INTERNATIONAL CONFERENCE

DAYS ON DIFFRACTION 2019

ABSTRACTS

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FOREWORD

"Days on Diffraction" is an annual conference taking place in May–June in St. Petersburg since 1968. The present event is organized by St. Petersburg Department of the Steklov Mathematical Institute, St. Petersburg State University, and the Euler International Mathematical Institute.

The abstracts of 139 talks to be presented at oral and poster sessions during 5 days of the conference form the contents of this booklet. The author index is located on the last pages.

Full-length texts of selected talks will be published in the Conference Proceedings. They must be prepared in LaTeX format and sent to `diffraction2019@gmail.com` not later than 24 June 2019. Format file and instructions can be found at `http://www.pdmi.ras.ru/~dd/proceedings.php`. The final judgement on accepting the paper for the Proceedings will be made by the Editorial Board after peer reviewing.

As always, it is our pleasure to see in St. Petersburg active researchers in the field of Diffraction Theory from all over the world.

Organizing Committee

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List of talks

Uniqueness and reconstruction in time-harmonic passive inverse problems with applications to helioseismology

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In this talk I will report on some recent uniqueness and reconstruction results for time-harmonic inverse problems from cross correlations of randomly excited waves. The main focus will be on recovering the solar sound speed, density and attenuation from the cross correlations of acoustic waves excited by turbulent convection.

The results presented in this talk are obtained in collaboration with T. Hohage and R. G. Novikov.

On the structure of the acoustic field in the sea containing a developed bubble layer near the surface

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Under the action of a strong wind and developed surface waves there are powerful bubble layers that can reach significant depths of tens of meters. Bubbles have a significant effect on the acoustic properties of water, leading to excessive absorption and dispersion of sound velocity. In a number of works it is stated that the surface layer of bubbles weakly affects the attenuation of sound in the sea up to high wind speeds (Weston, Novarini, et al., etc.). Other works (Willy, Schneider, etc.) show that the contribution of bubbles to sound attenuation is predominant at frequencies from 1 to 8 kHz in shallow sea.

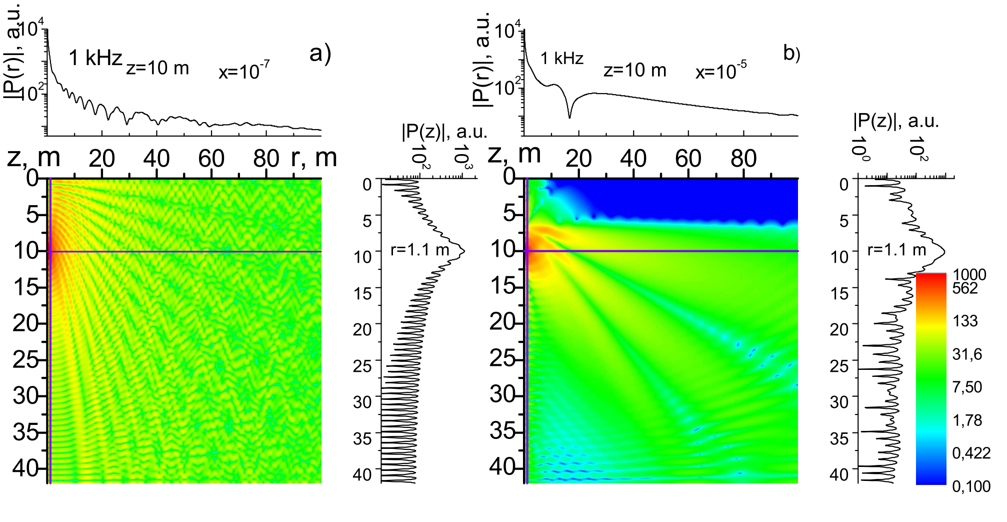

Fig. 1: Acoustic field with a frequency $f = 1$ kHz for a source at a depth of 10 meters in an isoscale channel with a depth of 42 meters in the presence of a near-surface bubble layer 7 meters thick with a different concentration of bubbles: a) $x = 10^{-7}$, b) $x = 10^{-5}$.

The influence of the near-surface layer of bubbles on the sound attenuation in the sea is analyzed with the use of the latest experimental results for the bubble size distribution function $g(R)$ obtained at different states of the sea surface. It is shown that the influence of the near-surface layer of bubbles on the sound propagation can be significant at typical concentrations of bubbles in the near-surface layers of the sea. The analytical estimation of coherent field attenuation length in the conditions of the surface of the underwater sound channel was conducted. For a detailed study of the numerical simulation we used the approximation of normal modes. It is shown that the influence of the near-surface layer of bubbles is an additional field decay at moderate distances caused by the attenuation of the sound energy propagating in the bubble layer. In the future, this energy is attenuated, which eventually leads to the absence of the contribution of the bubble layer in the exponential law. It should be noted that the presence of dissipation in the near-surface layer of bubbles with an increase in the concentration of $x > 10^{-7}$ can lead to a significant restructuring of the acoustic field structure.

The figure shows a 2D image of the acoustic field for different concentrations of bubbles in the near-surface layer. Calculations show a strong change in the structure of the acoustic field in excess of the concentration of bubbles equal to 10^{-7}. For this concentration of bubbles, the acoustic field in the bubble layer near the surface attenuates at a distance of about 100 m.

This work is performed under the state assignment No. 0271-2019-0009 and partially supported by RFBR grants No. 17-02-00561_a and the "Far East" No. 18-I-004.

Extreme scattering properties of small objects with compensated loss

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Scattering properties of small passive objects (we will focus on electromagnetic scattering, although most of the results are more general) are limited by the optical theorem and inevitable scattering loss even for lossless objects. We are interested in possibilities to overcome these limitations and create particles exhibiting extraordinary scattering response. Our approach is to compensate scattering loss generalizing the concept of parity-time symmetric structures for finite-size scatterers in infinite free space [1]. In particular, we introduce loss-compensated shadow-free scatterers. We show that compensation of loss for particles in free space does not correspond to a parity-time symmetric system. Although the forward scattering amplitude and the extinction cross section can vanish, these particles scatter incident energy into other directions, with controllable directionality. These loss-compensated scatterers possess extreme electromagnetic properties not achievable for passive and lossless scatterers. As an example, we study structures consisting of two or three closely positioned small dipole scatters and consider possible microwave realizations in the form of short dipole antennas loaded by lumped elements. The proposed particles with extraordinary response theoretically enable most unusual material properties when used as building blocks artificial media.

References

[1] M. Safari, M. Albooyeh, C. R. Simovski, S. A. Tretyakov, Shadow-free multimers as extreme-performance meta-atoms, *Phys. Rev. B*, **97**, 085412 (2018).

Localized solutions of two-dimensional linearized shallow water equation near a shore

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We discuss localized solutions for the two-dimensional linearized shallow water equation. These solutions correspond to the piston model of tsunami propagation. An asymptotic formula for the solution valid in a small neighborhood of the shore is proposed. We analyze this formula and study how parameters of the source affect a shape of an incoming and reflected wave profiles. This work was supported by the Russian Science Foundation (project no. 16-11-10282).

Asymptotic eigenfunctions of the 2D operator $\nabla D(x)\nabla$ degenerating on the boundary of the domain and billiards with semi-rigid walls

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We consider a spectral problem

$$\langle \nabla, D(x)\nabla\psi(x)\rangle = -\omega^2\psi(x), \quad x \in \Omega \subset \mathbb{R}^2,$$

where Ω is a finite domain with a smooth boundary $\partial\Omega$, $D(x) \in C^\infty(\overline{\Omega}) : D(x) > 0,\ x \in \Omega$, $D(x) = 0,\ \nabla D(x) \neq 0,\ x \in \partial\Omega$. We construct asymptotic eigenfunctions for large ω that can be represented with the use of a modified canonical operator ([1]). These eigenfunctions are associated with analogues of Liouville tori of integrable geodesic flows with a metric degenerating on $\partial\Omega$ defined by a Hamiltonian system with a Hamiltonian $D(x_1, x_2)(p_1^2 + p_2^2)$. This problem is different from the problem for integrable two-dimensional billiards ([2]) because the impulse components of the trajectories on such tori turn to infinity on the domain boundary. Such systems can be called billiards with semi-rigid walls.

Considered operator $\nabla D(x)\nabla$ arises in the problem of waves on water captured by a coast, where $D(x)$ is a basin depth function. In the talk we present a result for three types of the domain Ω defining an island in the ocean, a limited reservoir or an island in a limited reservoir.

The work was supported by the Russian Science Foundation (project no. 16-11-10282).

References

[1] A. Yu. Anikin, S. Yu. Dobrokhotov, V. E. Nazaikinskii, Simple asymptotics for a generalized wave equation with degenerating velocity and their applications in the linear long wave run-up problem, *Mathematical Notes*, **104**(4), 471–488 (2018).

[2] V. S. Matveev, The asymptotic eigenfunctions of the operator $\nabla D(x, y)\nabla$ corresponding to Liouville metrics and waves on water captured by bottom irregularities, *Mathematical Notes*, **64**(3), 357–363 (1998).

X-ray reflectometry and fluorescence analysis of gratings working in classical and conical diffraction

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Using the methods of grazing-incidence X-ray reflectometry and X-ray fluorescence (GIXRR and GIXRF), we experimentally and theoretically investigated the angular dependences of the specular-reflected and emitted radiation intensity of the gratings working in conical and classical diffraction mounts for the incident photon energy $\sim$ 8 keV and grazing-incidence angle range of 0.1–0.5 deg. The fluorescence intensity of the $\mathrm{Cr}_{\mathrm{K}_\alpha}$ line ($\sim$ 5.4 keV) was determined and investigated. The diffraction order efficiency as well as the fluorescence intensity have been found from a numerical solution of the respective Helmholtz equations [1, Ch. 12]. PCGrate-SX™ v. 6.7 computer code [2] has been used for solving the direct problem of GIXRR on gratings with polygonal groove profiles obtained from atomic-force microcopy measurements.

A variety of the fundamental parameters method [3] was developed for solving the inverse GIXRF problem on general multilayer gratings — restoring the concentration of analyzed chemical elements of layers (materials) and the shape of layer boundary profiles, taking into account the possibility of calculating the strength of electromagnetic fields and the absorbed energy and using the data of GIXRR.

References

[1] E. Popov (Ed.), *Gratings: Theory and Numeric Applications*, 2nd rev. ed., Presses Universitaires de Provence, Marseille (2014).

[2] The I. I. G., Inc. website: `http://pcgrate.com/` (accessed on March 12, 2019).

[3] L. I. Goray, V. E. Asadchikov, B. S. Roshchin, Yu. O. Volkov, A. M. Tikhonov, Fluorescence analysis of x-ray whispering gallery waves propagating along liquid meniscuses, *Proc. of the Int. Conf. Days on Diffraction 2018*, IEEE, 136–141 (2018).

Control and inverse for quantum graphs

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Quantum graphs are metric graphs with differential equations defined on the edges. Recent interest in control and inverse theories for quantum graphs is motivated by applications to important problems of classical and quantum physics, chemistry, biology, and engineering. For trees, i.e. graphs without cycles, various types of inverse and control problems were studied in the literature (see, e.g. [1–4] and references therein), but almost nothing (with several exceptions [5–7]) was done for graphs with cycles. In this talk we describe exact controllability and identifiability results for the wave and Schrödinger equations on general compact graphs.

We prove the exact controllability of the systems with the optimal number of controls and propose algorithms recovering the unknown coefficients of the equations, lengths of the edges, and the topology of the graph. Our central idea is a proper choice of observations in the form of the directional derivatives of the solutions to differential equations on graphs at some boundary and internal vertices. These observations guarantee observability and identifiability of the corresponding systems. Our approach is based on the boundary control method for inverse problems of mathematical physics, which uses deep connections between controllability/observability and identifiability of dynamical systems. Another component of the approach is a new version of the effective leaf peeling method [4].

References

[1] J. E. Lagnese, G. Leugering, E. J. P. G. Schmidt, *Modeling, Analysis and Control of Dynamic Elastic Multi-Link Structures*, Birkhäuser, Boston (1994).

[2] M. Belishev, A. Vakulenko, Inverse problems on graphs: recovering the tree of strings by the BC-method, *J. Inv. Ill-Posed Problems*, **14**, 29–46 (2006).

[3] S. Avdonin, Control problems on quantum graphs, in: "Analysis on Graphs and Its Applications", *Proceedings of Symposia in Pure Mathematics*, AMS, **77**, 507–521 (2008).

[4] S. Avdonin, P. Kurasov, Inverse problems for quantum trees, *Inverse Problems and Imaging*, **2**, 1–21 (2008).

[5] S. Avdonin, B. Belinskiy, L. Pandolfi, Controllability of a nonhomogeneous string and ring under time dependent tension, *Math. Model. Natur. Phenom.*, **6**, 4–31 (2010).

[6] P. Kurasov, Inverse scattering for lasso graph, *J. Math. Phys*, **54**, 042103 (2013).

[7] S. Avdonin, S. Nicaise, Source identification problems for the wave equation on graphs, *Inverse Problems*, **31**, 095007 (2015).

