the near-resonant regimes of the moving load.

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Holmgren–John unique continuation theorem for viscoelastic systems

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We consider Holmgren's uniqueness theorem for a partial differential equation with a memory term when the coefficients of the equation are analytic. This is a special case of the general unique continuation property (UCP) for the equation if its coefficients are analytic. As in the case in the absence of a memory term, the Cauchy–Kowalevski theorem is the key to prove this. The UCP is an important tool in the analysis of related inverse problems. A typical partial differential equation with memory term is the equation describing viscoelastic behavior. Here, we prove the UCP for the viscoelastic equation when the relaxation tensor is analytic and allowed to be fully anisotropic.

Numerical study of focusing ultrasonic spherical transducers from porous piezoceramics with multielectrode covering of same area

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The paper considers a focusing piezoelectric transducer in the form of a spherical segment with technological hole in the center. This transducer is designed to generate high-intensity ultrasonic fields with controllable characteristics in the focal spot in the external acoustic medium.

To enhance the efficiency of the acoustic wave excitation, porous piezoceramics is suggested as an active material for the transducer. As is well known, the best correspondence between impedances of the porous piezoceramics and the acoustic medium allows avoiding the use of the set of transition layers, which considerably complicate manufacturing and assembly of the spherical transducers. Partial control of the characteristics of the focal spot in the acoustic medium can be implemented by applying to one spherical surfaces the system of split electrodes with saw cuts and by setting different values of the electric potentials for individual electrodes of this system. The saw cuts were applied in such a way that all the split electrodes were had the equal areas. This split electrode system were allowed to shift the position of the initial focus on the pre-defined distance, by using the harmonically changing values of electric potentials with phase shifts on the electrodes.

The simulation of piezoelectric transducers was done with the use of multiscale finite element technologies. At microlevel we determined averaged properties of porous piezocermics on the base of the complex approach that includes the methods of the effective moduli and finite elements, different models of representative volumes and algorithms for the computations of the fields of inhomogeneous polarization [1]. The porous piezoceramic was further considered as a macrohomogeneous piezoelectric medium with effective characteristics. Then the solid and finite element models of spherical piezoelectric transducers with thickness polarization, multielectrode coverage and cuts between electrodes were built. Further, we carried out computations for determination of the electric resonances and antiresonances frequencies of the thickness modes, mode shapes and electric impedance frequency characteristics. At the next stage we investigated the models of piezoelectric transducers submerged into acoustic media. We also constructed finite element models of the whole system and computed the focal zone in acoustic medium for harmonic mode and for transient processes. All the models were implemented in computational package ANSYS. The calculation results allow us to conclude that the number of split electrodes substantially affect on the ability of the shift of the initial focus and on the intensity of the acoustic pressure in the shifted focus. It was noted that the multi-electrode coating allows to control the view of the focal area at the working acoustic medium and, thus, improve the efficiency of the transducer with a powerful ultrasound. Continuing research [2], here we carried out a comparative analysis of piezoelectric transducers made of porous piezoelectric ceramics of various porosities and studied non-stationary modes of operation of the considered piezoelectric transducers.

Author acknowledges the support of the Government of the Russian Federation (contract No. 075-15-2019-1928).

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Symmetric wedge wave in an elastic solid

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Along with bulk and surface waves (a fair example of the latter is the Rayleigh wave in half-space), wedge waves comprise a fundamental type of oscillations of solids and are intensively studied in geophysics, machine building, civil engineering etc.

First theoretical results on waves propagating along the edge of a wedge were obtained by numerical simulations. Then these waves were studied analytically at the physical level of rigor by many authors, mostly for small interior angles (slender wedge) and interior angles slightly less then π.

The first rigorous proof of existence of the wedge wave was obtained in the pioneering paper [1] by variational method for interior angles less then $\frac{\pi}{2}$. Then the idea of [1] was developed in [2] where the range of aperture angles was enlarged.

We prove the existence of a symmetric wedge mode in an elastic deformable wedge for all admissible values of the Poisson ratio $\sigma \in (-1, \frac{1}{2})$ and interior angles close to π, see Fig. 1, where $\varepsilon = \tan(\alpha)$.

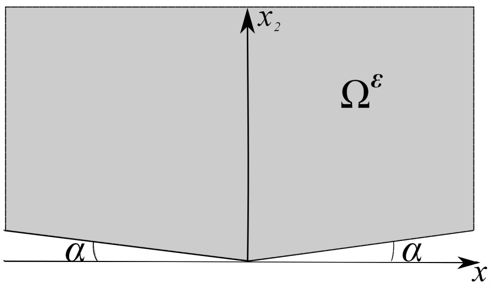

Fig. 1: The cross section of isotropic homogeneous elastic wedge.

Theorem. For any $\sigma \in (-1, \frac{1}{2})$, one finds ε^0 such that for any $0 < \varepsilon < \varepsilon^0$ there exists a symmetric wedge wave decaying exponentially w.r.t. the distance from the edge. This wave propagates along the edge with the velocity $\mathbf{c}_w$ which has the following asymptotics as $\varepsilon \to 0$:

$$\mathbf{c}_w^2 = \mathbf{c}_R^2(1 - \varepsilon^2\vartheta + O(\varepsilon^{\frac{5}{2}})), \tag{1}$$

where $\mathbf{c}_R$ is the velocity of the Rayleigh wave whereas $\vartheta > 0$ is an explicit coefficient depending on σ only.

The talk is based on the paper [3]. The study of the second author was supported by the Russian Science Foundation, grant no. 17-11-01003.

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Eigenvalues and threshold resonances in a quantum waveguide with a wide Neumann window

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In the unit strip the spectral mixed boundary-value problem

$$\begin{gathered} -\Delta u^\ell(y,z) = \lambda^\ell u^\ell(y,z),\ (y,z) \in \Pi = \mathbb{R} \times (0,1), \\ u^\ell(y,0) = 0,\ y \in \mathbb{R}, \quad u^\ell(y,1) = 0,\ |y| > \ell, \quad \partial_z u^\ell(y,1) = 0,\ |y| < \ell, \end{gathered} \tag{1}$$

is considered where $\ell > 0$ is a large parameter. This problem appears as a result of imposing artificial Neumann condition at the interval $(-\ell, \ell) \times \{1\} \subset \partial\Pi$, through which two unit planar quantum waveguides are coupled, and attracts attention for a long time, see [1–4]. The following asymptotic results will be presented and explained in the talk:

• Miscellaneous asymptotic forms for eigenvalues in the discrete spectrum $\sigma_d^\ell \subset (0, \pi^2)$ of problem (1) under variation of the parameter ℓ;

• Estimate of multiplicity $\#\sigma_d^\ell$ the discrete spectrum as $\ell \to +\infty$.

Since the multiplicity $\#\sigma_d^\ell$ grows unboundedly when $\ell \to +\infty$, there is the sequence $\{\ell_n^*\}_{n\in\mathbb{N}}$ of the critical lengths, for which problem (1) with the threshold parameter $\lambda = \pi^2$ gets a bounded solution and, therefore, the threshold resonance [5] occurs.

• Asymptotics of the critical lengths will be presented as $n \to +\infty$.

It is remarkable that the main tool to investigate the quality of the threshold resonances in problem (1) becomes differentiation along the boundary and manipulation with singularities of solutions at the endpoints $(\pm\ell, 1)$ of the Neumann window.

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Spectra of Steklov and Robin–Laplace-problems in bounded, cuspidal domains

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It is well-known by works of several authors that the spectrum of the Neumann–Laplace operator may be non-discrete even in bounded domains, if the boundary of the domain has some irregularities. In the same direction, in a paper in 2008 the authors considered the Steklov spectral problem in a bounded domain $\Omega \subset \mathbf{R}^n$, $n \geq 2$, with a peak and showed that the spectrum may be discrete or continuous depending on the sharpness of the peak. Later, we proved that the spectrum of the Robin Laplacian in non-Lipschitz domains may be quite pathological since, in addition to countably many eigenvalues, the residual spectrum may cover the whole complex plain.

We have recently complemented this study in two projects, where we consider the spectral Steklov- and Robin–Laplace-problems in a bounded domain Ω with a peak and also in a family Ω_ε of domains blunted at the small distance $\varepsilon > 0$ from the peak tip. The blunted domains are Lipschitz and the spectra of the corresponding problems on Ω_ε are discrete. We study the behaviour of the discrete spectra as $\varepsilon \to 0$ and their relations with the spectrum of case with Ω. In particular we find various subfamilies of eigenvalues which behave in different ways (e.g. “blinking” and “stable” families) and we describe a mechanism how the discrete spectra turn into the continuous one in this process.