On Okamoto-like symmetries of isomonodromic deformation equations

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The group of the birational symmetries of the isomonodromic equations for Painlevé VI case contain the transformation that mix the parameters that are the differences between the eigenvalues of the matrices-residues of the deforming system:

$$\Delta_k \to \Delta_k - \frac{\Delta_\Sigma}{2}, \quad \Delta_\Sigma = \sum_i \Delta_i.$$

It is the form of the famous Okamoto transformation. I will show how this transformation may be interpreted as the canonical transformation of the same P VI-Hamiltonian system. The multidimensional analogs of the transformation will be presented.

Third order operators with three-point conditions associated with Boussinesq's equation

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We consider a non-self-adjoint third order operator on the interval $[0, 2]$ with real 1-periodic coefficients and multi-point Dirichlet conditions at the points 0, 1 and 2. The eigenvalues of this operator consist an auxiliary spectrum for the inverse spectral problem associated with the good Boussinesq equation. We determine eigenvalue asymptotics at high energy and the trace formula for the operator.

This is a joint work with E. L. Korotyaev.

Types of mode dispersion of optical vortices in twisted optical fibers

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The orbital angular momentum (OAM)-bearing beams [1], in particular optical vortices (OV) [2], are commonly recognized as highly perspective carries of information encoded in the orbital degrees of the freedom of light [3] in optical fibres [4, 5]. Surely, such a OAM-technologies require the analysis of the dispersion generated from the difference of OV's velocities.

We have studied the mutual effect of the spin-orbit interaction and the twisted mechanical stress in the step-index circular fibers to the dispersion of OVs with topological charge $|\ell| \geq 1$. It has been shown that there is three possible types of mode dispersion: standard polarization mode dispersion (PMD), new topological mode dispersion (TMD) and hybrid mode dispersion (HMD) (Fig. 1a). It has been established that for OVs with topological charge $|\ell| = 1$ there is only the HMD, while for $|\ell| > 1$ all three types of dispersion exist: PMD, TMD and HMD. It has been established that as a result of the influence of TMS, the values of PMD, TMD and HMD become different for each pair of OVs with $|\ell| > 1$. Although PMD and TMD are going down to zero, it is not implemented simultaneously (Fig. 1b).

(a) (b)

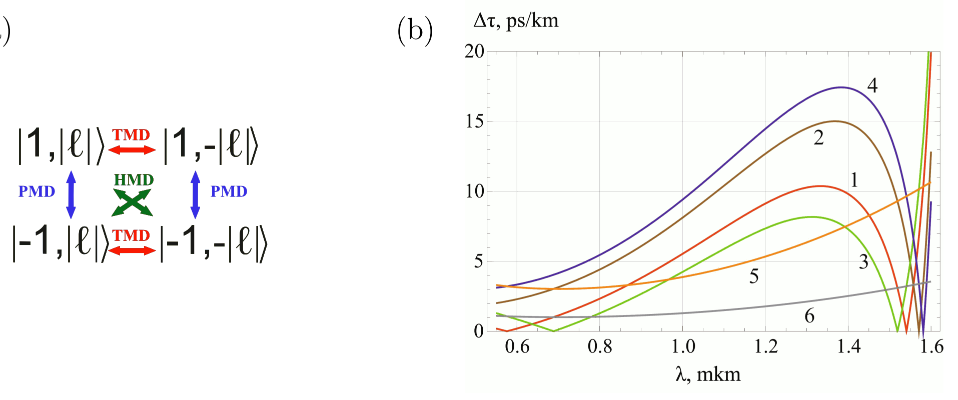

Fig. 1: a) Types of dispersion. b) The spectral dependence of dispersion of the OV with the azimuthal number $|\ell| = 2$. Curve numbers indicate the following types of dispersion: PMD $(1, 2)$, TMD $(3, 4)$, HMD $(5, 6)$. Core — 3.1% GeO_2, 96.9% SiO_2; cladding — SiO_2; core radius $r = 8.55$ mkm; at the $\lambda_{\text{He-Ne}}$ the waveguide parameter $V = 10.2$ and the normalized index difference $\Delta = 3.3 \cdot 10^{-3}$.

The research was supported by V. I. Vernadsky Crimean Federal University, project ВГ10/2018.

References

[1] A. Yao, M. Padgett, Orbital angular momentum: origins, behavior and applications, *Advances in Optics and Photonics*, **3**, 161–204 (2011).

[2] M. Soskin, M. Vasnetsov, Singular optics, *Progress in Optics*, Ed. E. Wolf, **42**, 219–276 (2001).

[3] A. E. Willner, H. Huang, Y. Yan, et al, Optical communications using orbital angular momentum beams *Advances in Optics and Photonics*, **7**, 66–106 (2015).

[4] N. Bozinovic, S. Golowich, P. Kristensen, S. Ramachandran, Control of orbital angular momentum of light with optical fibers, *Opt. Lett.*, **37**, 2451–2453 (2012).

[5] E. B. Barshak, C. N. Alexeyev, B. P. Lapin, M. A. Yavorsky, Twisted anisotropic fibers for robust orbital-angular-momentum-based information transmission, *Phys. Rev. A*, **91**, 033833 (2015).

Measuring the dynamic characteristics of Coriolis vibratory gyroscopes via the Hilbert transform technique

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An operating principle of the Coriolis vibratory gyroscopes (CVG) is based on inertial properties of elastic standing waves excited in axisymmetric resonators, such as rings or shells [1]. In imperfect resonators, the effect of splitting the oscillatory frequency takes place with a corresponding standing wave drift. Some kinds of trimming (balancing) procedures allows one to compensate the imperfectness of CVG resonator characteristics but these procedures must follow the measurement of effects caused by inhomogeneous distributions and identification of their harmonics amplitude and phase parameters. Some methods of such identifications are known [2]. They are based on the fact that the oscillatory process in an imperfect resonator can be represented as a sum of two harmonic oscillations with different frequencies. However, for small values of imperfectness of resonators, the difference between these two frequencies is small and thus we deal with a narrow-band process which can be studied effectively by the Hilbert transform technique [3, 4]. The Hilbert transform provides a method for determining the instantaneous amplitude and the instantaneous frequency of a signal. The results of numerical simulations, both under free and forced vibration regimes, show that this method displays good performance and may be effectively used instead of or together with conventional approaches of studying dynamics behavior of imperfect CVG resonators.

References

[1] G. Bryan, On the beats in the vibrations of a revolving cylinder or bell, *Proc. Camb. Phil. Soc., Math. Phys. Sci.*, **7**(3), 101–111 (1890).

[2] V. Apostolyuk, *Coriolis Vibratory Gyroscopes. Theory and Design*, Springer (2016).

[3] Y. Wei, Y. Dong, X. Huang, Z. Zhang, A stepped frequency sweeping method for nonlinearity measurement of microresonators, *Sensors*, **16**, 1700 (2016).

[4] M. Feldman, S. Braun, Nonlinear vibrating system identification via Hilbert decomposition, *Mechanical Systems and Signal Processing*, **84**, 65–96 (2017).

The wave model of a metric space with measure

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We consider a metric space (Ω, d) with a Borel measure μ on it. Under certain conditions on the space and on the measure, starting with some family of open sets from the space, using lattice theory we construct an isometric copy $(\widetilde{\Omega}, \widetilde{d})$ of the space (Ω, d), which is called its *wave model*. This construction differs from the wave model of the metric space (without a specified measure) proposed earlier by us. It has direct applications to inverse problems of mathematical physics. For instance, the wave model can be used to reconstruct a Riemannian manifold with the boundary from its spectral data.

On algebraic and uniqueness properties of harmonic quaternion fields on 3d manifolds

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Let Ω be a smooth compact oriented 3-dimensional Riemannian manifold with boundary. A quaternion field is a pair $q = \{\alpha, u\}$ of a function α and a vector field u on Ω. A field q is *harmonic* if α, u are continuous in Ω and $\nabla\alpha = \operatorname{rot} u$, $\operatorname{div} u = 0$ holds into Ω. The space $\mathscr{Q}(\Omega)$ of harmonic fields is a subspace of the Banach algebra $\mathscr{C}(\Omega)$ of continuous quaternion fields with the point-wise multiplication $qq' = \{\alpha\alpha' - u \cdot u',\, \alpha u' + \alpha' u + u \wedge u'\}$. We prove a Stone–Weierstrass type theorem: the subalgebra $\vee\mathscr{Q}(\Omega)$ generated by harmonic fields is dense in $\mathscr{C}(\Omega)$. Some results on 2-jets of harmonic functions and the uniqueness sets of harmonic fields are provided.

Comprehensive study of harmonic fields is motivated by possible applications to inverse problems of mathematical physics.

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Parametric amplification of a longitudinal wave by shear ones in an anisotropic layer

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The six-beam scattering of elastic waves by a crystal film is investigated. Cases of symmetric diffraction of waves inside the crystal layer are analyzed. In this scattering geometry, the projections of the wave vectors on the normal to the film surface are pairwise equal to each other up to the sign. Under certain conditions, quasi-shear waves modulate the structure in the path of quasi-longitudinal waves. If the period of the modulated structure is close to the length of a quasi-longitudinal wave, the amplitude of oscillations of the latter is amplified. The mathematical description of this effect and parametric resonance (see, for example, [1]) are similar. This effect is illustrated by examples of calculations of the scattering spectra of elastic waves by cubic and tetragonal crystals in a three-layer model. Calculations are based on the transfer matrix method [2].

References

[1] M. B. Vinogradova, O. V. Rudenko, A. P. Sukhorukov, *The Wave Theory*, Nauka Publishers, Moscow (1990).

[2] Yu. N. Belyayev, Calculation of the six-beam diffraction in layered media using polynomials of principal minors, *Journal of Theoretical and Computational Acoustics*, **26**, 1850017-1–21 (2018).

The analysis of bending thin periodically perforated plates

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We rigorously derive the correct Γ-limit that describes the energy of a thin, periodically perforated elastic plate as the period of the perforations and thickness of the plate tend to zero simultaneously. The limit energy functional is obtained via simultaneous homogenisation and dimension reduction from the fully non-linear elastic energy functional for a 3D perforated elastic plate. The limit of the bending energy regime studied results in Kirchhoff's nonlinear model for plates. The form of the limit homogenised functional depends on the scaling between the period of the perforations and the thickness of the plate.

Bateman-type non-complexified solution of the wave equation with two spatial variables

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The classical Bateman solution of the wave equation with 3 spatial variables

$$u_{xx} + u_{yy} + u_{zz} - c^{-2} u_{tt} = 0, \quad c = const > 0, \tag{1}$$

is

$$u = f\left(\alpha + (x^2 + y^2)/\beta\right)/\beta, \quad \alpha = z - ct, \quad \beta = z + ct, \tag{2}$$

where the waveform $f(\cdot)$ is an arbitrary function [1]. This solution is remarkable for several reasons and was generalized in various directions. Its complexified versions (where, e.g., β is replaced with $\beta - i\epsilon$, $\epsilon > 0$) allow, with proper choice of the waveform, a description of highly localized solutions of (1) (see, e.g., [2]). They also provide examples of solutions that have a single singularity at a running spacial point $\{x = y = 0, z = ct\}$, see [3]. Complexified versions are generalized to the case of a wave equation with an arbitrary number $m \geq 2$ of spatial variables [4].

Obvious generalization of the non-complexified solution (2) to m-dimensional case (see, e.g., [5]) does not satisfy the corresponding homogeneous wave equation for even m, it solves a running point source problem. We present a solution of the homogeneous wave equation $u_{xx} + u_{yy} - c^{-2}u_{tt} = 0$, which involves an arbitrary function and is an analog of (1). We prove that it has a single singularity at a running point if $f(\cdot)$ is smooth and compactly supported. The construction is based on the classical descent method (see, e.g., [6]), the proof employs the results of [7].

The research was supported by RFBR grant 17-01-00529.

References

[1] H. Bateman, The conformal transformations of space of four dimensions and their applications to geometrical optics, *Proc. London Math. Soc.*, **7**, 70–89 (1909).

[2] A. P. Kiselev, Localized light waves: Paraxial and exact solutions of the wave equation (a review), *Opt. Spectrosc.*, **102**, 603–622 (2007).

[3] A. S. Blagovestchenskii, A. P. Kiselev, A. M. Tagirdzhanov, Simple solutions of the wave equation with a singularity at a running point, based on the complexified Bateman solution, *J. Math. Sci.*, **224**, 73–82 (2017).

[4] A. P. Kiselev, M. V. Perel, Relatively distortion-free waves for the m-dimensional wave equation, *Differ. Equ.*, **38**(8), 1206–1207 (2002).

[5] K. Shimomura, Generalizations of Bateman's transformation for general indefinite metrics, *Math. J. Ibaraki Univ.*, **45**, 7–13 (2013).

[6] R. Courant, H. Hilbert, *Methods of Mathematical Physics*, Interscience, New York (1962).

[7] L. Hörmander, *The Analysis of Linear Partial Differential Operators. I. Distribution Theory and Fourier Analysis*, Springer, Berlin (1983).

On hyperbolization of an unbounded Schrödinger type operator

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A lot of problems in plasma physics (e.g., resonant absorption of electromagnetic waves in an inhomogeneous plasma) is governed by a nonlinear Schrödinger type equation as follows [1]:

$$-2i\frac{\partial E}{\partial t} + \frac{\partial^2 E}{\partial x^2} + \left(-x + |E|^2\right)E - D = 0, \tag{1}$$

where $\frac{\partial^2 E}{\partial x^2} - xE$ is the unbounded operator, the continuous spectrum of which covers the entire real axis. Eq. (1) without the Kerr nonlinearity (the term $|E|^2E$) is represented by the Airy functions. Consequently, Fourier components of E in Eq. (1) are varied according to the e^{ik^3t} rule in time. This fact yields a stiff restriction for a time step ($\Delta t \sim \Delta x^3$).