S. N. acknowledges the partial support of the grant RNF 17-11-01003.

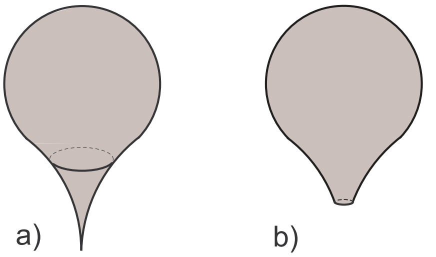

Fig. 1: Cuspidal peak and blunted domains.

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Edge diffraction of acoustic waves by periodic composite metamaterials: the hollow wedge

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Normally in metamaterial research, some form of infinite periodicity is assumed which allows us to restrict the study to a small portion known as the unit cell. This has led to many studies which increase the complexity of the unit cell and reconstruct the global scattering using the periodicity of the metamaterial. An alternative approach looks into the case where infinite periodicity is no longer assumed. This means that the metamaterial will have well-defined boundaries that can symbolise many different interfaces such as edges and corners.

In this presentation, the scattering of an acoustic pressure wave by a hollowed out wedge is studied where, for simplicity, the unit cells will be sound-soft cylinders with infinite height and a small radius. This configuration can also be viewed as two separate semi-infinite gratings with two sets of scattering coefficients to determine. We will construct an iterative scheme from the resulting infinite system of equations and find a solution using the discrete Wiener–Hopf technique. We shall also discuss some tools that are useful for computations such as tail-end asymptotics and rational approximations.

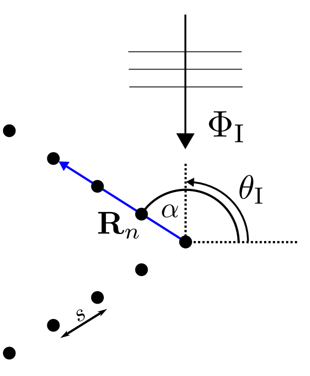

Fig. 1: Diagram of the hollow wedge with scatterers located at $\mathbf{R}_n$ and the incident wave Φ_{I} (in terms of the spacing s, the wedge angle α and the incident angle θ_{I}).

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Multipoint formulas for inverse scattering at high energies

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We consider inverse scattering for the multidimensional Schrödinger equation with smooth compactly supported potential v. We give explicit asymptotic formulas for the Fourier transform $\hat{v}(p)$ at fixed p in terms of the scattering amplitude f at n points at high energies. The precision of these formulas is proportional to n. To our knowledge these formulas are new for $n \geq 2$, whereas they reduce to the Born formula at high energies for $n = 1$. This talk is based, in particular, on references [1] and [2].

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Time-domain wave propagation in layered media

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This talk will consider some models for wave propagation in layered composites when the layers are thin compared to the propagation distance. Such problems have been addressed by homogenisation theory over the past 30 years. We will describe some examples in which the asymptotic behaviour can be deduced from exact but complicated integral representations and, in the case of 'Goupillaud' materials, the asymptotic solution is compared with exact numerical solutions.

Most of the models will be amenable to asymptotic analysis using the method of multiple scales in appropriate regions of space and time. In one-dimensional periodic problems the Goupillaud property allows operational calculus to be used to derive integral representations of the solution. These confirm that, over long enough times, the dominant response to an initial impulse is described by a dispersed wave form involving Airy functions.

When homogenisation theory is applied to waves propagating along periodic layers in a two-dimensional material, it reveals how energy exchange between the layers again eventually results in a dispersed wavefield.

The talk will conclude with the numerical solution of one-dimensional waves in a random Goupillaud medium.

Finite element methods for inverse initial source and control problems subject to the wave equation

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There is a well-known duality between inverse initial source problems and control problems for the wave equation, and analysis of both these boils down to the so-called observability estimates (see [1–3]). I will present recent results on numerical analysis of these problems.

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On resolvent approximations for elliptic high order operators with periodic coefficients

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We study homogenization of a fourth-order elliptic operator A_ε with measurable ε-periodic coefficients in d space dimensions ($d \geq 2$), where ε is a small positive parameter tending to zero. Such kind of operators arise, for instance, in elasticity theory on thin plates made of composite materials with ε-periodic structure; the case $d = 2$ is the most important for applications. We find an approximation for the resolvent $(A_\varepsilon + 1)^{-1}$ in the operator $(L^2(R^d)\to L^2(R^d))$-norm with remainder term of order ε^3. This result relies in an essential way on the approximation of the resolvent $(A_\varepsilon + 1)^{-1}$ in the energy operator norm, that is, $(L^2(R^d)\to H^2(R^d))$-norm, with ε^2 order remainder term. The latter approximation (in the energy operator norm) itself is of interest; it has been recently proved in [1]. Formerly, resolvent approximations for high order elliptic operators in the energy operator norm were known only with ε order remainder term (see, e.g., [2]). The larger preciseness of the resolvent approximation in the energy operator norm can be attained thanks to the new type of ansatz in powers of the parameter ε, which is proposed in [1]. To obtain the resolvent approximation of ε^3 order in the $(L^2\to L^2)$-norm from the resolvent approximation of ε^2 order in the energy operator norm, we follow the scheme applied in [3] to derive the resolvent approximation of ε^2 order in the $(L^2 \to L^2)$-norm from the resolvent approximation of ε order in the energy operator norm. All the above approximations are constructed under minimal regularity assumptions on the data; and that is why they make sense, generally, due to smoothing operators involved in them. Among the smoothing operators, we exploit mostly Steklov's averaging operator and its iterations. Besides, the usage of smoothing in our ansatz plays a crucial role in the proof of estimates for the remainder terms. This approach was firstly proposed in [4, 5] and is widely used now to obtain operator-type estimates for homogenization error.

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A strict mathematical problem for quasiphotons

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Quasiphotons are asymptotic localized solutions of the wave equation with a smoothly varying wave speed constructed in [1, 2]. These solutions are localized according to the Gauss law near a point running along the geometrical-optical rays with the speed of the waves. In [3], any solution of the wave equation in the half-plane with boundary data from a wide class was decomposed in terms of quasiphotons. The crucial point was the expansion of the boundary data of solutions in terms of the boundary data of quasiphotons. However quasiphotons do not vanish as time tends to infinity, and thus the respective boundary value problem was not posed well-posed as a mathematical one. However, as shown in [4], localized solutions with a proper localized behavior exist for a homogeneous medium and are not unique.

The problem here is to state a well-posed mathematical problem for the solution of the initial-boundary value problem for the wave equation in the half-plane with initial data specified as time tends to minus infinity. We describe the functional class in which the solution of this problem should be sought. We describe a class of data for which the existence and uniqueness are ensured. Some stability conditions are found.

The technique is based on the known results for initial-boundary value problems on a bounded time interval [5].

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Angular momentum of Airy beams under diffraction

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In the paraxial approximation, it is usually assumed that an elliptically polarized electromagnetic wave has an electric and magnetic field perpendicular to the direction of propagation. This means neglecting the small longitudinal components of these fields which are of the first order of smallness in the angle of divergence with respect to the transverse components. Only when they are taken into account the electric field strength and magnetic induction satisfy two scalar Maxwell's equations. The longitudinal components of the electric and magnetic fields lead to the appearance of the first-order corrections for such quantities as the Umov–Poynting vector and the angular momentum of the electromagnetic field.

In this paper, the orbital and spin components of the angular momentum of the electromagnetic field are found for a two-dimensional elliptically polarized Airy beam. The choice of the beam is associated with its diffraction-free propagation in a homogeneous linear medium in the paraxial approximation. Due to this, diffraction effects appear only as corrections of the first order of smallness [1]. For a linearly polarized beam, the appearance of the spin angular momentum is solely due to diffraction and is absent in the main order solution.

The propagation of a linearly polarized Airy beam in a photorefractive medium with diffusion nonlinearity [2–4], in which it propagates without diffraction, is also considered. The dependence of the spin angular momentum on the characteristics of the photorefractive medium and on the conditions of diffusion nonlinearity creating is analyzed in detail.

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Solving diffraction gratings synthesis problems with multiperiodic profile using gradient-based methods

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Diffraction gratings are widely applied in laser technology, spectroscopy, space research and many other fields of modern science and technology [1]. Being used in different optical systems, diffraction gratings have to meet different requirements. In most cases a grating should have maximal possible diffraction efficiency in certain diffraction orders. This required characteristic can be obtained by choosing the grating profile shape, since it is the profile geometry that mostly determines the efficiency with which light is diffracted in each order. To find the optimal parameters before a diffraction grating is produced a diffraction grating synthesis problem should be solved.

In last decades, diffraction gratings with more complex profile, in particularly, with miltiperiodic profile when grating period is divided into several number of equals parts, are often applied for various purposes. For example, the binary blazed grating with such profile geometry can be used to imitate properties of the transmission blazed grating with a sawtooth profile for protecting the optical imaging systems of infrared guided missiles from damage due to its simplicity of producing [2]. At terahertz range such profile geometry of a metallic reflection grating allows to significantly reduce the backscattering [3].

This work focuses on the problems of diffraction grating synthesis with multiperiodic profile. From mathematical point of view the diffraction grating synthesis problems are optimal control problems and are formulated as a minimization problem for the cost functional which depends on grating geometry parameters called control parameters. Herewith, the cost functional is constructed in a way such that its minimum corresponds to a maximal value of the diffraction efficiency in a desired reflection or transmission diffraction order. To minimizing the cost functional a gradient-based method with a gradient being computed through obtaining the solution of the adjoint problem [4] is applied. It is the most stable method with respect to the increase of the number of the control parameters, and its convergence to the optimal solution is mathematically justified [5]. In prospect this allows one to consider a rather general formulation of the synthesis problem, in which the optimal shape of the grating profile can be obtained directly as a result of solving the problem without any special a priori assumptions (besides practical realization).

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New concept of the surface waves of interference nature: creeping waves

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In the paper [1] a new concept of the surface waves of interference nature, propagating in a smooth, strictly convex surface Σ embedded in $\mathbb{R}^3$, was proposed and described for the whispering gallery waves. Present report is an extension of this paper and it is devoted to the creeping waves, propagating on the convex side of Σ embedded in 3D Euclidean space. Similar to whispering gallery waves, they slide along geodesic flow on a Σ and inherit the 3D problems: torsion of the geodesic lines and the wavefield singularities on their caustics.

The main ideas of our theory remain, in principle, the same. Let $\vec{r} = \vec{r}(s, \gamma)$ be the flow of geodesic lines associated with the creeping wave, where $\vec{r}$ is radius-vector in $\mathbb{R}^3$, s is arc length of the geodesic line and γ is a parameter, which specifies the geodesic line in the flow. For each geodesic we construct a specific solution of the Helmholtz equation, concentrated in a tubular vicinity of the selected geodesic line but which has no singularities on the caustics. At this step we make use of the main ideas of the theory of creeping waves in 2D case: scaling of local coordinates in the neighborhood of the geodesic and main oscillating factors of the ansatz, see [2]. The asymptotics of total wavefield is presented as a superposition, or better to say, an integral over γ of the localized solutions. Evidently the integral has no singularities on caustics and arising algorithm of the wavefield calculations does not depend on an observation point.

We would like to emphasize the resemblance of our theory of creeping waves and Gaussian beam summation method, see [3, 4].

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Spectrum of Laplacians on periodic graphs with waveguides

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We consider discrete Laplace operators on periodic graphs perturbed by guides (i.e., graphs which are periodic in some directions and finite in other ones). For example, for planar graphs, the guide is a periodic graph embedded into a strip. The spectrum of the Laplacian on the unperturbed periodic graph is a union of a finite number of non-degenerate bands and eigenvalues of infinite multiplicity. It is known [1] that the spectrum of the perturbed Laplacian consists of the unperturbed one and the guided spectrum created by the perturbation. This additional guided spectrum is also a union of a finite number of bands and the corresponding wave-functions are mainly located along the guides. The guided spectrum may partly lie above the spectrum of the unperturbed operator, on this spectrum and in gaps of the unperturbed problem. The guided spectrum lying *above* the spectrum of the unperturbed operator was studied in [1]. Our goal is to study the guided spectrum *in gaps* of the unperturbed problem. We estimate the number and the positions of the guided bands in gaps in terms of eigenvalues of Laplacians on some finite graphs. We also determine sufficient conditions for the perturbations of the periodic graphs under which the guided bands do not appear in gaps of the unperturbed problem.

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Study of the Doppler spectrum of surface reverberation using numerical simulation

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Previously, we have proposed an original modification of the boundary element method (in 2D), which makes it possible to simulate the scattering of sound on the surface of water in a deep sea without imposing restrictions on the shape of the surface. It was demonstrated that the method cope with not small Rayleigh parameters, shading and multiple scattering [1, 2]. This time, we use the surface obtained by hydrodynamic modeling as a soft boundary in acoustic modeling.

To simulate the sea surface we use the direct method for numerical simulation of potential flows with a free surface of two-dimensional fluid proposed by Zakharov and Dyachenko [3]. As input, we take the empirical energy spectrum of wind-generated waves $S(f)$, which sets the initial surface. The system of transformed Euler equations is numerically integrated to get how the surface evolves from the initial state.

The subject of our interest is the sound of the middle frequency range from 1 to 5 kHz, which scattering significantly depends on the state of the sea surface.The purpose of this study is to reproduce some of the effects that are observed in experiments (such as not observing the Bragg peak with sufficiently developed waves [4]). We get the pressure field, the backscattering strength, Doppler spectrum depending on the angle of incidence and scattering angle at different wind speed and compare our data with the predictions of classical models such as resonant scattering of sound or the two-scale model and with empirical dependences.

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On some spectral problems in relative elliptic theory

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Relative elliptic theory is the theory of elliptic operators associated with pairs (M, X), where X is a submanifold in an ambient manifold M. Many aspects of relative elliptic theory were studied systematically by B. Yu. Sternin and his coauthors (e.g., see [1,2]). In this talk, we focus on self-adjoint extensions of symmetric problems in relative theory. In particular, we describe explicitly the Friedrichs extension.

The work is supported by RFBR (grant No. 19-01-00574a).

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Asymptotic solution for the explicit difference scheme for the wave equation

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We study solution of the explicit difference scheme for following wave equation with localized initial data

$$u_{tt}(x,\,t) - \langle \nabla,\, c^2(x)\nabla\rangle u(x,\,t) = 0, \quad u|_{t=0} = V(x/\mu), \quad u_t|_{t=0} = 0, \quad x \in R^2.$$

Function $c(x)$ is smooth, bounded and strictly positive function, parametr $\mu \ll 1$ is the localization parameter.

Let us denote the lattice step by both directions as h. The solution of the difference scheme crucially depends on the ratio h/μ. We want to study the behavior of this solution depending on this ratio.

It is well known that for difference scheme one can match the pseudo-differential equation [1–3]. Solution of such equation is the continuous function and its reduction on the lattice gives us the solution of the difference scheme. Maslov canonical operator provides analytical asymptotic formulas for the solution of such equation.

In our case we restrict ourself to the leading wave front and the leading wave, which is localized in the vicinity of this front [2]. Asymptotic formulas for such wave can be expressed via Airy functions and their derivations. In some cases these asymptotic formulas can be presented with the help of Jacobi theta-functions.

Such analytical representation of the asymptotic formulas provide convenient method for analysis of the solution of the difference scheme.

This study is supported by RSF grant 16-11-10282.

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Short-wave asymptotic solutions of the wave equation with localized perturbations of the velocity

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To describe the propagation of waves in media containing localized rapidly changing inhomogeneities (e.g., narrow underwater ridges or pycnoclines in the ocean, layers with sharply changing optical or acoustic density, etc.), it is natural to use the wave equation with a small parameter characterizing the ratio of the scales of the localized inhomogeneity and of the general change of velocity (e.g., of the thickness of a pycnocline to the external typical scale of changes in ocean density). We describe the propagation of wave packets whose characteristic wavelength is comparable with the scale of inhomogeneity. The results are published in [1].