To avoid such a computational problem two ideas are suggested. The first one is the heuristic substitution

$$E = Ue^{it(x+(1/\mu^2-\mu)/2)} \tag{2}$$

which leads to a semi-bounded operator in x coordinate. Another one is a hyperbolization procedure [2]: adding of the second derivative of an unknown function in time with a special small parameter. It permits us to obtain explicitly the Green function for the linear part of the operator considered:

$$4\mu^2 \frac{\partial^2 U}{\partial t^2} + 2i\frac{\partial U}{\partial t} + \frac{\partial^2 U}{\partial x^2} - it\frac{\partial U}{\partial x} + \left(\frac{1}{\mu^2} - \mu - \frac{t^2}{4} + |U|^2\right) U - De^{it(x+(1/\mu^2-\mu)/2)} = 0. \tag{3}$$

As a result one can use a split-step procedure [3], where exact solutions both for a linear part and a nonlinear one exist. Another advantage is the essentially increased time step.

References

[1] L. D. Landau, E. M. Lifshits, *Teoreticheskaya Fizika*, Vol. 8, Fizmatlit, Moscow (2005) (in Russian).

[2] B. N. Chetverushkin, *Kinetic Schemes and a Quasi-Gas-Dynamic System of Equations*, MAKS Press, Moscow (2004) (in Russian).

[3] A. D. Yunakovsky, *Modeling of the Nonlinear Schrödinger Equation*, Institute of Applied Physics, N. Novgorod (1996).

Continuous operator method application for direct and inverse scattering problems

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It is known [1] that numerous problems in scattering theory can be modeled by integral equations of the first kind $Ax = f$, where A is a compact operator. For instance, boundary problems for the Helmholtz equation lead to such equations. In order to solve such problems we employ a continuous operator method [2], proved well in solving ill-posed problems for linear and nonlinear equations [3, 4]. We will demonstrate the method by solving the Dirichlet problem for the Helmholtz equation

$$\frac{1}{4\pi}\int\limits_{\partial D} \frac{e^{ik|x-y|}}{|x-y|}\varphi(y)ds(y) = f(x), \quad x \in \partial D, \tag{1}$$

where D is a bounded closed domain. An approximate solution of equation (1) is sought in the form of a function $\varphi_n(y) = \sum\limits_{k=0}^{n} \alpha_k \psi_k(y)$, where $\psi_k(y)$ are fundamental functions defined on ∂D.

An approximate system of equations is associated with equation (1)

$$\sum_{j=0}^{n} \alpha_j \frac{1}{4\pi}\int\limits_{\partial D} \frac{e^{ik|x_l-y|}}{|x_l-y|}\psi_j(y)ds(y) = f(x_l), \quad l = 0, 1, \ldots, n, \tag{2}$$

where x_l, $l = 0, 1, \ldots, n$, are the collocation points. The integral in the left hand side of the equation (2) is replaced by a cubature formula. Within the continuous operator method, the following system of ordinary differential equations is associated with equation (2)

$$\frac{d\alpha_l(t)}{dt} = (\beta_l)\left(\sum_{j=0}^{n} \alpha_j(t) \frac{1}{4\pi}\int\limits_{\partial D} \frac{e^{ik|x_l-y|}}{|x_l-y|}\psi_j(y)dy - f(x_l)\right), \quad l = 0, 1, \ldots, n, \tag{3}$$

where $\beta_l = \pm 1$, $l = 0, 1, \ldots, n$. The signs are chosen so that the logarithmic norm of the right hand side of the system (3) is negative. The system (3) is solved numerically. Similar approach is used for solving inverse scattering problems, in particular — the nonlinear problem of signal detection.

References

[1] D. Colton, R. Kress, *Integral Equation Methods in Scattering Theory*, Society for Industrial and Applied Mathematics (SIAM) (2013).

[2] I. V. Boikov, On a continuous method for solving nonlinear operator equations, *Differ. Equ.*, **48**(9), 1308–1314 (2012).

[3] I. V. Boikov, A. I. Boikova, O. A. Baulina, Continuous method for solving of gravity prospecting problems, In: D. Nurgaliev, N. Khairullina (Eds.), *Practical and Theoretical Aspects of Geological Interpretation of Gravitational, Magnetic and Electric Fields*, Springer Proceedings in Earth and Environmental Sciences, Springer (2019).

[4] I. V. Boykov, V. A. Roudnev, A. I. Boykova, O. A. Baulina, New iterative method for solving linear and nonlinear hypersingular integral equations, *Applied Numerical Mathematics*, **127**, 280–305 (2018).

Decomposition of eigenfunctions of the continuous spectrum of a locally perturbed discrete Schrödinger operator by eigenfunctions of the continuous spectrum of an unperturbed operator

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We consider the representation of generalized Chebyshev polynomials corresponding to the local perturbation of the discrete Schrödinger operator as linear combinations of classical Chebyshev polynomials of the second kind. Explicit formulas relating the coefficients of this expansion to the perturbation parameters of the discrete Schrödinger operator are obtained. This report is a continuation of the authors' work (see [1]).

References

[1] V. V. Borzov, E. V. Damaskinsky, Generalized Chebyshev oscillator connected with a point interaction for the discrete Schrödinger equation, *VI International Conference MQFT-18 dedicated to prof. A. N. Vasiliev*, Saint Petersburg State University, 27–30 August 2018.

Internal gravity waves in stratified medium with shear flows: analytical solutions and asymptotics

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Among the large variety of observed wave processes of various physical nature in the ocean and the atmosphere of the Earth, a special place causes the interaction of excited waves with hydrodynamic flows. Under real oceanic conditions, it is necessary to consider internal gravity waves propagating against the background of medium currents with a vertical velocity shift. The scale of the change in speed along the vertical is of the same order as the maximum speeds of the internal gravity waves. If the scale of change of currents horizontally is much larger than the lengths of internal gravity waves, and the scale of temporal variability is much greater than the periods of internal waves, then the natural mathematical model is the case of stationary and horizontal homogeneous mean shear flows. The problem of the internal gravity waves fields from an oscillating point source of disturbances in a stratified medium with an average shear flow is solved. A model distribution of the shear flows in depth is considered and an analytical solution of the problem is obtained in the form of the characteristic Green function, which is expressed in terms of the modified Bessel functions of the imaginary argument. Expressions for the dispersion relations are obtained by using of Bessel functions Debye asymptotics. Integral representations of solutions are constructed. The dependences of the wave characteristics of the excited fields on the main parameters of the used stratification models, shear flows, and generation modes are investigated [1–6]. The research was carried out in the framework of the Federal target program AAAA-A17-117021310375-7.

References

[1] V. Bulatov, Yu. Vladimirov, *Wave Dynamics of Stratified Mediums*, Nauka Publishers, Moscow (2012).

[2] C. Mei, M. Stiassnie, D. Yue, *Theory and Applications of Ocean Surface Waves*, World Scientific Publishing, London (2017).

[3] V. Bulatov, Yu. Vladimirov, *Fluid Dynamics*, **53**, 616–622 (2018).

[4] V. Bulatov, Yu. Vladimirov, *Oceanology*, **58**, 796–801 (2018).

[5] V. Bulatov, Yu. Vladimirov, *Russian Journal of Earth Science*, **18**, ES2004 (2018).

[6] R. Tarakanov, A. Marchenko, M. Velarde (Eds.), *The Ocean in Motion*, Springer Oceanography, Springer International Publishing AG, part of Springer Nature (2018).

Solutions of the nonlinear nonautonomous Klein–Fock–Gordon equation. Choice of ansatz

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Methods of finding of exact analytical solutions of the stationary nonlinear nonautonomous Klein–Fock–Gordon (KFG) equation

$$U_{xx} + U_{yy} + U_{zz} = p(x,y,z)\, F(U). \tag{1}$$

are suggested [1, 2]. Here v is constant, $p(x,y,z)$ and $F(U)$ are arbitrary functions of the arguments, the subscript means the derivative with respect to the corresponding variable. We are finding solutions of the eq. (1) in the form of complex function

$$U = f(\theta). \tag{2}$$

The argument $\theta(x,y,z)$ is called ansatz. It is proved that (2) will be the solution of (1) if $f(\theta)$ satisfies to the nonlinear ordinary differential equation of the first or second order, and $\theta(x,y,z)$ is a root of the special algebraic equation [3]. Function of $f(\theta)$ can be found in the form of integral for arbitrary $F(U)$. For some $F(U)$ the corresponding integrals can be inverting and then $f(\theta)$ is in an explicit form.

It is proved that $\theta(x,y,z)$ can be chosen as an root of the algebraic equations which define family of surfaces of curvilinear coordinates. So the general ellipsoidal coordinates set family of ellipsoids and one and two sheets hyperboloids. These families of coordinate surfaces are described by the equation

$$\frac{x^2}{a^2+\theta}+\frac{y^2}{b^2+\theta}+\frac{z^2}{c^2+\theta}=1. \tag{3}$$

The geometrical type of the family determines root $\theta(x, y, z)$ of the equation (3). In general case this is a cubic equation. It is proved that any root $(\theta_1, \theta_2, \theta_3)$ of the equation (3) can be chosen as ansatz for solution (2). Moreover, it is proved that ansatz are also roots of the equations of the following coordinate surfaces: elliptic and hyperbolic paraboloids, cones, flattened ellipsoids of rotation, two sheets hyperboloids of rotation, confocal paraboloids of rotation, etc.

The found solutions of the equation (1) demonstrate that it is possible to find exact analytical solutions of the nonlinear nonautonomous KFG equation in various orthogonal coordinates. It considerably expands the class of problems of mathematical physics in which perhaps successful application of the KFG equation.

References

[1] E. L. Aero, A. N. Bulygin, Yu. V. Pavlov, Exact analytical solutions for nonautonomic nonlinear Klein–Fock–Gordon equation, In: *Advances in Mechanics of Microstructured Media and Structures*, Eds.: F. dell'Isola, V. A. Eremeyev, A. Porubov, Springer, Cham, Switzerland, 21–33 (2018).

[2] A. N. Bulygin, Yu. V. Pavlov, Methods of finding of exact analytical solutions of nonautonomous nonlinear Klein–Fock–Gordon equation, In: *Dynamical Processes in Generalized Continua and Structures*, Eds.: H. Altenbach, A. Belyaev, V. A. Eremeyev, A. Krivtsov, A. V. Porubov, Springer, Cham, Switzerland, 1–15 (2019).

[3] H. Bateman, *The Mathematical Analysis of Electrical and Optical Wave–Motion on the Basis of Maxwell's Equations*, Cambridge University Press, Cambridge (1915).

Performance improvements of the boundary integral-equation method for diffraction gratings using classic concurrency approaches

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The oblique-incident (off-plane) radiation scattering from one-periodical gratings (2D structures) with arbitrary conductivity is considered. A border profile of the grating can vary. Incident plane waves are assumed to be time-harmonic. The initial consideration of the diffraction problem is the solution of Helmholtz equations for a 2-dimensional case, that is coupled by transmission conditions at the interfaces between different materials.

Even though a number of techniques such as caches for exponential and kernel functions, different approximations and discretization schemes were suggested by the authors ([1], Ch. 12), many diffraction problems still remain time-consuming tasks. In particular case, concurrency methods can be considered to increase the performance for solving diffraction problems. The parallel scheme exploits parallelization for both planar incident waves and grating plane sections on different levels. The paralleling technique is implemented using semaphores and mutexes using Windows API technology [2].

The numerical experiments were performed for a number of gratings in the extreme ultraviolet soft-x-ray wavelength ranges. The results show achieved improvements in the performance up to four times (Fig. 1). The developed technique allows one much faster solving grating synthesis tasks by parameters variations. The obtained solutions can find their applications in dispersive elements in spectral instruments, beam splitters, etc.

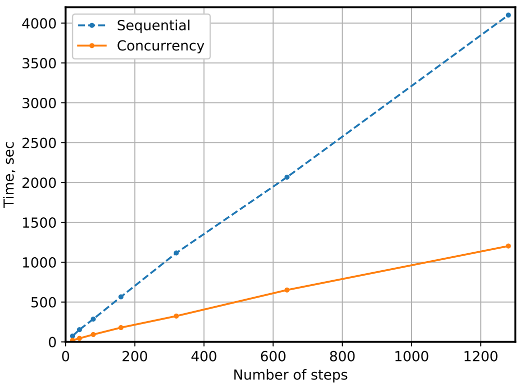

Fig. 1: Execution times of sequential and concurrent implementations for solving diffraction task vs the number of values of the trapezoid ridge of the lamellar Au grating.

References

[1] E. Popov (Ed.), *Gratings: Theory and Numeric Applications*, 2nd rev. ed., Presses Universitaires de Provence, Marseille (2014).

[2] *MS Windows API: Windows API Description*, `https://docs.microsoft.com/en-us/windows/desktop/apiindex/windows-api-list`, Accessed: 2019-11-03.