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Semiclassical asymptotics of the solution to the problem of scattering of Gaussian beams on a delta-potential

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In the present work we consider the Cauchy problem for the Schrödinger equation with a delta potential localized on a surface M of codimension 1. In this problem the Schrödinger operator is defined as a self adjoint extension of the Schrödinger operator with a smooth potential restricted to the functions vanishing on M. The domain of the operator requires boundary conditions for the solution on the surface M $(y \in M)$:

$$\begin{cases} \psi(y-0,t) = \psi(y+0,t), \\ h(\frac{\partial\psi}{\partial m}(y-0,t) - \frac{\partial\psi}{\partial m}(y+0,t)) = q(y)\psi(y,t). \end{cases}$$

We describe the reflection of a Lagrangian manifold with complex germ and construct an asymptotic solution, using Maslov complex germs theory. The asymptotic is expressed as a linear combination of WKB asymptotic solutions with complex phases, corresponding to the incident, reflected, and transmitted waves. The main terms of the asymptotics are obtained.

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A wave-related integral identity for solutions of Heun equation and for other special functions

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The aim of this talk is to demonstrate that a known integral identity for solutions of the Heun's equation can be interpreted in terms of wave processes described in separated variables. As an example, we consider a 2D problem of diffraction by a segment. We separate the variables for the Helmholtz equation in the elliptic coordinates. For each separation constant, the field is a product of two Mathieu functions. We make an observation that the field on a segment is expressed in terms of a Mathieu function, and, beside, the directivity of the field is expressed in terms of the same function. As it is known, the function on the obstacle is connected with the directivity via the Fourier transform, and this provides the integral relation for the Mathieu function.

Carcasses of dispersion diagrams and pulses in waveguides

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The problem of finding the pulse response of a thin-layered waveguide is studied. The waveguide is described by the matrix Klein–Gordon equation. The field is represented as a sum of Fourier integrals. This representation is interpreted as a contour integral on the complex manifold, that is the dispersion diagram of the waveguide. The saddle-point method is applied to the integral. All possible positions for the saddle points form the so-called carcass of the dispersion diagram. The carcass is the set of points at which the group velocity is real. We demonstrate different types of the carcass for the simplest non-trivial bi-layered waveguide. The complex branches of the carcass correspond to different types of transient pulses.

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Theoretical and experimental analysis of the piezo-induced Lamb waves for identification of defects between a waveguide and surface-mounted objects

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Continuous monitoring of the structural health using acousto-ultrasonic methods is based on the employment of the piezo-induced guided waves, propagating in the structure and interacting with all kinds of structural defects. Applicability of the acousto-ultrasonic methods in damage detecting is confirmed by numerous scientific investigations; methods for signal processing and corresponding damage indices have been developed. However, to proceed from the research field to the industrial field, some problems are to be addressed. Particularly, the reliability of the monitoring system must be estimated in terms of the probability of detection of any damage with the stated size with a certain confidence level. The performance of such analysis is either related to enormous expenses or impossible due to the uniqueness of the inspecting structure. Therefore, it is necessary to employ mathematical models to evaluate responding signal alteration due to various defects. A debonding between the structure and surface-mounted objects, such as piezoelectric transducers and structural stiffeners is investigated and analysis of wave motion is performed using a mathematical model.

A hybrid mathematical model [1] is extended here and employed to simulate Lamb wave excitation and sensing via rectangular piezoelectric-wafer active transducers mounted on the surface of an elastic plate with rectangular surface-bonded obstacles (stiffeners) with interface defects. The model is validated experimentally using laser Doppler vibrometry for fully bonded and semi-debonded rectangular obstacles. A numerical analysis of fundamental Lamb wave scattering via rectangular stiffeners in different bonding states is presented. Two kinds of interfacial defects between the surface-mounted obstacle and the plate are considered: the partial degradation of the adhesive at the interface and an open crack. Damage indices calculated using the data obtained from a sensor are analyzed numerically. The choice of an input impulse function applied at the piezoelectric actuator is discussed from the perspective of the development of guided-wave-based structural health monitoring techniques for damage detection.

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A Sobolev problem with nonlocal conditions on a Riemannian manifold

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A *Sobolev problem* is a (pseudo)differential problem on a closed smooth manifold for which the boundary conditions are posed on a certain submanifold of arbitrary codimension. We deal with a Sobolev problem on a Riemannian manifold whose boundary conditions include the weighted spherical means operator. We study the Fredholm solvability of this problem. As part of the solution, we obtain an expression of the weighted spherical means operator in the form of a Fourier integral operator associated with two-sided geodesic flow.

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New class of the finite-gap Fuchsian equations

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About 20 years ago [1, 2], a class of the finite-gap Heun equations have been proposed

$$\frac{d^2y}{dz^2} + P(z)\frac{dy}{dz} + Q(z)y = 0, \tag{1}$$

where

$$P(z)=\frac{1}{2}\left(\frac{1-2m_1}{z}+\frac{1-2m_2}{z-1}+\frac{1-2m_3}{z-a}\right),\quad Q(z)=\frac{N(N-2m_0-1)z+\lambda}{4z(z-1)(z-a)},\tag{2}$$

$N=m_0+m_1+m_2+m_3$, $m_i\in\mathbb{N}_0$, $\lambda,z\in\mathbb{C}$.

The algebraic genus of the corresponding spectral curve is

$$g=\max\left\{\max_{0\le j\le 3}m_j,\frac{N}{2}-\min_{0\le j\le 3}m_j\right\}\quad\text{for even } N,$$
$$g=\max\left\{\max_{0\le j\le 3}m_j,\frac{N+1}{2}\right\}\quad\text{for odd } N.$$

In 2006 [3], the coefficients (2) were slightly changed: an additional false singular point appeared. The position of the fifth point $z=b$ depends on the parameters a and m_j, $j=0,\dots,3$.

In present work we consider the Fuchsian equation (1) with five regular singular points

$$P(z)=\frac{1}{2}\left(\frac{1}{z-1}+\frac{1}{z+1}+\frac{1}{z-k_1}+\frac{1}{z+k_1}\right),\quad Q(z)=\frac{-\lambda^2-M\lambda z+A_1z^2+A_2z+A_3}{(z^2-1)(z^2-k_1^2)},\tag{3}$$

where λ is a spectral parameter,

$$M=g,\quad A_1=-\frac{g(g+2)}{4},\quad A_2=0,\quad A_3=\frac{g(1+k_1^2)}{4},$$

and g is an algebraic genus of the corresponding hyperelliptic spectral curve.

Examples of the finite-gap solutions to the equation (1), (3) for $g\le 3$ are given.

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Whispering gallery waves' diffraction by boundary inflection: searchlight asymptotics, wave operators, and an integral equation

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We consider a long-standing problem of diffraction of a high-frequency whispering gallery wave by boundary inflection. Like Airy ODE and associated Airy function are fundamental for describing transition from oscillatory to exponentially decaying asymptotic behaviors, the boundary inflection problem leads to an arguably equally fundamental canonical inner boundary-value problem for a special PDE describing transition from a "modal" to a "scattered" high-frequency asymptotic behaviors. This is a Schrödinger equation on a half-line with a potential linear in both space and time. The latter problem was first formulated and analysed by M.M. Popov starting from 1970-s, and has been intensively studied since then (see recent paper [1] for a review and some further references).

The associated solutions have asymptotic behaviors with a discrete spectrum at one end and with a continuous spectrum at the other end, and of central interest is to find the map connecting the above two asymptotic regimes. We report recent result in [1] proving that the solution past the inflection point has a "searchlight" asymptotics corresponding to a beam concentrated near the limit ray. This is achieved by a a non-standard perturbation analysis at the continuous spectrum end, and the result allows interpretations in terms of associated wave operators and of a scattering operator connecting the modal and the scattered asymptotic regimes.

We also report some most recent progress on a reduction of the problem to one-dimensional boundary integral equations and on their further analysis. The integral equations are of improper weakly singular Volterra type of both first and second kinds (with appropriate jump conditions for the latter), and can be shown to be well-posed. Their subsequent regularization allows to express the solution in term of uniformly convergent Neumann series with some further benefits for the problem's asymptotic analysis.

Some parts of the reported work are joint with Ilia Kamotski, and with Shiza Naqvi.

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Simple unidirectional electromagnetic few-cycle pulses

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We present a family of simple electromagnetic finite-energy pulses, which are free of backward-propagating components. The mathematical framework comprises using Hertz's vector potential built on the basis of the axisymmetric exact solution of the three-dimensional wave equation in a linear isotropic medium [1, 2]

$$u_+ = \frac{1}{S(z_* - S)}\,, \quad \text{where} \quad S = \sqrt{c^2 t_*^2 - x^2 - y^2}\,, \quad z_* = z + i\zeta, \quad t_* = t + i\tau\,, \tag{1}$$

with free real parameters ζ and τ. If fixing the branch of the square root so that $S|_{x=0,y=0} = \sqrt{c^2 t_*^2} = ct_*$, the pulses propagate along the positive z-direction. The absence of singularities is ensured by the condition $\frac{\zeta}{\tau} < c$, where $c > 0$ is the speed of wave propagation in the medium.

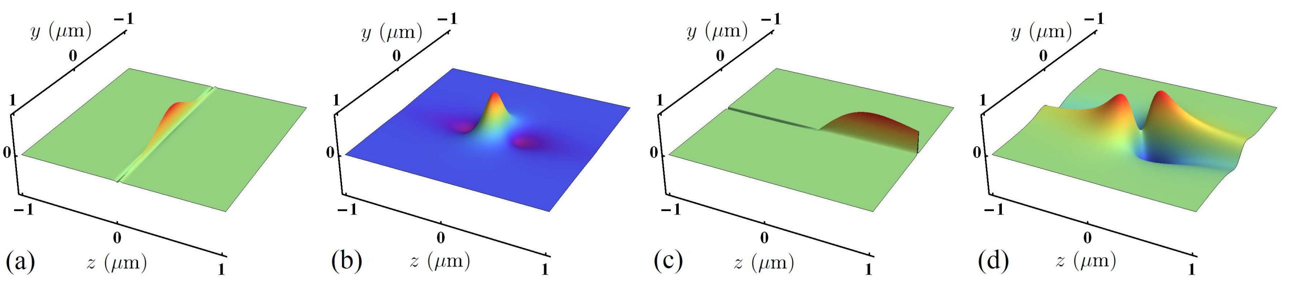

Fig. 1: Transverse components of electric fields for (a) pancake, (b) ball, (c) needle, and (d) doughnut pulses at $x = 0$ corresponding to (b),(d) real and (a),(c) imaginary parts of complex wavefields obtained via Hertz's potential based on u_+ (1).

Different ratios between the free parameters ζ and τ can yield, in particular, pancake, ball, and needle pulses if Hertz's potential is orthogonal to the z-axis. Furthermore, doughnuts are obtained via longitudinally oriented Hertz's potential – the technique feasible for any axisymmetric solution (see, e.g., [3]).

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Propagation on broadband signals in shallow-to-deep scenario: preservation and eventual loss of modal components' identity

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In the presented study the modelling of acoustic field of propagating from the shelf to the deep water is performed. Consider an model experimental path of length of 200 km was located in the Sea of Japan, a source of signals was deployed on the shelf and a receiver was located in deep-water part of the Sea of Japan. The propagation path consists of a 30 km long shallow-water part and a 150 km long deep ocean part.

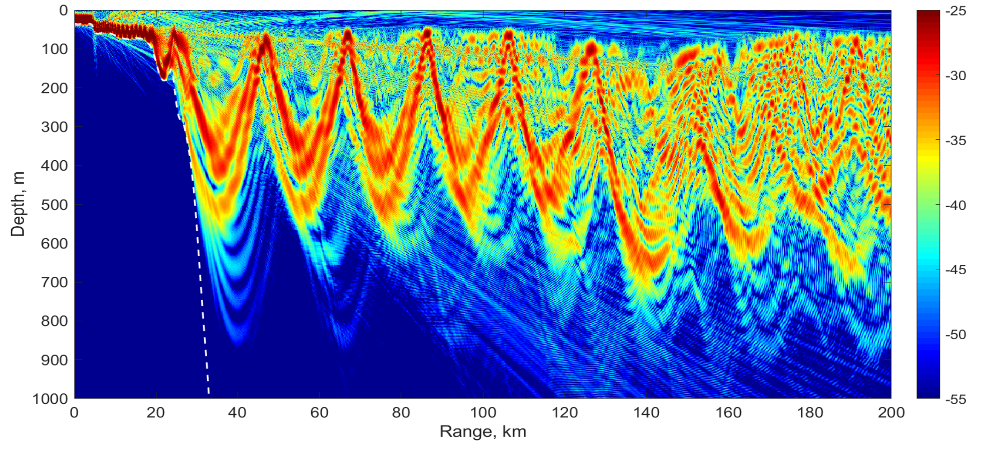

Fig. 1: Acoustic field of separate mode No. 1.

In order to analyze modal arrival times and study the physics of propagation in this scenario we performed separate simulation of the propagation of different modes formed by the source on the shelf. Individual acoustic modes were used as an initial condition for wide-angled parabolic equation, and acoustic acoustic field along the considered path was calculated. Interaction between the modes was studied by performing modal decomposition of the resulting field. It is shown that acoustic modes propagates almost adiabatically along the shallow-water part of the path, while in the relatively small transitional area of the continental slope modal identity is erased due to strong modal coupling. Despite that, modal components propagate with almost equal velocities along the deep-water part of the path, and arrival times of individual modes can be associated, for example, with individual peaks of impulse response of hydrophone.

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Heun's differential equation, integral transformation and q-deformation

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Kazakov and Slavyanov have contributed greatly to the study of Heun's differential equation and related equations. In particular, they established Euler integral symmetries for the Heun equation [1] and investigated relationship between the (deformed) Heun equation and the Painlevé VI equation [2].

On the other hand, the Euler integral symmetries for the Heun equation and the deformed Heun equation were studied from the perspective of the middle convolution [3, 4].

A q-deformation of the Heun's differential equation was found by Hahn [5], and it was rediscovered in [6] by considering degenerations of the Ruijsenaars-van Diejen system. The q-Heun equation is written as

$$\{a_0 + a_1x + a_2x^2\}g(x/q) + \{b_0 + b_1x + b_2x^2\}g(x) + \{c_0 + c_1x + c_2x^2\}g(qx) = 0,$$

under the condition $a_0a_2c_0c_2 \neq 0$. Some basic properties of the q-Heun equation and its variants were investigated in [7].

Recently we discovered a q-integral symmetry of the q-Heun equation [8], which was obtained by applying a q-deformation of the middle convolution established by Sakai and Yamaguchi [9].

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On a two dimensional inverse source problem in a scattering medium with partial boundary data

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This talk concerns an inverse source problem for the linearized Boltzmann equation in two dimensions. The medium is assumed known. The outgoing radiation is measured on an arc of the boundary. For scattering kernels dependent on the angle of scattering, we show that a source can be recovered in the convex hull of the measuring arc.

This is joint work with K. Sadiq of Radon Institute, Linz, Austria and H. Fujiwara of Kyoto University, Kyoto, Japan.

Solution methods for the Helmholtz equation within and across thin layers

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Wave propagation within layered media and circumstances where rapid variations in the local amplitude profile of a wave field occur across spatially 'thin' layers are both ubiquitous in nature and have been the subject of close scrutiny for centuries.

Associated experimental and theoretical developments have gone hand-in-hand and some of the mathematical challenges that have arisen have led to the formulation of mathematical techniques and methodologies with wider-reaching applications, for example Keller's celebrated Geometrical Theory of Diffraction.

The purpose of this talk is to mention some of these developments and applications that have been prominent over the last few decades and, wherever possible, to identify links between them. Those chosen — which include (i) complex ray theory (ii) scattering by slender bodies (iii) Friedlander–Keller ray expansions (iv) cloaking and (iv) scattering at points of inflection — do not form an exhaustive list and these particular topics are chosen in part because they are amongst those most familiar to the presenting author. It is hoped that this will engender an ongoing research dialogue within the context of modern and applicable wave theory.

Nonlinear guided electromagnetic waves in circle cylindrical waveguide

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The report focuses on the study of monochromatic electromagnetic wave propagation in a circle cylindrical waveguide filled with nonlinear medium. The waveguide is a nonlinear dielectric layer on a metallic rod with perfectly conducting coating on its outer surface. The metallic rod is of thickness R_1, the dielectric layer is of thickness R. Thus the waveguide can be considered as the so called Goubau line of radii $R_2 = R_1 + R$ with perfectly conducting coating. The dielectric permittivity within the waveguide is described by the Kerr law and is characterized by the nonlinearity coefficient α; coefficient α is assumed to be dependent on the radius of the waveguide. We look for guided transverse-electric (TE) wave propagating in the dielectric layer. The guided waves are characterized by propagation constants (PCs) that are solutions to a boundary eigenvalue problem for Maxwell's equations with appropriate boundary conditions. Statement of the problem is similar to that given in [1], where one just needs to replace interfaces at $\rho = R_1, R_2$ with perfectly conducted boundaries.