Reconstruction of solution to a hyperbolic equation from boundary data

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We deal with the problem of determining solution to the equation

$$\partial_t^2 u - \Delta u + \sigma \partial_t u = 0$$

considered in $\Omega \times \mathbb{R}$ ($\Omega \subset \mathbb{R}^3$, σ is a real parameter). The solution u is to be recovered in some part of the cylinder $\Omega \times \mathbb{R}$ from Cauchy data $u, \partial_\nu u$ given on $S \times I$, where $S \subset \partial\Omega$ is a part of the boundary, $I \subset \mathbb{R}$ is a time interval. We provide a reconstruction algorithm for this problem based on analytic expressions. The equation in consideration may describe acoustic waves or electromagnetic waves in a conducting medium. Our result may have applications in geophysics (ground-penetrating radar) and photoacoustic tomography with limited data.

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On range condition of the tensor x-ray transform in $\mathbb{R}^n$

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Consider the problem of the range description of the tensor x-ray transform. Specific issue is to minimize the number of differential equations that describe the range.

In this paper a geometrical interpretation of the range condition and related John differential operator is given. As a corollary, it is proved that the range of the m-tensor x-ray transform in $\mathbb{R}^n$ can be described by $\binom{n+m-2}{m+1}$ differential equations.

Diffraction of flexural-gravity waves from a vertical cylinder of non-circular cross section

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Diffraction of uni-directional hydroelastic waves by a vertical cylinder of non-circular cross section frozen in an ice cover of infinite extent is studied within the linear wave theory. The problem is formulated in a polar coordinate system (r, θ, z) where the z-axis points vertically upwards. The plane $z = -H$ corresponds to the sea bottom, and the plane $z = 0$ corresponds to the ice-fluid interface. The rigid cylinder extends from the sea bottom to the ice-fluid interface. The cross section of the vertical cylinder is described by the equation $r = R[1 + \varepsilon f(\theta)]$, where R is the mean radius of the cylinder and ε is a small non-dimensional parameter of the problem.

The problem of flexural-gravity waves diffraction by a circular cylinder was solved by both the vertical mode method and the Weber integral transform in [1] for water of finite depth. Here we present the solution of this problem for a non-circular vertical cylinder. The problem is formulated for a velocity potential $\phi(r, \theta, z, t)$ (see [1]),

$$\phi_{rr} + \frac{1}{r}\phi_r + \frac{1}{r^2}\phi_{\theta\theta} + k^2\phi = 0 \quad (-H < z < 0,\ r > R[1 + \varepsilon f(\theta)]), \tag{1}$$

$$\phi_z = 0, \quad (z = -H,\ r > R[1 + \varepsilon f(\theta)]), \tag{2}$$

$$\frac{\partial \phi}{\partial n} = 0 \quad (r = R[1 + \varepsilon f(\theta)]), \tag{3}$$

$$\phi \sim e^{ikr\cos\theta} \quad (r \to \infty), \tag{4}$$

$$\phi_z = w_t(r, \theta, t) \quad (z = 0,\ r > R[1 + \varepsilon f(\theta)]), \tag{5}$$

$$m w_{tt} + D\nabla^4 w = -\rho\phi_t(r, \theta, 0, t) - \rho g w(r, \theta, t) \quad (r > R[1 + \varepsilon f(\theta)]), \tag{6}$$

$$w = 0, \quad \frac{\partial w}{\partial n} = 0 \quad (z = 0,\ r = R[1 + \varepsilon f(\theta)]), \tag{7}$$

where $w(r, \theta, t)$ is the deflection of the ice cover, $\mathbf{n}$ is the unit outward normal vector to the surface of the cylinder, $m = \rho_i h_i$ is the mass of the ice cover per unit area, h_i is the ice thickness, ρ_i is the ice density, $D = Eh_i^3/[12(1 - \nu^2)]$ is the rigidity coefficient for an elastic plate of constant thickness, E is the Young module of the ice, ν is the Poisson ratio, and ρ is the water density. In this study, the edge of the ice plate is clamped to the cylinder which is given by the condition (7).

An asymptotic technique for the diffraction problem [2] and the vertical mode method [1] are combined to obtain an approximate solution of the hydroelastic diffraction problem (1)–(7).

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References

[1] A. A. Korobkin, S. Malenica, T. Khabakhpasheva, The vertical mode method in the problems of flexural-gravity waves diffracted by a vertical cylinder, *Applied Ocean Research*, **84**, 111–121 (2019).

[2] N. B. Dişibüyük, A. A. Korobkin, O. Yilmaz, Linear wave interaction with a vertical cylinder of arbitrary cross section: an asymptotic approach, *Journal of Waterway, Port, Coastal Engineering*, **143**(5), (2017).

Using specialized artificial neural networks to generate time series similar to human electroencephalograms

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The aim of this work is to use specialized artificial neural networks to generate time series that reproduce the properties of real human electroencephalograms (EEG). The focus of the study is on manifestations of nonlinearity and deterministic chaos in EEG signals. The study deals with the question open place of modeling in the study of complex systems and processes. Particular attention is paid to the definition of indicators of the studied time series. The goal is to make meaningful judgments about the intrinsic properties of the generated EEG signals compared to real signals. Here we present and evaluate simulation results with the help of specialized artificial neural networks, namely, multilayer perceptrons with chaotic neurons. We call a chaotic neuron a neuron with the activation function generated by known mappings from the region of nonlinear chaotic dynamics. In the framework of this study, quantitative indicators that were chosen to compare the real EEG signal with the model EEG signal include: Hjorth mobility, Hjorth complexity, information entropy, algorithmic complexity, and largest Lyapunov exponent. The conclusion is that under certain conditions, neural networks with chaotic neurons can reproduce properties of real EEG signals fairly well in terms of most widely used quantitative indicators. But at the same time, for some other quantitative indicators, the similarity between generated and real signal is not sufficient. In other words, the concept of an artificial neural black box has some limitations due to the complexity of real human electroencephalograms.

Asymptotic solutions to the Cauchy problem with localized initial data for linear systems of evolution equations with real characteristics

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We consider the Cauchy problem for the system of differential and pseudodifferential equations

$$ihu_t = L\Big(x, -ih\frac{\partial}{\partial x}, h\Big)u, \quad u|_{t=0} = V^0\Big(\frac{x-\xi}{\mu}\Big), \quad x \in \mathbb{R}^n, \quad t \in [0, T].$$

Here $0 < h \le \mu \ll 1$ are small parameters, ξ is a constant vector in $\mathbb{R}^n$, $L(x, ih\nabla, t, h)$ is $m\times m$ matrix-valued differential or pseudodifferential operator acting to $m - D$ vector functions with components $u_1(x,t), u_2(x,t), \dots, u_m(x,t)$, $V(y)$ is a smooth $m - D$ vector function with components decaying faster than $C/(1+y^2)^{(n+1)/2}$ as $|y| \to \infty$. Also we assume that the matrix-valued symbol $L(x,p,t,h)$ of the operator $L(x, ih\nabla, t, h)$ is a smooth function of all arguments and all eigenvector ("effective Hamiltonians") $H_j(p,x,t)$, $j = 1,\dots,r \le m$, of the matrix-function $L(x,p,t,0)$ are real-valued smooth functions and $H_j(p,x,t) \ne H_l(p,x,t)$ for $p \ne 0$ and $x \in \mathbb{R}^n$, $t \in [0,T]$. The considered problems include the wide class of many important physical equations (with constant and variable coefficients) like the Schrödinger equation, the linearized Navier–Stokes equations, the wave equations and hyperbolic systems of Petrovskii type, the Maxwell equations, etc.

We discuss the general approach to construction of asymptotic solution to this problem in a form of the modified Maslov canonical operators constructed on the Lagrangian manifolds $\Lambda^j_t = g^t_{H^j}\Lambda_0$

where $g^t_{H^j}$ are the Hamiltonian flows associated with the Hamiltonians H^j and the initial Lagrangian manifold $\Lambda_0 = \{(p, x) \in \mathbb{R}^{2n}_{px} : p = \alpha \in \mathbb{R}^n_x,\ x = \xi\}$. Generally speaking the manifolds Λ^j_t could be not smooth and standard Maslov scheme (suggested in 1965) for construction of asymptotic solutions to considered problem is not applicable in many important situations (for instance in the case of the wave equation). We show that the obtained recently new integral representation for the Maslov canonical operator in the singular charts [1] allows one to extend the definition of the canonical operator to such irregular cases and apply the standard Maslov scheme (algorithm) to the problems under consideration. We discuss the relationship between this construction and Van Vleck formula in quantum mechanics and asymptotic solutions of different equations presented in the papers [2–21].

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References

[1] S. Dobrokhotov, V. Nazaikinskii, A. Shafarevich, New integral representations of Maslov canonical operator in singular charts, *Izv. Math.*, **81**(2), 286–328 (2017).

[2] V. A. Borovikov, M. Ya. Kelbert, The field near the wave front in a Cauchy–Poisson problem, *Mechanics of Fluids and Gases, Izvestiya of Academy of Science USSR*, **2**, 173–174 (1984).

[3] V. P. Maslov, M. V. Fedorjuk, Logarithmic asymptotic of rapidly decreasing solutions of Petrovskii hyperbolic equations, *Math. Notes*, **45**(5), 382–391 (1989).

[4] S. Yu. Dobrokhotov, P. N. Zhevandrov, V. P. Maslov, A. I. Shafarevich, Asymptotic fast-decreasing solutions of linear, strictly hyperbolic systems with variable coefficients, *Math. Notes*, **49**(4), 355–365 (1991).

[5] S. Yu. Dobrokhotov, A. I. Shafarevich, Some asymptotic solutions of linearized Navier–Stokes equations, *Math. Notes*, **53**(1), 19–26 (1993).

[6] S. Dobrokhotov, S. Sekerzh-Zenkovich, B. Tirozzi, T. Tudorovski, Description of tsunami propagation based on the Maslov canonical operator, *Dokl. Math.*, **74**(1), 592–596 (2006).

[7] M. V. Berry, Focused tsunami waves, *Proc. R. Soc. Lond. Ser. A, Math. Phys. Eng. Sci.*, **463**(2087), 3055–3071 (2007).

[8] S. Yu. Dobrokhotov, P. N. Zhevandrov, V. V Kuzmina, Asymptotics of the solution of the Cauchy–Poisson problem in a layer of nonconstant thickness, *Math. Notes*, **53**(6), 657–660 (1993).

[9] S. Yu. Dobrokhotov, A. I. Shafarevich, B. Tirozzi, Localized wave and vortical solutions to linear hyperbolic systems and their application to linear shallow water equations, *Russ. J. Math. Phys.*, **15**(2), 192–221 (2008).

[10] S. Yu. Dobrokhotov, B. Tirozzi, C. A. Vargas, Behavior near the focal points of asymptotic solutions to the Cauchy problem for the linearized shallow water equations with initial localized perturbations, *Russ. J. Math. Phys.*, **16**(2), 228–245 (2009).

[11] S. Yu. Dobrokhotov, D. A. Lozhnikov, V. E. Nazaikinskii, Wave trains associated with a cascade of bifurcations of space-time caustics over elongated underwater banks, *Math. Model. Nat. Phenom.*, **8**(5), 32–43 (2013).

[12] S. Yu. Dobrokhotov, S. A. Sergeev, B. Tirozzi, Asymptotic solutions of the Cauchy problem with localized initial conditions for linearized two-dimensional Boussinesq-type equations with variable coefficients, *Russ. J. Math. Phys.*, **20**(2), 155–171 (2013).

[13] A. I. Allilueva, S. Yu. Dobrokhotov, S. A. Sergeev, A. I. Shafarevich, New representations of the Maslov canonical operator and localized asymptotic solutions for strictly hyperbolic systems, *Dokl. Math.*, **92**(2), 548–553 (2015).

[14] S. Yu. Dobrokhotov, A. A. Tolchennikov, B. Tirozzi, Asymptotics of shallow water equations on the sphere, *Russ. J. Math. Phys.*, **21**(4), 430–449 (2014).

[15] S. Yu. Dobrokhotov, V. E. Nazaikinskii, Punctured Lagrangian manifolds and asymptotic solutions of the linear water wave equations with localized initial conditions, *Math. Notes*, **101**(5), 1053–1060 (2017).

[16] S. Yu. Dobrokhotov, V. E. Nazaikinskii, Propagation of a linear wave created by a spatially localized perturbation in a regular lattice and punctured Lagrangian manifolds, *Russ. J. Math. Phys.*, **24**(1), 127–133 (2017).

[17] S. Yu. Dobrokhotov, S. Ya. Sekerzh-Zen'kovich, A. A. Tolchennikov, Exact and asymptotic solutions of the Cauchy–Poisson problem with localized initial conditions and a constant function of the bottom, *Russ. J. Math. Phys.*, **24**(2), 310–321 (2017).

[18] S. Yu. Dobrokhotov, V. E. Nazaikinskii, A. A. Tolchennikov, Asymptotics of linear water waves generated by localized source near the focal points on the leading edge, *Russ. J. Math. Phys.*, **24**(4), 544–552 (2017).

[19] A. I. Allilueva, A. I. Shafarevich, Nonstandard characteristics and localized asymptotic solutions of a linearized magnetohydrodynamic system with small viscosity and drag, *Theor. Math. Phys.*, **190**(1), 164–175 (2017).