If $\alpha = 0$, then the nonlinear problem degenerates into the linear one. It is well known that there is only a finite number of guided TE waves in the linear problem, that is there is only a finite number of PCs in this case.

If α is sufficiently small, then one can apply a classical perturbation approach to find "nonlinear" PCs that are close to the corresponding linear solutions [1, 2]. Although this approach is quite simple and commonly used in practice, it has at least two significant disadvantages. Firstly, this approach is based on the usage of linear solutions (PCs) and, therefore, it cannot be applied if one wishes to determine nonlinear solutions without linear counterparts. Secondly, it can be applied only for small values of the nonlinearity coefficient.

In the paper we suggest a modern approach called the integral characteristic equation method for studying the considered problem [3], which is essentially based on asymptotical evaluation of the integral characteristic function. This approach does not have disadvantages mentioned above. To be more precise, the method can be applied for arbitrary positive values of the nonlinearity coefficient α and it allows one to prove existence of "nonlinear" PCs that do not have linear counterparts. In the problem under consideration this approach allows proving existence of a novel type of guided waves without linear counterparts. We also present numerical results and provide comparison between the nonlinear and linear cases.

The work was financially supported by the Russian Science Foundation (grant no. 18-71-10015).

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Regularization of the boundary control method via the mean curvature flow

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The most remarkable feature of the Boundary Control Method (BCM) is that a nonlinear inverse problem for a dynamical system is reduced to solving a system of algebraic linear equations with the Gram matrix. However, the latter is extremely ill-conditioned in high dimensions. A novelty is that a regularized mean curvature flow model is introduced and developed to stabilize numerical implementations of the BCM.

Let G be a linear operator generated by the Gram matrix. Consider an operator equation $Gu = v$ in Banach spaces and assume that u is defined in a bounded domain Ω and it is smooth. To find the regularized solutions, we minimize the Tikhonov functional $T_\alpha(u) = \frac{1}{2}\|Gu - v\|^2_{L^2} + \alpha S(u)$, $\alpha > 0$, in which the stabilizer $S(u)$ is the norm in the space of functions of bounded variation, i.e., $S(x) = \|u\|_{BV(\Omega)} = \|u\|_{L^1(\Omega)} + E(u)$, and $E(u) = \int_\Omega |\nabla u|$ is the total variation of u. Based of the Euler–Lagrange equation for T_α, the mean curvature flow problem

$$\partial_\tau u_{\alpha\varepsilon} = g_{\alpha\varepsilon}\nabla \cdot \left(\left[g^{-1}_{\alpha\varepsilon} + \alpha\right] \nabla u_{\alpha\varepsilon}\right) - G^*(Gu_{\alpha\varepsilon} - v),\ g_{\alpha\varepsilon} = (|\nabla u_{\alpha\varepsilon}|^2 + \varepsilon^2)^{1/2},\ \tau > 0,\ u_{\alpha\varepsilon} = u_0,\quad \tau = 0$$

subject to the zero Neumann boundary condition is introduced. It is shown that for sufficiently small $\varepsilon > 0, \alpha \in (0,1)$ and at sufficiently large τ the regularized solutions $u_{\alpha\varepsilon}$ approximate u. The computational effectiveness of the proposed approach is demonstrated in the numerical experiments. One example of reconstruction of the mass density is shown below in comparison with the standard Tikhonov H^1-regularization.

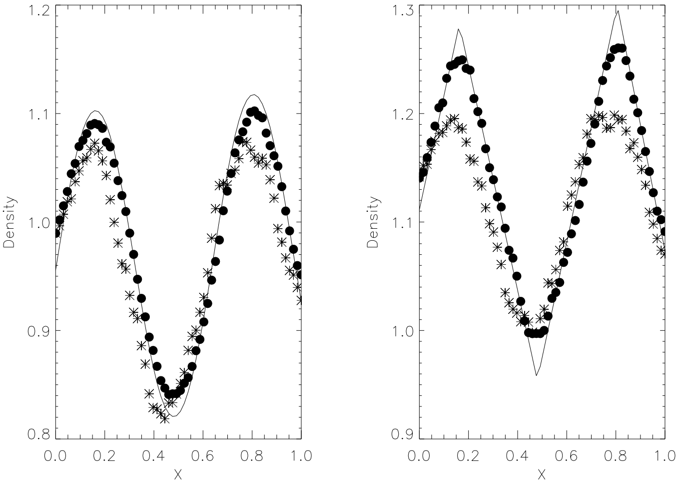

Figure: Means of reconstructions for the smooth (left) and piecewise linear (right) model density with the 10% level noise: the solid line – for the model density; asterisks – for the Tikhonov H^1-regularization; bullets – for the total variation minimization.

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Stability of equilibrium states of the Protosphera

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We analyze the equilibrium states of a plasma with axial-symmetry located in a cylindrical vessel V with an entrance for the gas ("gun") and a positive and negative electrodes located respectively at the top and the bottom of the vessel V. An example of such system is the Protosphera [1]. A magnetic field $\mathbf{B}$ acts in the plasma with magnetic energy $\int_V dV\mathbf{B}^2/2\mu_0$ much larger than the pressure forces (force-free magnetic field). $\mathbf{B}$ satisfies the following condition inside V and is zero outside

$$\langle \nabla \times \mathbf{B} \rangle = \lambda \mathbf{B}. \tag{1}$$

These states are named Taylor states, they are characterized by a parameter λ. They are local minima of the magnetic energy so their stability gives the stability of the plasma. We analyze the case of nonlinear dependence of $\lambda(\psi)$ with respect to the poloidal flux function $\psi = \int_S dS\mathbf{B} \cdot \mathbf{n}$ where S is a surface orthogonal with respect to the z axis. $\mathbf{B}$ can be represented by the formula

$$\mathbf{B} = \frac{1}{2\pi}(\nabla\psi \times \nabla\phi + \mu_0 I \nabla\phi), \tag{2}$$

μ_0 is the magnetic permeability, ϕ is the toroidal angle, I is a suitable functional of ϕ satisfying the relation $\mu_0 \frac{dI}{d\psi} = \lambda(\psi)$.

In particular we find conditions on the magnetic field and on ψ such that the energy loss due to dissipation due to Ohmic resistance is compensated (Dynamo effect). We construct non axis-symmetric perturbation $\tilde{\phi}, \tilde{\mathbf{B}}, \tilde{\mathbf{A}}$, such that the local minima are stable. We use also the concept of helicity

$$H = \int_V \mathbf{A} \cdot \mathbf{B} dV, \tag{3}$$

where $\mathbf{A}$ is the vector potential of $\mathbf{B}$.

The flux of helicity brought by the incoming gas can be described

$$\mathbf{h} = -D_K \nabla\lambda, \tag{4}$$

where D_K is an experimental constant and

$$\mathbf{h} = -\left\langle 2\tilde{\phi}\tilde{\mathbf{B}} + \tilde{\mathbf{A}} \times \frac{\partial \tilde{\mathbf{A}}}{\partial t} \right\rangle. \tag{5}$$

From the minimization of the energy functional with an applied current J one gets, where η is the resistivity

$$-\frac{1}{2\mu_0}\mathbf{h} \cdot \nabla\lambda = \eta J^2 \tag{6}$$

([2]). Using the equation (4) we get the equation to be solved

$$\frac{D_K}{2\mu_0}|\nabla\lambda|^2 = \eta J^2. \tag{7}$$

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On the effect of intersection of characteristics in a two-dimensional massless Dirac equation with linear potential and localized initial data

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We consider the Cauchy problem with localized initial data for the two-dimensional massless Dirac equation (it describes quantum states in graphene [1]) with linear potential.

Applying the h-Fourier transform in spatial variables to the solution Ψ, we can write the expansion $\widetilde{\Psi}(p,t)$ of the form $\widetilde{\Psi} = \sum_{\sigma\in\pm} e^{iS^{\pm}/h}\eta^{\pm}$ (mode expansion $\eta^{\pm}$). The specificity of this problem (associated with the effect of changing the multiplicity) is that the phases and modes of this decompositions have different form depending on the value of $p_2/h^{\frac{1}{2}}$. For $p_2 \gg h^{\frac{1}{2}}$, the asymptotics is constructed in the standard form WKB, where the modes are expanded in powers of h ([2]). For $|p_2| \ll h^{\frac{1}{2}}$ the asymptotics is constructed by the Kucherenko method [3] using Duhamel principle (although a different situation with a smooth intersection of characteristics).