[20] A. I. Allilueva, A. I. Shafarevich, Localized asymptotic solutions of the linearized system of magnetic hydrodynamics, *Math. Notes*, **102**(6), 737–745 (2017).

[21] A. I. Allilueva, A. I. Shafarevich, Evolution of Lagrangian manifolds and asymptotic solutions to the linearized equations of gas dynamics, *Regul. Chaotic Dyn.*, **24**(1), 80–89 (2019).

Asymptotics of long surface waves generated by a localized source moving along the bottom of the basin

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We consider the linear problem of the excitation of surface waves by a localized source moving along the bottom of the basin in a one-dimensional case. We suppose that the source is bottom perturbation moving at a variable speed and the time of the movement is finite. Thus, one of the mechanisms for the generation of landslide tsunami waves can be described [1–3]. In the shallow water approximation the elevation of the free surface is described by a pseudodifferential equation in dimensionless variables [4]

$$h^2\frac{\partial^2\eta}{\partial t^2}+\hat{\mathbf{L}}\eta=h\frac{\partial^2}{\partial t^2}\left[\frac{1}{\cosh(|\hat{p}|D(x))}D_1\Big(\frac{x-\xi(t/t_1)}{\mu},t\Big)\right],\qquad \eta|_{t=0}=\frac{\partial\eta}{\partial t}\Big|_{t=0}=0\,, \tag{1}$$

where $\hat{\mathbf{L}}=L\Big(\frac{\overset{1}{\hat{p}}+\overset{3}{\hat{p}}}{2},\overset{2}{x},h\Big)$, $L=|p|\tanh\big(|p|D(x)\big)+O(h^2)$, $\hat{p}=-ih\frac{\partial}{\partial x}$, h and μ are small parameters, $\xi(t/t_1)$ is the trajectory of the source, t_1 is the time of the movement, $D(x)$ is the basin depth, the function $D_1(y,t)$ describes the shape of the bottom perturbation.

We construct the asymptotic solution of the equation (1). It is shown that under the condition that the speed of the source does not exceed the characteristic speed of long waves, the profile of a wave excited on the liquid surface is determined by the acceleration of the moving center of the bottom perturbation, and its length is the product of the characteristic speed of long waves and the time of source movement. During movement, the source generates a perturbation of the free surface, the principal term of the asymptotic expansion of this perturbation in dimensional variables has the

form
$$\tilde{\eta}(x,t_1) = \sum_{+,-}\Big[\frac{Q\ddot{\mathbf{X}}_D(\frac{ct\pm x}{c})}{c^2} + \frac{\dot{Q}(\frac{ct\pm x}{c})}{c}\Big], \tag{2}$$
where $\mathbf{X}_D = \xi_{tt}$ is the acceleration of the source center, Q is the 2D volume of the source, c is the characteristic speed of long waves. After the end of the movement, the answer to the problem is the solution of a homogeneous equation (1) with initial conditions (2). The influence of dispersion effects on the resulting surface wave is discussed.

References

[1] S. Tinti, E. Bortolucci, C. Chiavettieri, Tsunami excitation by submarine slides in shallow-water approximation, *Pure and Applied Geophysics*, **158**(4), 759–797 (2001).

[2] I. Didenkulova, E. Pelinovsky, Analytical solutions for tsunami waves generated by submarine landslides in narrow bays and channels, *Pure and Applied Geophysics*, **170**(9-10), 1661–1671 (2013).

[3] G. A. Papadopoulos, et al., Numerical modeling of sediment mass sliding and tsunami generation: The case of February 7, 1963, in Corinth Gulf, Greece, *Marine Geodesy*, **30**(4), 315–331 (2007).

[4] S. Yu. Dobrokhotov, V. E. Nazaikinskii, Waves on the free surface described by linearized equations of hydrodynamics with localized right-hand sides, *Russian Journal of Mathematical Physics*, **25**(1), 1–16 (2018).

Homogenization of the periodic Schrödinger-type equations with the lower order terms

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In $L_2(\mathbb{R}^d;\mathbb{C}^n)$, we consider a selfadjoint matrix strongly elliptic second order differential operator
$$\mathcal{B}_\varepsilon = \mathcal{A}_\varepsilon + \sum_{j=1}^{d}\big(a_j(\mathbf{x}/\varepsilon)D_j + D_j a_j(\mathbf{x}/\varepsilon)^*\big) + \mathcal{Q}(\mathbf{x}/\varepsilon) + \lambda I, \quad \varepsilon > 0.$$
Here the principal term $\mathcal{A}_\varepsilon = b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$ is given in a factorized form, where g is a periodic, bounded, and positive definite $m\times m$ matrix-valued function, and $b(\mathbf{D}) = \sum_{l=1}^{d} b_l D_l$, where b_l are $m\times n$ matrices. It is assumed that $m \geq n$ and that $\operatorname{rank} b(\boldsymbol{\xi}) = n$ for $\boldsymbol{\xi} \in \mathbb{R}^d \setminus \{0\}$. The a_j are periodic matrix-valued functions; in general, they are unbounded. The potential $\mathcal{Q}$ is a distribution generated by some periodic measure (with values in the class of Hermitian matrices). The parameter λ is subject to some restriction ensuring the positive definiteness of the operator $\mathcal{B}_\varepsilon$.

We study the behavior of the operator $e^{-it\mathcal{B}_\varepsilon}$, $t\in\mathbb{R}$, for small ε. The following sharp order error estimate is obtained:
$$\|e^{-it\mathcal{B}_\varepsilon} - e^{-it\mathcal{B}^0}\|_{H^3(\mathbb{R}^d)\to L_2(\mathbb{R}^d)} \leq C_1(1+|t|)\varepsilon. \tag{1}$$
Here $\mathcal{B}^0$ is the effective operator with constant effective coefficients. We show that under some additional assumptions on the operator (which are formulated in terms of the spectral characteristics near the bottom of the spectrum), this result can be improved:
$$\|e^{-it\mathcal{B}_\varepsilon} - e^{-it\mathcal{B}^0}\|_{H^2(\mathbb{R}^d)\to L_2(\mathbb{R}^d)} \leq C_2(1+|t|)\varepsilon.$$
Finally, we confirm that (1) is sharp with respect to the type of the norm: in the general case the estimate $\|e^{-it\mathcal{B}_\varepsilon} - e^{-it\mathcal{B}^0}\|_{H^s\to L_2} = O(\varepsilon)$ is not true if $s<3$. The supporting examples are given.

The results are applied to study the solution $\mathbf{u}_\varepsilon$ of the Cauchy problem for the Schrödinger-type equation $i\partial_t \mathbf{u}_\varepsilon = \mathcal{B}_\varepsilon \mathbf{u}_\varepsilon + \mathbf{F}$. Applications to the magnetic Schrödinger equation with a singular electric potential and to the two-dimensional Pauli equation with singular potentials are given.

References

[1] M. A. Dorodnyi, Homogenization of the periodic Schrödinger-type equations with the lower order terms, *St. Petersburg Math. J.* (to appear).

Enhanced existence time of solutions to the fractional Korteweg – de Vries equation

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We consider the fractional Korteweg – de Vries equation $u_t + uu_x - |D|^\alpha u_x = 0$ in the range of $-1 < \alpha < 1$, $\alpha \neq 0$. Using basic Fourier techniques in combination with the normal form transformation and modified energy method, we extend the existence time of classical solutions in Sobolev space with initial data of size ε from $1/\varepsilon$ to a time scale of $1/\varepsilon^2$.

References

[1] M. Ehrnström, Y. Wang, Enhanced existence time of solutions to the fractional Korteweg – de Vries equation, *arXiv*: 1804.06297 (2018).

A method for constructing an orthogonal system of eigenwaves of a cluster of open gyrotropic cylindrical waveguides

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Multi-element waveguide systems are used in many applications ranging from the laboratory radio-frequency plasma sources [1] to optical transmission systems for classical and quantum information technologies [2, 3]. It has been shown that a simple accurate method for solving the problems of diffraction, excitation, and propagation of electromagnetic waves in the presence of an open single-element waveguide is that based on expansion of the field in terms of orthogonal eigenwaves of such a guiding structure [4]. Although methods for constructing an orthogonal system of eigenwaves of open cylindrical waveguides have long been known [4, 5], a similar approach for an open multi-cylindrical waveguide is yet to be developed.

In this paper, a method for constructing an orthogonal system of eigenwaves of an open waveguide cluster, which consists of identical parallel circular cylinders, is presented. The case where the cylinders are filled with a gyrotropic medium and located in free space has been considered. The fields are expanded in terms of azimuthal harmonics in the individual coordinate system of each cylinder. In this representation, the fields in the outer region of the cylinders are the superpositions of the fields of waves that are scattered from and incident on the cylinders. Each azimuthal harmonic of both the scattered fields and the fields in the inner regions of the cylinders is represented as described in [5]. The continuity conditions for the tangential field components at the surfaces of the cylinders allow one to obtain a system of equations, which yields the coefficients of the eigenwaves of the considered waveguide cluster. The dispersion relation is derived for determining the longitudinal wave numbers of the discrete-spectrum waves which are guided by the cluster. It is also shown that there exist the continuous-spectrum waves of two kinds with different polarizations. It is established that these waves of both kinds and the discrete-spectrum waves are mutually orthogonal. The norms of the continuous-spectrum waves have been obtained in closed form. A relatively simple approach is also proposed for calculation of the norms of the discrete-spectrum waves.

The results obtained can be useful in studying the diffraction, excitation, and propagation of electromagnetic waves in the presence of optical transmission lines and open plasma or metamaterial waveguides.

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References

[1] F. F. Chen, H. Torreblanca, Permanent-magnet helicon sources and arrays: a new type of rf plasma, *Physics of Plasmas*, **16**, 057102 (2009).

[2] C. Markos, J. C. Travers, A. Abdolvand, B. J. Eggleton, O. Bang, Hybrid photonic–crystal fiber, *Reviews of Modern Physics*, **89**, 045003 (2017).

[3] Y. Chen, J. Gao, Z.-Q. Jiao, K. Sun, W.-G. Shen, L.-F. Qiao, H. Tang, X.-F. Lin, X.-M. Jin, Mapping twisted light into and out of a photonic chip, *Physical Review Letters*, **121**, 233602 (2018).

[4] V. V. Shevchenko, *Continuous Transitions in Open Waveguides*, Golem Press, Boulder (1971).

[5] V. A. Es'kin, A. V. Kudrin, A new method for constructing an orthogonal system of eigenwaves of an open cylindrical waveguide surrounded by an isotropic medium, *Proceedings of PIERS 2017*, 843–848 (2017).

Solution of the electrostatic problem for a core-mantle particle with non-confocal spheroidal boundaries of the layer

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Light scattering by small non-spherical particles can be met in various fields of science. In nano-optics one often uses the Rayleigh approximation based on solution of the electrostatic problem and calculation of the particle polarizability. This problem, being a particular case of the wave problem under the condition of zero wave number [1], is suitable for analysing the light scattering methods.

In this work we consider the electrostatic problem for a two-layer particle with the mantle having non-confocal spheroidal boundaries. The fields inside the mantle are expanded in terms of spheroidal harmonics related to the outer and internal boundary systems. To match these two expansions, we utilize the earlier obtained relations between the spheroidal harmonics of the Laplace equation written in the systems with different focal distance [2]. Further, we apply both the separation of variables (SVM) and T-matrix (TMM) methods and find that the solutions they provide are equivalent.

We study the range of applicability of our approach by numerically analysing the T-matrix. We find that in contrast to the often used spherical basis TMM and SVM methods [3, 4], our approach practically does not have limitations as concern the non-confocal spheroid parameters.

The polarizability of a small particles, being determined by the dipole component of an analogue of the scattered field, is proportional to the element T_{11} of the electrostatic T-matrix which is a product of the infinite matrices whose elements are either explicit expressions, or finite sums. We suggest an approximate way to calculate the core-mantle spheroid polarizability, when all the fields are represented just by one-term expansions. Our numerical computations have demonstrated that this approximation has the accuracy of about 1%.

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References

[1] P. C. Waterman, Matrix method in potential theory and electromagnetic scattering, *J. Appl. Phys.*, **50**, 4550–4566 (1979).

[2] V. G. Farafonov, A new solution to the problem of scattering of a plane wave by a multilayer confocal spheroid, *Opt. Spectrosc.*, **114**, 421–431 (2013).

[3] V. G. Farafonov, V. I. Ustimov, M. V. Sokolovskaya, The condition of applicability for the extended boundary conditions method for small multilayer particles, *Opt. Spectrosc.*, **120**, 448–460 (2016).

[4] V. G. Farafonov, V. I. Ustimov, Light scattering by small multilayer particles: a generalized separation of variables method, *Opt. Spectrosc.*, **124**, 252–261 (2018).

Topological 3D-laser solitons and their transformations

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Dissipative localized structures of light in laser media with fast saturable absorption — laser solitons — are attractors and have increased stability due to a dynamic balance of energy influx and energy outflow in dissipative media or systems. Their dynamics are described by the generalized Ginzburg–Landau complex equation for a slowly varying envelope of electric field strength E (quasi-optical approximation, radiation is locally close to a monochromatic plane wave with linear polarization)

$$\frac{\partial E}{\partial z} - \sum\nolimits_{n=1}^{3} c_n \frac{\partial^2 E}{\partial x_n^2} = f(|E|^2)E. \tag{1}$$

Here $x_1 = x$ and $x_2 = y$ with the transverse Cartesian coordinates x and y, $x_3 = \tau = t - z/v_g$ is the running time in the framework moving along axis z with group velocity v_g, and t is time. The coefficients c_n are complex, $\mathrm{Re}\, c_n \geqslant 0$. For zero frequency detuning, function f of intensity $I = |E|^2$ is real:

$$f(I) = -1 - \frac{a_0}{1+I} + \frac{g_0}{1+I/\beta}. \tag{2}$$

In the right-hand side of equation (2), a_0 is the small-signal resonance absorption, g_0 is small-signal gain, and β is the ratio of saturation intensities for gain and absorption.

The important elements of the solitons' topology are three-dimensional vortex lines, on which the field strength vanishes and, when traversing in a closed loop, the radiation phase changes by $2\pi m$, where the integer m is the topological charge. The vortex lines are directed towards increasing phase in a small vicinity of the line. They can be either open or closed. Their interesting property is that the longitudinal component (along the line) of the flow of electromagnetic energy can change its direction along the line to the opposite (alternating lines) or not to change it (non-alternating lines). Closed vortex lines can be knotted or unknotted, unlinked or linked.

The tangle laser solitons found in [1–3] exist in overlapping regions of parameters. Therefore, when the latter change, hysteresis phenomena occur. We demonstrate that the hysteresis is reversible, that is, when parameters return to their original values, the radiation structure is restored if, during the hysteresis cycle, there is no change in its skeleton topology. Otherwise, the final structure differs from the original by simpler topology (reduced topological indices), decrease of the radiation energy and increase of the medium energy.

References

[1] N. A. Veretenov, N. N. Rosanov, S. V. Fedorov, Rotating and precessing dissipative-optical-topological-3D solitons, *Phys. Rev. Lett.*, **117**, 183901 (2016).

[2] N. A. Veretenov, S. V. Fedorov, N. N. Rosanov, Topological vortex and knotted dissipative optical 3D solitons generated by 2D vortex solitons, *Phys. Rev. Lett.*, **119**, 263901 (2017).

[3] S. V. Fedorov, N. A. Veretenov, N. N. Rosanov, Irreversible hysteresis of internal structure of tangle dissipative optical solitons, *Phys. Rev. Lett.*, **122**, 023903 (2019).

Two coalescing turning points for difference equation

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Consider the difference Schrödinger equation

$$\psi(z+h)+\psi(z-h)+v(z)\psi(x)=E\psi(z), \tag{1}$$

where z is a complex variable, v is an analytic function, $h>0$ is a translation parameter, and E is a spectral parameter. As $h\to 0$ analytic solutions to this equation have semiclassical behavior as well as solutions to the differential Schrödinger equation

$$-h^2\psi''(z)+v(z)\psi(z)=E\psi(z). \tag{2}$$

For equation (1), the points where $v(z)-E=\pm 2$ play the same role as the turning points for the equation (2) do. So we call them turning points too. Using ideas of the complex WKB method, we obtain for equation (1) asymptotics of a transition matrix relating solutions having simple semiclassical behavior in complex domains separated by two coalescing turning points. A similar result was obtained by V. Buslaev and A. Fedotov for the Harper equation, i.e., equation (1) with $v(z)=\cos(z)$, but this never was published.

The values of the integrated density of states in the spectral gaps of the Harper operator

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We study spectral properties of the Harper operator acting on $L_2(\mathbb{R})$ by the formula

$$(H\psi)(x)=\psi(x+h)+\psi(x-h)+2\cos(2\pi x)\psi(x), \quad x\in\mathbb{R}, \tag{1}$$

where $h>0$ is a translation parameter. This operator was intensively studied by many mathematicians because of its intriguing spectral properties and, in particular, the Cantor structure of the spectrum for irrational values of h, see, e.g., [1].

B. Helffer and J. Sjöstrand, and, later, V. S. Buslaev and A. A. Fedotov studied the geometrical structure of the spectrum of the Harper operator in the case when h can be represented by a continuous fraction with sufficiently large elements, see review [2] and references therein. In this case, in the semiclassical approximation, the renormalization methods of B. Helffer and J. Sjöstrand, and of V. S. Buslaev and A. A. Fedotov allow to describe step by step sequences of smaller and smaller gaps in the spectrum, i.e., to get a description similar to one of the classical Cantor set.

It is well known that, in the spectral gaps of one-dimensional differential Schrödinger operators with 1-periodic potentials, the integrated density of states takes integer values. For the Harper operator (1), the possible values of the integrated density of states in the spectral gaps are determined by the Gap Labeling theorem, see, e.g., [3]. They may be equal to positive linear combinations of h and 1 with integer coefficients. We compute the values of the integrated density of states in the gaps in the case considered by B. Helffer and J. Sjöstrand, and by V. S. Buslaev and A. A. Fedotov. Though we use semiclassical constructions, we compute the exact values of the integrated density of states. Therefore we heavily use the monodromization renormalization method suggested by V. S. Buslaev and A. A. Fedotov.

References

[1] A. Avila, S. Jitomirskaya, The ten martini problem, *Annals of Mathematics*, **170**(1), 303–342 (2009).

[2] A. A. Fedotov, Monodromization method in the theory of almost-periodic equations, *St. Petersburg Mathematical Journal*, **25**(2), 303–325 (2014).

[3] L. Pastur, A. Figotin, *Spectra of Random and Almost-Periodic Operators*, Springer-Verlag, Berlin, Heidelberg (1992).

Recovery of time dependent coefficients from boundary data for hyperbolic equations

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We study uniqueness of the recovery of a time-dependent magnetic vector-valued potential and an electric scalar-valued potential on a Riemannian manifold from the knowledge of the Dirichlet to Neumann map of a hyperbolic equation. The Cauchy data is observed on time-like parts of the space-time boundary and uniqueness is proved up to the natural gauge for the problem. The proof is based on Gaussian beams and inversion of the light ray transform on Lorentzian manifolds under the assumptions that the Lorentzian manifold is a product of a Riemannian manifold with a time interval and that the geodesic ray transform is invertible on the Riemannian manifold. This talk is based on recent work with Joonas Ilmavirta, Yavar Kian and Lauri Oksanen.

Breast ultrasound tomography problem: 3D simulation

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The work is devoted to the problem of determining small sound speed fluctuations in glandular tissue for specific 3D breast model [1, 2] Fig. 1.

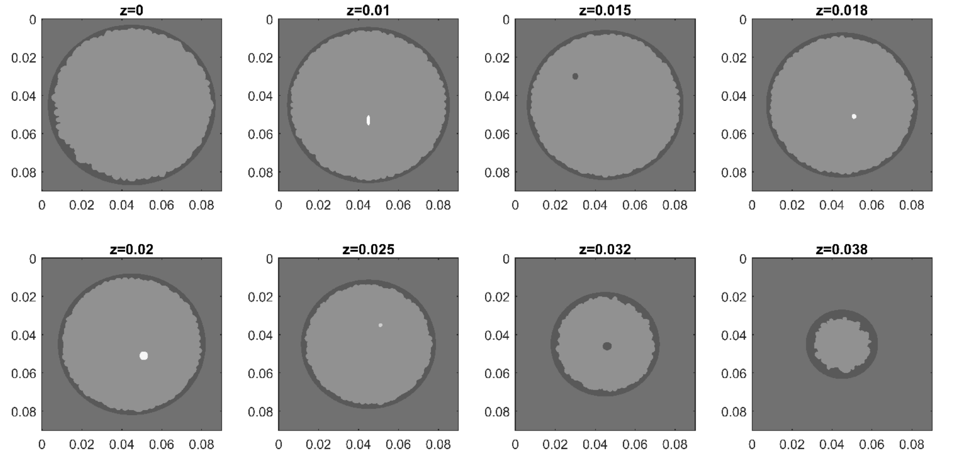

Fig. 1: Breast acoustical model (3D).

Our approach, proposed in [3, 4], is based on visualization of inclusions and unknown inner boundary between fat and glandular tissues and determination of sound speeds in inclusions using kinematic argument. The results of numerical modeling in 3D are presented. This work is supported by the Russian Science Foundation under grant 16-11-10027.

References

[1] N. Duric, P. Littrup, Breast Ultrasound Tomography, In: C. M. Kuzmiak (Ed.), *Breast Imaging*, IntechOpen, Ch. 6, `https://www.intechopen.com/books/breast-imaging/breast-ultrasound-tomography`.

[2] N. Duric, P. Littrup, L. Poulo, A. Babkin, R. Pevzner, E. Holsapple, C. Glide, Detection of breast cancer with ultrasound tomography: First results with the Computed Ultrasound Risk Evaluation (CURE) prototype, *Medical Physics*, **34**(2), 773–785 (2007).

[3] V. M. Filatova, V. V. Nosikova, L. N. Pestov. Application of Reverse Time Migration (RTM) procedure in ultrasound tomography, numerical modeling, *Eurasian Journal of Mathematical and Computer Applications*, **4**(4), 5–13 (2016).

[4] V. M. Filatova, V. V. Nosikova, Determining sound speed in weak inclusions in the ultrasound tomography problem, *Eurasian Journal of Mathematical and Computer Applications*, **6**(1), 11–20 (2018).

The energy flux analysis of the wave processes in the cylindrical shell with spring-type boundary condition on the outer surface

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The problem of free oscillations of an infinite circular cylindrical shell with spring-type boundary condition on the outer surface, analogous to Winkler foundation for a plate, are studied. The shell is considered of Kirchoff–Love type. The statement of the problem is considered in the rigorous statement. The dispersion equation is found on the base of exact analytical solution. The propagating waves are analyzed. Energy analyses are studied with special attention to the specific points of dispersion curves. The comparison of different mechanisms of energy transmission in the shell is fulfilled. The energy fluxes and its components as functions of parameters of the shell are discussed. The possible fields of applicability of the gained effects are established.

Multideck structures of boundary layers in compressible flows

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The multideck (double- and triple-deck) structures of the boundary layer are encountered in various problems of flow of an incompressible viscous fluid along various surfaces with small irregularities for large Reynolds numbers. However, they have not been studied well in the case of compressible flows. The triple-deck structure was also considered for compressible flows along a hump, but only in the stationary case, and the double-deck structure has not been studied in detail for compressible fluids.

Our goal is to use the results of our approach [3] to construct asymptotic solutions of fluid flow problems in the compressible case. Namely, we consider a compressible viscous fluid flow along a semi-infinite plate with small periodic or localized (hump-type) irregularities on the surface for large Reynolds numbers, see Fig. 1. We assume that the upstream flow is plane-parallel with velocity $U_0 = (u_\infty, 0)$ and density ρ_∞.

We construct formal asymptotic solutions with multi-deck structure for the understudy problems in stationary [1, 2] and nonstationary cases, and we obtain that the equations for the terms of asymptotic solutions are similar to the incompressible case [3]. Moreover, if we let $M \to 0$ (M is the Mach number) in the obtained equations, then we obtain equations for the incompressible case [3].

Numerical simulation of a flow in the near-wall region shows that its behavior is similar to that in the incompressible case (because the density enters the equations as a coefficient): if the amplitude of irregularities is less than critical, then we observe a laminar flow, otherwise we observe a vortex flow which becomes one stationary vortex after some (large) time (see Fig. 2). The critical amplitude varies irreversibly with upstream density and upstream velocity, but the flow character remains unchanged.

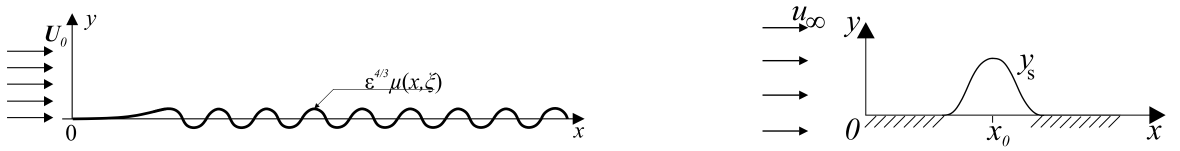

Fig. 1: The plate: periodic irregularities (left fig.) and localized hump (right fig.).

Fig. 2: The flow in the near-wall region : vortex dynamics (left fig.) and after some time — stationary vortex (right fig.) [2].

References

[1] R. K. Gaydukov, Double-deck structure in the problem of a compressible flow along a plate with small localized irregularities on the surface, *Eur. J. Mech. – B/Fluids*, **71**, 59–65 (2018).

[2] R. K. Gaydukov, Double-deck structure of the boundary layer in the problem of a compressible flow along a plate with small irregularities on the surface, *Eur. J. Mech. – B/Fluids*, **66**, 102–108 (2017).

[3] R. K. Gaydukov, V. G. Danilov, Equations for velocity oscillations in problems of a fluid flow along a plate with small periodic irregularities on the surface for large Reynolds numbers, *Proc. Int. Conf. DAYS on DIFFRACTION 2018*, 118–123 (2018).

Period-doubling biurcations in the generalized FitzHugh–Nagumo system

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We study a FitzHugh–Nagumo-like system of three ODEs with one fast variable corresponding to the membrane potential and two slow gating variables:

$$
\begin{aligned}
\varepsilon \dot{x} &= x - x^3/3 - y - z, \\
\dot{y} &= a + x, \\
\dot{z} &= a + x - z,
\end{aligned}
$$

where ε is a small parameter and the parameter a is assumed to be slightly less than one.