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An application of the method of real-valued semiclassical approximation for the asymptotics with complex-valued phases to multiple orthogonal Hermite polynomials

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The multiple orthogonal Hermite polynomials $H_{n_1,n_2}(z,a)$ are defined by the following recurrence relations ([1]):

$$H_{n_1+1,n_2}(z,a) = (z+a)H_{n_1,n_2}(z,a) - \frac{1}{2}\left(n_1 H_{n_1-1,n_2}(z,a) + n_2 H_{n_1,n_2-1}(z,a)\right),$$
$$H_{n_1,n_2+1}(z,a) = (z-a)H_{n_1,n_2}(z,a) - \frac{1}{2}\left(n_1 H_{n_1-1,n_2}(z,a) + n_2 H_{n_1,n_2-1}(z,a)\right).$$

We construct the uniform Plancherel–Rotach-type asymptotics of diagonal polynomials $H_{n,n}(z,a)$ as $n \to \infty$. To obtain the result we develop the method of the real-valued semiclassical approximation for the asymptotics with complex-valued phases, using some ideas from [2]. We reduce the system that defines the polynomials to a pseudo-differential equation. The feature of the problem is that the symbol (the Hamiltonian) of the corresponding operator is complex-valued. We suggest a method that reduces the problem to three equations with real-valued symbols (Hamiltonians). This allow us to get rid of the complex-valued Hamiltonians and obtain uniform asymptotics of $H_{n,n}(z,a)$ in the form of the Airy function Ai of a complex argument.

The talk is based on the joint work with A. I. Aptekarev, S. Yu. Dobrokhotov and D. N. Tulyakov.

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On the relation between the *T*-matrices for different spheroidal and spherical bases in the axisymmetric problem of light scattering by a spheroid

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We consider T-matrices in the axisymmetric light scattering problems, i.e. when the fields do not depend on the azimuthal angle. It occurs when the radiation source is a dipole located at the symmetry axis of a spheroid and the dipole moment is parallel to this axis. The axisymmetric problem is also a part of the plane wave scattering problem within the approach suggested in the case of a spheroid in [1]. In the best known formulation of the light scattering problem for a spheroid that is a generalization of the Mie theory for a sphere [2], the axisymmetric problem corresponds to the separate problem for the azimuthal number $m = 0$. In the field expansions used in the both mentioned papers, the spheroidal wave functions were utilized as the scalar basis. Such a basis corresponds to the problem geometry but in numerical realization causes some problems related to difficulties in calculations of these functions in a wide range of parameter values.

In another approach (see, e.g., [3, 4]) the basis is formed by the spherical wave functions whose computations do not meet problems. However, in this case the infinite systems of linear algebraic equations relative to the unknown field expansion coefficients become ill-conditioned for large aspect ratios ($a/b \le 10 - 20$) [5].

Our analysis of the relations between the spheroidal and spherical T-matrices derived in different bases opens the way to overcome the difficulties of both approaches. Namely, for strongly elongated or flattened spheroids, we solve the problem applying a proper spheroidal basis [1] in which case there is no limitations of the aspect ratio a/b, and then make the transition from the spheroidal wave functions to the spherical ones. As a final step, we transform the spherical T-matrix in the basis of [4] into the standard T-matrix in the basis of [3]. The result can be further used for solution of application problems, which is demonstrated by our numerical illustrations.

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Superwaves create complicated phenotypes by short genotypes

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In this paper, we consider excitable media. We show that in these media the propagation of waves with chaotic and time periodic fronts is possible, as well as waves, which transfer associative memory. The time evolution of medium state at the wave fronts are determined by attractors. We can completely control these attractors by initial data choice. By those results we show that the number of genes needed for morphogenesis weakly depends on organism size but sharply depends on the number of cell types involved in morphogenesis.

Such waves and chaotic regimes arise in many systems including ecosystems and systems of chemical kinetics.

The paper is conjoint with Ivan Sudakov (Dayton and Novgorod University), Dmitry Grigoriev (Lille) and John Reinitz (University of Chicago).

Propagation of global analytic singularities for Schrödinger equations with quadratic hamiltonians

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We study the propagation in time of 1/2-Gelfand–Shilov singularities, i.e. global analytic singularities, of tempered distributional solutions of the initial value problem

$$\begin{cases} \partial_t u + q^w(x, D)u = 0, \\ u|_{t=0} = u_0, \end{cases}$$

on $\mathbb{R}^n$, where u_0 is a tempered distribution on $\mathbb{R}^n$, $q = q(x,\xi)$ is a complex-valued quadratic form on $\mathbb{R}^{2n} = \mathbb{R}^n_x \times \mathbb{R}^n_\xi$ with nonnegative real part $\operatorname{Re} q \geq 0$, and $q^w(x,D)$ is the Weyl quantization of q. We prove that the 1/2-Gelfand–Shilov singularities of the initial data that are contained within a distinguished linear subspace of the phase space $\mathbb{R}^{2n}$, called the *singular space of* q, are transported by the Hamilton flow of $\operatorname{Im} q$, while all other 1/2-Gelfand–Shilov singularities are instantaneously regularized. Our result extends the observation of Hitrik, Pravda–Starov, and Viola '18 that this evolution is instantaneously globally analytically regularizing when the singular space of q is trivial.

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Dispersion and attenuation of fundamental edge waves in a two-layered plate: 3D solution and approximate theories

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Edge waves (EW) are guided waves in thin-walled structures, which are similar to Lamb waves but strongly localized in the vicinity of edges. They are thoroughly studied in the case of a homogeneous plate (see [1]). It is shown theoretically and experimentally [2, 3], that the EW-family contains two fundamental waves and infinitely many high-order waves. In the case of a laminated plate, only the fundamental waves were studied up to recent time on the basis of 2D plate theories, which reduce the laminate to a homogeneous plate with some effective stiffnesses.

In this work, the 3D theory of elastodynamics is used to study fundamental EWs in a semi-infinite two-layered plate composed of two dissimilar isotropic plates. The semi-analytical method based on modal expansion is employed for numerical investigation of dispersion properties of waves, propagating along the edge ot the plate under consideration. The most interesting property of EWs in a two-layered plate is the attenuation of the second fundamental wave, caused by coupling between quasi-symmetric and propagating quasi-antisymmetric modes.

In the first part of the work the plate composed of two dissimilar plate with elastic moduli and thicknesses being of the same order (e.g. aluminium 3 mm and steel 1 mm). The dispersion curves obtained on the basis of 3D theory are compared to those calculated with the use of the classical laminated plate theory [4]. The limits of applicability of the latter are discussed.

In the second part the isotropic plate coated by a thin soft film is considered as a two-layered plate. The limits of applicability of the effective boundary condition [5] by describing of edge waves are studied by comparison with the 3D solution.

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Inverse boundary problems for biharmonic operators in transversally anisotropic geometries

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We study inverse boundary problems for first order perturbations of the biharmonic operator on a conformally transversally anisotropic Riemannian manifold of dimension $n \geq 3$. We show that a continuous first order perturbation can be determined uniquely from the knowledge of the set of the Cauchy data on the boundary of the manifold provided that the geodesic X-ray transform on the transversal manifold is injective.

Exact formulation of nonlocal transparent boundary conditions in wave problems for a layered or FG elastic strip, plate, and cylinder

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The problem how to formulate the exact non-reflecting boundary conditions (NBC) is investigated in context of NDT needs. For this purpose, an elastic region consisting of a certain core of finite size (S1) and attached infinite solid (S2) in the form of 2D semi-strip or 3D infinite plate or semi-cylinder is considered. Normally the solid S1 is the solid of “inspection” and the impact of S2 is finally replaced by the respective NBC. The infinite solid S2 is assumed to be free from any source of energy providing the energy flow from the infinity, i.e., its energy exchange is possible only via the virtual boundary of continuous contact in between S1 and S2. Thus, the statement of the total problem can be subdivided into respective subproblems due to the superposition principle when necessarily. The faces of S2 can be stress-free, rigidly clamped, or combine these homogeneous boundary conditions on different sides or in the different directions. The materials of S2 can be transversely isotropic, layered or functionally gradient. For the time harmonic problem under such assumptions, the wave field of S2 is firstly presented as a series of eigenfunctions satisfying the radiation principle, then the sought NBC are formulated as the demand of zero magnitudes of non desirable waves using the generalized orthogonality relations. Secondly, the NBC obtained above are reduced to the system of the boundary integral equations (BIE) of special kind with two singular and one hyper singular equation. The properties and potentials use of resulting BIE is discussed.