The system possesses a unique equilibrium, which is stable for sufficiently large a. However, decrease of a leads to the supercritical Andronov–Hopf bifurcation at a value $a_H = 1 - \frac{1}{4}\varepsilon + O(\varepsilon^2)$ (see e.g. [1]). Immediately after the bifurcation the amplitude of the newborn stable periodic orbit is small and lies below the threshold of spiking. In contrast, for $a \ll a_H$ the system exhibits large-scale periodic oscillations: continuous spiking known as "canards".

In [1] the author found numerically that dynamics near the slow surface can effectively become three-dimensional. As a result, the initial periodic state may lose stability already before the canard transition via a sequence of period-doubling bifurcations. Studying numerically the period doubling cascades for small but fixed values of the parameter ε, M. Zaks observed that the cascade follows the Feigenbaum law with the Feigenbaum constant 4.67, which is typical for dissipative systems. On the other hand for smaller values of ε the process switches to the Feigenbaum constant of a conservative map.

In this paper we derive the asymptotic formula for the Poincaré return map and calculate the parameter values for the first period-doubling bifurcation. We also discuss more general 3d model with a similar bifurcation scenario.

References

[1] M. Zaks, On chaotic subthreshold oscillations in a simple neuronal model, *Math. Model. Nat. Phenom.*, **6**(1), 149–162 (2011).

On the non-existence of low-frequency traveling A0 mode in a fluid-immersed elastic plate

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As is well known, the traveling Lamb waves that carry wave energy along a free elastic plate become guided leaky waves if the plate is immersed into acoustic fluid of lower density (e.g., a metal or composite plate in air or water). Since their phase velocities are greater than the sound velocity in the fluid, they re-emit wave energy into the environment during propagation and, therefore, cease to be able to transfer the energy to infinity in an ideally elastic plate. Instead, two new traveling Scholte–Stoneley waves (referred to as A and S modes) appear at the fluid-solid interfaces. Their real wavenumbers are greater than that of the bulk acoustic wave in the fluid, and, consequently, their velocities are less than the sound velocity in the entire frequency range. Therefore, they propagate without the leakage of wave energy delivering it to infinity, while the wavenumbers of the former Lamb waves shift from the real axis into the complex plane, wherein their imaginary parts specify the attenuation decrements due to energy leakage.

The only exception is the fundamental antisymmetric A0 mode. Its dispersion curve comes out vertically from the origin so that there is always a low-frequency range in which it passes over the straight dispersion line of the acoustic bulk wave, as in the case of the Scholte–Stoneley waves. In this range, its phase velocity is also less than that in the environment, and it is expected that its wavenumber must be real to describe the propagation without energy leakage. However, attempts to find the corresponding real root of the characteristic equation of a fluid-loaded plate have failed. In many papers, the authors just leave this place empty without comments or remain the curve for the fluid-free plate.

We decided to study in detail the frequency-dependent root movement in the complex plane. Normally, the root of every leaky Lamb mode has a counterpart on the unphysical sheet. It turned out that at low frequencies both such A0 roots lay on the unphysical sheet, and one of them comes out into the physical sheet only when its real part becomes less than the acoustic wavenumber, i.e., after crossing a vertical cut that selects the sheets. Since the cut path can be arbitrarily traced from the branch point to infinity, various scenarios of the A0 root appearance on the physical sheet are possible. It even can exist in the entire frequency range if the cut is deflected to the left going from the branch point to the origin along the real axis and then upward. But in any case, the physical A0 root remains complex yielding a guided leaky wave not carrying energy to infinity. Note that the total wave field generated by a source does not depend on the choice of the cut path.

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Weighted Radon transforms and their inversion

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In this talk, we consider the problem of inversion of weighted Radon transforms for general weights. We discuss old and new theoretical results on this problem and also present new numerical results of our iterative inversion algorithm of [1].

References

[1] F. O. Goncharov, An iterative inversion of weighted Radon transforms along hyperplanes, *Inverse Problems*, **33**, 124005 (2017).

Vectorial focus wave modes with elliptic and parabolic cross-section at planar interface between two dielectrics

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Focus wave modes (FWM) are solutions of scalar Helmholtz equation, which are localized and can ideally propagate long distances without spreading due to diffraction and dispersion. This type of pulsed beams is understood in terms of superposition of individual monochromatic Bessel beams with a given angular dispersion. Here, we extend the concept of FWM's to another two classes of nondiffracting beams — parabolic Weber-type and elliptic Mathieu beams. We aim to construct a scalar analogues to the classic FWM's and investigate their propagation inside a dielectric material, when their temporal durations can be up to few cycles. Furthermore, due to the large spatial angles introduced by the angular dispersion required for those durations, vectorial theory is introduced. We overcome this problem by introducing vector parabolic and elliptic nondifracting beams with controllable axial polarization, which can be either linear, circular and radial or azimuthal. Those newly obtained vector monochromatic beams are substituted into expressions for nondiffracting and nondispersive pulsed beams and, as a result, spatio-temporal distortions are successfully compensated. Lastly, we dive into problems arising while applying these vectorial pulsed beams for laser-microprocessing applications. Finally, we briefly report here on generation and implementation of the considered pulsed beams in scenarios involving a laser-micromachining of transparent dielectric materials [1].

References

[1] S. Orlov, V. Vosylius, P. Gotovski, A. Grabusovas, J. Baltrukonis, T. Gertus, Vector beams with parabolic and elliptic cross-sections for laser material processing applications, *Journal of Laser Micro/Nanoengineering*, **13**, 280–286 (2018).

Linear homogeneous isotropic viscoelastic constrained reduced Cosserat medium as an acoustic metamaterial

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We consider a special kind of a linear reduced Cosserat medium, where each point of the medium is characterized by a translational displacement $\mathbf{u}$ and micro-rotation $\boldsymbol{\theta}$, but no stresses in the medium work on $\nabla\boldsymbol{\theta}$ and, in addition, there is a constraint relating the micro-rotation and the translational field:

$$\boldsymbol{\theta} = \nabla \times \mathbf{u}/2. \tag{1}$$

In such a medium couple stress is zero, stress tensor $\boldsymbol{\tau}$ is asymmetric, but its antisymmetric part is determined by the balance of moment, and not by constitutive equations (due to the existence of the kinematic constraint). Equations of such a medium [1] are

$$\nabla \cdot \boldsymbol{\tau}^S = \rho\ddot{\mathbf{u}} + I(\nabla\nabla \cdot \ddot{\mathbf{u}} - \triangle\ddot{\mathbf{u}})/4. \tag{2}$$

Our motivation is to investigate the simplest viscoelastic, though specific, model, including rotations. In case of the reduced viscoelastic non-constrained Cosserat continuum we see a rich variety in wave behaviour. We consider the constrained isotropic model, where the number of parameters is limited, and it is easier to investigate the dispersion relation analytically. For this case the equations in displacements in the Fourier domain look as

$$-(\lambda(1+i\kappa\omega)+2\mu(1+i\nu\omega))\mathbf{kk}\cdot\mathbf{u}+\mu(1+i\nu\omega)\mathbf{k}\times(\mathbf{k}\times\mathbf{u}) = -\omega^2\rho\mathbf{u}+I\omega^2\mathbf{kk}\cdot\mathbf{u}/4-I\omega^2k^2\mathbf{u}/4, \tag{3}$$

where ω is the frequency, and $\mathbf{k}$ is the wave vector. The longitudinal wave is the same as in the classical medium, and the shear wave has a dispersion relation

$$\frac{\mu(1+i\nu\omega)}{\omega^2} = \frac{\rho + Ik^2/4}{k^2} = \frac{I}{4} + \frac{\rho}{k^2}, \tag{4}$$

$$\frac{1}{k^2} = c_s^2\frac{1+i\nu\omega}{\omega^2} - \frac{I}{4\rho}. \tag{5}$$

If we represent $k = K + i\kappa$, we obtain

$$K^4 - K^2\frac{\omega^2(c_s^2 - \frac{I}{4\rho}\omega^2)}{z} - \nu^2\frac{c_s^4\omega^2}{z^2} = 0, \qquad \kappa = -\nu\frac{c_s^2\omega}{zK}, \tag{6}$$

where

$$z = \left(c_s^2 - \frac{I}{4\rho}\omega^2\right)^2 + \nu^2c_s^4\omega^2 > 0. \tag{7}$$

We investigate dispersion properties of this medium and its difference with the classical elastic and reduced Cosserat continua (elastic and viscoelastic). For non-zero viscosity $K(\omega)$ has a decreasing part at large ω, therefore this material is a double negative acoustic metamaterial in this domain.

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References

[1] E. F. Grekova, R. Abreu, Isotropic linear viscoelastic reduced Cosserat medium: an acoustic metamaterial and a first step to model geomedium, Chapter 13 in *Generalized Continua*, 165–185, Springer (2019).

On the solution of the problem of low-frequency acoustic signal propagation in a shallow-water waveguide with three-dimensional random inhomogeneities

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Based on the local mode method, the problem of the propagation of tonal acoustic signals of a frequency $f < 1$ kHz in a shallow water waveguide typical for the shelf zones of an ocean is considered in the presence of three-dimensional random fluctuations of the sound speed in the water column. Three-dimensional linear acoustics equations with random coefficients are reformulated into causal equations of the first order, for which, by analogy with the two-dimensional case, the solution can be written in quadratures in a convenient exponential representation. As applied to the problem of fluctuations of the speed of sound in a waveguide, a solution is written in the one-way propagation approximation (forward scattering), on the basis of which it is possible to perform statistical modeling. In the particular case, an example illustrating the statistical calculations of the average transmission loss of the signal frequency of 500 Hz is presented.

Beyond the Bakushinskii veto part II: Discretisation and white noise

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We deal with the solution of an ill-posed equation $K\hat{x} = \hat{y}$ for a given compact linear operator K on separable Hilbert spaces. Often, one only has a corrupted version y^δ of $\hat{y}$ at hand and the Bakushinskii veto tells us, that we are not able to solve the equation if we do not know the noise level $\|y^\delta - \hat{y}\|$. But in applications it is ad hoc unrealistic to know the error of a measurement. In practice, the error of a measurement is usually estimated through averaging of multiple measurements. In [1] we introduced the natural approach to solve such a problem: Consider noisy but multiple measurements $Y_1, \ldots, Y_n$ of the right hand side $\hat{y}$. Furthermore, assuming that the noisy measurements are unbiased and independently and identically distributed according to an unknown distribution, the natural approach would be to use $(Y_1 + \ldots + Y_n)/n$ as an approximation of $\hat{y}$ and the estimated error $\sigma_n/\sqrt{n}$ (where σ_n is the square root of the sample variance) together with a deterministic regularisation method. We showed that the approach converges in probability for the discrepancy principle and in L^2 for a priori strategies.

Here we deal with the problem of discretisation and unbounded noise: in practice, we cannot measure $\hat{y}$ directly, which is in general an object with infinitely many components, but only linear functionals $l_1(\hat{y}), l_2(\hat{y}), \ldots$ We show how one obtains convergence against the true solution $\hat{x}$ when one measures more and more components (linear functionals) again and again.

References

[1] B. Harrach, T. N. Jahn, R. Potthast, Beyond the Bakushinskii veto: Regularising without knowing the noise distribution, *arXiv*: 1811.06721 [math.NA] (2018).

Computer simulation of torsional transducer from porous piezoceramics with twisted rod

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Piezoelectric devices are widely used in current technology, converting the electrical energy into the mechanical energy, and vice versa. Fairly often, the piezoelectric transducers operate at the resonant frequencies of a certain types. Among such transducers there is a separate class of the devices operating on torsional modes, or on connected longitudinal–torsional vibration modes.

In the literature we can find a description of different types of the torsional transducers. Such piezoelectric transducers are typically a combination of elastic and piezoelectric parts with different geometry, polarization direction, and electrode location. Thus, there are the bolt-clamped Langevin devices consisting of a pair of piezoceramic discs polarized along a circumferential directions, or a pair of piezoceramic elements, one of which is radially polarized, and the second part is polarized by thickness; in different models the hollow piezoceramic cylinder with wire helical electrodes coated on the side surface is proposed to use. Other authors considered piezoelectric transducers of helical forms, and including taking into account the nanoscale factors and the surrounding liquid medium.

This paper considers a piezoelectric transducer operating on torsional modes. The transducer consists of a piezoelectric cylinder rigidly coupled with a elastic twisted rod. Along the central axis in the cylinder and in the rod there are the cylindrical hollows with the same radius. The piezoelectric cylinder is made of PZT ceramics, polarized in the circumferential direction. It has two electrodes at the end surfaces and it is rigidly fixed at the bottom end. When a harmonically varying voltage or electric current is applied to the electrodes, in the cylinder predominantly torsional oscillations are generated. The naturally twisted shape of the elastic rod promotes to an additional amplification of torsional vibrations. The analysis of the piezoelectric transducer was carried out in the ANSYS finite element package.

A comparison was made of the amplitude-frequency characteristics of the transducer near the first modes of thickness oscillations for the cases of dense and porous PZT ceramics, for the cases of twisted and non-twisted forms of an elastic rod, and for the cases when the oscillations are excited by voltage or by current generators.