Edge-scattering of Gaussian beams and geometrical theory of diffraction

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In this talk we discuss application of geometrical theory of diffraction (GTD) to solving scattering problems of Gaussian beams by edges of perfectly conducting (PEC) screens in a frequency domain in case of the short-wave approximation. Basically our asymptotic analysis is confined by studying two canonical problems, namely, constructing uniform GTD asymptotic solutions to scattering of Gaussian beams of Gauss–Hermit type by flat PEC disk and diffraction by circular aperture in a PEC screens. A well-known scalar wave propagation model based on Helmholtz equation is being used to develop the asymptotic analysis. For both problems it is assumed that the wave length of the incident beam is much less than the geometric parameters of the problems: waist of the incident

beam, radius of curvature of disk or aperture edges, and corresponding distance as we pay a particular interest to description of the diffracted fields in the near-filed zone. We consider a case when waist of the beam is comparable to the size of a disk or aperture.

From the point of view of historical background it is worth remarking that all the basic points of mathematical foundations of GTD could be found in the famous monograph [1]. The presented analysis is based on the principles of uniform GTD described in [1]. A well-known short-wave asymptotic GTD solution of point source diffraction by similar disks or aperture was employed in order to develop the current analysis as the Gaussian beam paraxial approximation is obtained by complexification of a point source solution. Thus, we utilize the short-wave asymptotic GDT solution for plane wave scattering by a disk that was developed in [2]. In this talk we also derived short-wave asymptotic solutions for the corresponding problems using physical optics (PO) methods (Kirchoff approximation). In numerical analysis interference pictures of intensity distributions of the diffracted fields in the near-field zone to compare GDT and PO results are presented. Details of accuracy analysis in applicability of GTD and PO methods are discussed for both canonical problems. In future prospectives, it is assumed to study a short-wave scattering of vectorial electromagnetic beams of Gauss–Laguerre type (optical vortexes) by edges of screens, wedges and apertures, by applying uniform GTD analysis. As it is of great interest to analyse the evolution of corresponding topological charges of the incident beams due to a diffraction process [3].

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Over-reflection of acoustic waves by compressible boundary layer flows

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The acoustic wave equation for inviscid compressible boundary layer flows, i.e. the Pridmore–Brown equation, is studied for the over-reflection of acoustic waves based on the exact solution in terms of the confluent Heun function. As shown in figure 1(a), an acoustic wave with frequency $\widetilde{\omega}$, wavenumber vector $\widetilde{\boldsymbol{k}}$ propagates at an angle Θ in the free stream and incident on an exponential boundary layer bounded by a rigid wall, giving rise to a reflected wave, characterized by the reflection coefficient R. The projection of $\widetilde{\boldsymbol{k}}$ in the horizontal direction is the streamwise wavenumber $\widetilde{\alpha}$, and their angle is denoted by ϕ (incident angle). In our study, the reflection coefficient R is given as a function of the dimensionless streamwise wavenumber α, the Mach number M and the incident angle of acoustic waves ϕ, and computed numerically.

It is shown that the over-reflection ($R > 1$) of acoustic waves arises in boundary layer flows, i.e. reflected amplitudes are greater than incident amplitudes. The phenomenon has been validated to be closely related to the critical layer y_c, at which there is a jump in the energy flux across the critical layer. In figure 1(b), a special acoustic phenomenon, the resonant over-reflection, is observed and proved to be caused by resonant frequencies ω_r induced by unstable modes of the temporal stability. At resonant frequencies induced by the first unstable mode, the over-reflection coefficient has an unusual peak in an extremely narrow frequency interval, as shown in figure 1(b).

The maximum values of these peaks are largely synchronised with the variation of the growth rate ω_i of the unstable modes. In addition, the resonant over-reflection appears also at resonant frequencies caused by higher unstable modes, but their peaks of the over-reflection coefficient are always smaller than that caused by the first unstable mode.

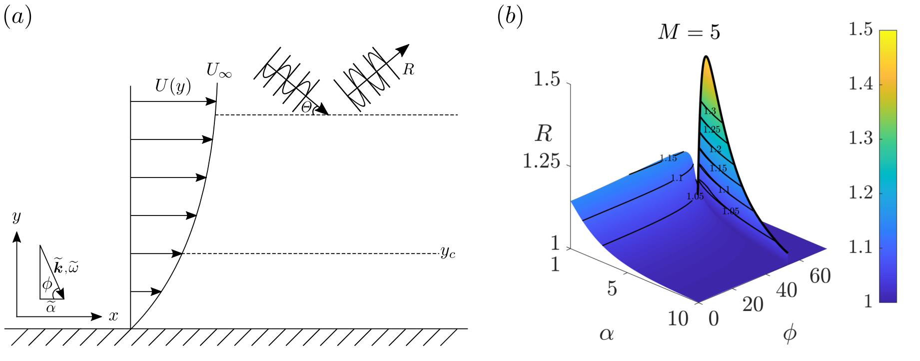

Fig. 1: (a) Illustration of an exponential boundary layer flow. (b) Over-reflection coefficient R as a function of α and ϕ in propagation domains at $M = 5$.

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Alexey Popov's diffraction by a jump of curvature revised

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We are concerned with construction of high-frequency asymptotic formulas for the wavefield described by

$$u_{xx} + u_{yy} + k^2 u = 0, \quad \partial_n u|_C = 0, \tag{1}$$

where $k \to \infty$ is the wavenumber and ∂_n denotes the derivative along the normal to a contour C. The boundary C is a sum of the half-line C_- and the piece of smooth contour C_+, as shown in Fig. 1, with a jump in curvature at the conjugation point O. The incident field is a plane wave $u^{\text{inc}} = e^{ikx}$, and $u = u^{\text{inc}} + u^{\text{out}}$, where u^{out} is the outgoing wave. The problem has been earlier addressed in [1] by Alexey Vladimirovich Popov who ingeniously used a combination of parabolic-equation approach, Kirchhoff-type heuristics and Malyuzhinets technique to derive an expression for the cylindrical wave arising at the non-smoothness point O.

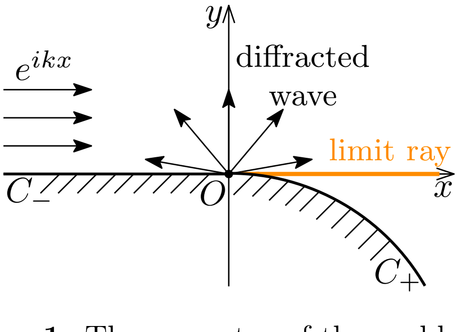

Fig. 1: The geometry of the problem.

We describe the outgoing wavefield u^{out} in a neighborhood of O through a formal employment of the Leontovich–Fock parabolic-equation method [2, 3]. With this we continue our work on the systematic application of boundary-layer techniques to diffraction by a jump of curvature and similar problems [4, 5]. We introduce the standard stretched coordinates [2, 3]

$$S = k^{\frac{1}{3}} s, \quad N = k^{\frac{2}{3}} n,$$

where s is the arc length of the contour C measured from O and n is the length of normal to C, and seek the outgoing field in the form of the Leontovich–Fock Anzatz:

$$u^{\text{out}} = e^{iks} W(S, N).$$

Substitution of the Anzatz in (1) gives, to the main order, the parabolic equation

$$W_{NN} + 2iW_S + 2\varkappa H(S) N W = 0 \tag{2}$$

with the boundary condition

$$W_N|_{N=0} = -i\varkappa H(S) S e^{-i\varkappa \frac{S^3}{6}}. \tag{3}$$

Here, $H(S) = \{1, S > 0;\, 0, S \leq 0\}$ is the Heaviside function, $\varkappa$ is the value of curvature of C_+ at the point O.

We explicitly solve the problem (2)–(3). The solution is, in a sense, similar to that presented in [2, 3] for a smooth contour, but with some classical Airy functions replaced by incomplete Airy functions. We derive formulas for the wavefield in the vicinity of the limit ray (Fig. 1). Similarly to [2, 3], the wavefield is a sum of the classical Fresnel field and a background field which is described by a novel special function. An expression for the diffracted wave which we obtain agrees with the one found in [1].

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