The simulation results allowed us to calculate the first electrically active resonance frequencies and to carry out the harmonic analysis of torsional vibrations of the transducer near to resonance frequencies. The computational experiments have shown that the torsional vibrations on the first electrically active resonance frequency are the largest for the transducer made of porous piezoceramics, with the twisted rod, and when the source of oscillations was a current generator.

By changing the geometrical dimensions and the piezoceramic porosity it is possible to vary the parameters of the transducer within a fairly wide range and obtain optimal performance in various piezotechnical applications.

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Riemann–Hilbert approach to scattering problems in elastic media

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We are developing Riemann–Hilbert (RH) approach to scattering problems in elastic media. The approach is based on the unified transform method of A. S. Fokas for solving boundary problems for linear and integrable nonlinear PDEs. The main topics of this project are 2D elastodynamic equation in an isotropic quarter-space with the stress free boundary conditions, Helmholtz equation in quarter space with general boundary conditions and Helmholtz equation in general sector. In this talk we report the recent results (and difficulties) on RH approach to Helmholtz in general sector. The second part of the talk will be related to the elastodynamic equation. In the series of papers of the author and her collaborators the global relation for this problem was reduced to a certain, so called, *supplementary* vector Riemann–Hilbert problem with a shift posed on a torus. However, it is notoriously difficult to solve explicitly such factorization problems and there is a possibility that the formulated non-local Riemann–Hilbert problem does not admit an explicit solution. A more realistic expectation, which we are currently trying to fulfill, is to obtain the connection formulae between the asymptotic parameters of the Rayleigh waves along the horizontal and vertical parts of the boundary.

The talk is based on the joint works with A. Its and Yu. Kaplunov. It is also a part of a large joint project with Yu. Antipov, A. Its and M. Lyalinov.

Heat kernel for Laplace operator with covariant derivative: expansions, path integral and gauges

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Previously, in [1] the diagram technique and matrix formalism were constructed to work with the Sealy–DeWitt coefficients. In this work we study the generalization of the formalism, its connection with the representation by using the path integral and its dependence on gauge fixing.

References

[1] A. V. Ivanov, Diagram technique for the heat kernel of the covariant Laplace operator, *Theor. Math. Phys.*, **198**(1), 100–117 (2019).

Multibump trajectories of adiabaically perturbed periodic Hamiltonian systems with pitchfork bifurcations

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We study a Hamiltonian system with a Hamiltonian

$$H(q,p,\varepsilon t) = \frac{p^2}{2} - \frac{1}{2}\varphi(\varepsilon t)q^2 + \frac{1}{4}q^4, \quad \varepsilon \ll 1, \tag{1}$$

slowly depending on time. It is assumed the function φ to be a periodic C^2-function with period 1, which satisfy the following condition

(H_1): there exist $\tau_k \in [0,1), k = 1,\dots,2m$, such that $\varphi(\tau_k) = 0$, $\varphi'(\tau_k) \neq 0$.

Systems of type (1) appear in many branches of physics, for example, they describe dynamics of charged particles in the Earth's magnetospheric tail, where the Larmor radius of the particles is larger than the minimum radius of curvature of the magnetic field lines [2].

One may consider (1) as the simplest example of the slow-fast systems. The Hamilton's equations associated to (1) and considered in the extended phase space are

$$\dot{q} = p, \quad \dot{p} = \varphi(\tau)q - q^3, \quad \dot{\tau} = \varepsilon.$$

Thus, (q, p) can be treated as fast variables and τ as a slow one. The "slow" manifold Γ of this system is defined by equations $p = 0$, $\varphi(\tau)q - q^3 = 0$ and consists of one point $(q, p) = (0, 0)$ if $\varphi(\tau) \leq 0$ and of three points $\{(0, 0), (\pm\sqrt{\varphi(\tau)}, 0)\}$ in the case $\varphi(\tau) > 0$. Each time the parameter τ passes through the point τ_k the pitchfork bifurcation occurs. When $\varphi(\tau) > 0$ the origin becomes a hyperbolic equilibrium of the "frozen" system and possesses the figure-eight separatrix $\mathcal{S}$ consisted of two loops $\mathcal{S}_\pm$.

In the present work, using the ideas similar to [1], we show that the sequence of bifurcations leads to appearance of multi-bump trajectories of the system. In particular we show that there exists $\varepsilon_0 > 0$ and a subset $\mathcal{E}_h \subset (0, \varepsilon_0)$ such that

1. for any $\varepsilon_1 < \varepsilon_0$ the Lebesgue measure $\mathrm{leb}\big((0, \varepsilon_1) \setminus \mathcal{E}_h\big) = O\left(\mathrm{e}^{-c/\varepsilon_1}\right)$ with some positive constant c;
2. for any $\varepsilon \in \mathcal{E}_h$ the origin is a hyperbolic equilibrium of the system (1);
3. for any natural N and any sequence $\{a_k\}_{k=1}^N$ with $a_k = 0$ if $\varphi'(\tau_k) < 0$ and $a_k \in \{-1, 0, 1\}$ if $\varphi'(\tau_k) > 0$ there exists a multi-bump trajectory which stays during the time between τ_k and τ_{k+1} in a small neighborhood of the origin if $a_k = 0$ or follows one of the branches of the "frozen" separatrix $\mathcal{S}_\pm$ if $a_k = \pm 1$.

References

[1] A. V. Ivanov, Connecting orbits near the adiabatic limit of Lagrangian systems with turning points, *Reg. & Chaotic Dyn.*, **22**(5), 479–501 (2017).

[2] D. L. Vainshtein, L. M. Zelenyi, A. I. Neishtadt, B. V. Savenkov, Jumps in an adiabatic invariant with small initial values, *Plasma Physics Reports*, **25**(4), 299–303 (1999).

Ocean bathymetry as an inverse problem for the radiative transfer equation

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We consider the problem of ocean bathymetry by using a side-scan sonar. Process of acoustic waves propagation on frequencies on the order of tens kHz is described by the integral-differential radiative transfer equation [1]. The propagation media is semi space which is bounded by a seabottom surface. The bathymetric function is included in the boundary condition. We obtain the solution of the initial-boundary value problem in the double-scattering approximation and some assumptions for the directivity pattern of the receiving antenna:

$$I^\pm(t) = \chi_{[0,\mp\infty]} \frac{8\sigma_d J_0 \exp(-\mu ct)\big(u'_{y_1} \cdot y_1 + l - u(y_1)\big)^2}{c^2 t^3 \sqrt{1 + (u'_{y_1})^2}\,\big|u'_{y_1}(l - u(y_1)) - y_1\big|}$$
$$+ 4\sigma_d^2 J_0 \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} S^\pm(\mathbf{k}) \frac{\exp(-\mu|\mathbf{y}|)}{|\mathbf{y}|}\ \exp(-\mu|\mathbf{y}-\mathbf{z}|)\ \frac{\exp(-\mu|\mathbf{z}|)}{|\mathbf{z}|}\ \left|\mathbf{n}(\mathbf{y}) \cdot \frac{\mathbf{y}}{|\mathbf{y}|}\right|^2\ \left|\mathbf{n}(\mathbf{z}) \cdot \frac{\mathbf{z}}{|\mathbf{z}|}\right|^2$$
$$\times\, \delta\left(t - \frac{|\mathbf{y}-\mathbf{z}|}{c} - \frac{|\mathbf{z}|}{c} - \frac{|\mathbf{y}|}{c}\right)\sqrt{1 + u'_{y_1}}\sqrt{1 + u'_{z_1}}\, dy_1 dz_1. \quad (1)$$

Here, $I^{\pm}$ denotes a sonar input signal, c denotes the constant velocity, $t \in [0, T]$, $\mathbf{y}, \mathbf{z} \in \mathbb{R}^2$ denote points of the seabottom, J_0 is the source power, the function u describes changes of the ocean bottom relief, σ_d denotes the constant seabottom reflection coefficient, $\mathbf{n}(\mathbf{y})$ and $\mathbf{n}(\mathbf{z})$ denote external normals, wave vector $\mathbf{k}$ belongs to the unique sphere $\Omega = \{\mathbf{k} \in \mathbb{R}^2 : |\mathbf{k}| = 1\}$, δ is the Dirac delta function and the function $S^{\pm}(\mathbf{k})$ is the radiation pattern of the receiving antenna on the starboard and the portside, respectively.

In paper [2] we found the solution of the inverse problem for the function which described relief changes from the middle-level l. It was obtained by using a single-scattering approximation.

$$u_i' = \frac{1}{y_{1,i}} \left(u_{i-1} - l + \sqrt[4]{1 + v_{0,i-1}^2} \left(\frac{I(t_i) c^2 \, t_i^3 \, |v_{0,i-1}(l - u_{i-1}) - y_{1,i}|}{8 \, \sigma_d \, J_0 \, \exp(-\mu c t_i)} \right)^{\frac{1}{2}} \right), \tag{2}$$

where $t_i = 2\frac{\sqrt{y_{1,i}^2 + (l - u_{i-1})^2}}{c}$, $v_{0,i} = u'(t_{i-1})$.

In Fig. 1 the seabottom relief and the absolute error are shown. In this paper we analyze an influence of **re**reflected signal on the reconstruction of the bathymetry function by using (2).

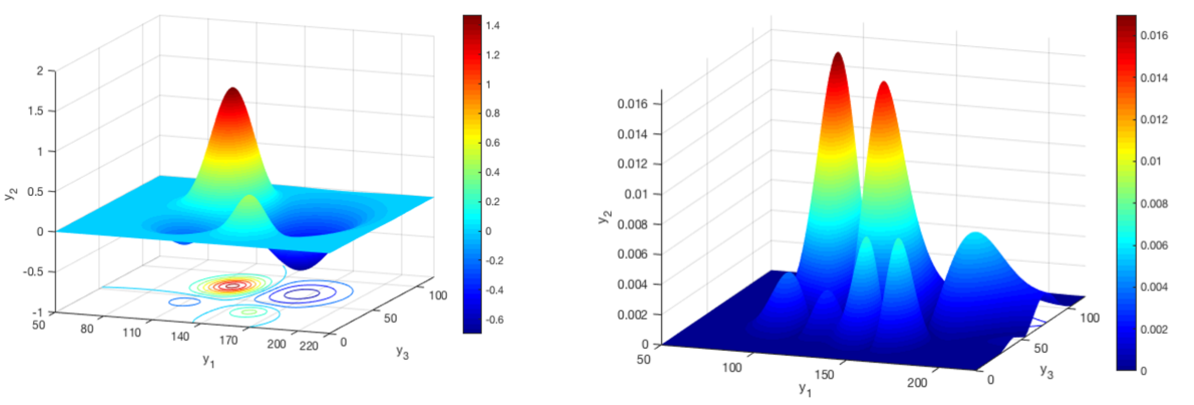

Fig. 1: Reconstruction of the seabottom relief (u) and absolute error (Δu).

References

[1] I. V. Prokhorov, A. A. Sushchenko, Studying the problem of acoustic sounding of the seabed using methods of radiative transfer theory, *Acoustical Physics*, **61**(3), 368–375 (2015).

[2] E. R. Lyu, V. A. Kan, I. V. Prokhorov, A. A. Sushchenko, Determination of the bottom surface profile, *Proceedings of SPIE – The International Society for Optical Engineering*, **10466**, Art. no. 1046622 (2017).

Eikonal algebra on metric graph

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An eikonal algebra is a C*-algebra related to a metric graph. It is determined by trajectories and reachable sets of a dynamical system associated with the graph [1, 2]. The system describes the waves, which are initiated by boundary controls and propagate into the graph with finite velocity. Motivation and interest to eikonal algebras comes from the inverse problem of reconstruction of the graph via its dynamical and/or spectral boundary inverse data [3]. Eikonal algebra is determined by these data. Its structure and spectrum (the set of irreducible representations) is related with the graph topology. The topology is visualized using the coordinatization of the spectrum via the eikonal operators which generate the eikonal algebra.

References

[1] M. I. Belishev, N. Wada, A C*-algebra associated with dynamics on a graph of strings, *Journal of the Mathematical Society of Japan*, **67**(3), 1239–1274 (2015).

[2] M. I. Belishev, A. V. Kaplun, Eikonal algebra on a graph of simple structure, *Eurasian Journal of Mathematical and Computer Applications*, **6**(3), 4–33 (2018).

[3] M. I. Belishev, N. Wada, On revealing graph cycles via boundary measurements, *Inverse Problems*, **25**(10), 105011 (2009).

Time frequency analysis of the sound field modal decomposition in shallow water in the presence of internal waves

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In the paper analysis of the wideband sound field modal decomposition in shallow water waveguide using single hydrophones and vertical line arrays (VLAs) are carried out. For this goal time-frequency analysis (for single hydrophones) in the form of the Warping Transform and standard methods (for VLA) are applied for the data of measurements obtained in the SWARM'95 experiment, carried out in the Atlantic shelf of USA (depth ∼80–90 m). This area is characterized by remarkable activity of nonlinear internal waves moving approximately twice per day in the form of trains, containing up to 10–12 separate peaks, toward the coast. Low frequency wide-band sources (J15, generating LFM signals 50–300 Hz, and airgun with band ∼30–250 Hz) were used at the 15 km acoustic tracks with the angle about 30 degrees between them. Sound signals were recorded by two vertical line arrays (WHOI VLA and NRL VLA). In the presence of moving nonlinear internal waves significant sound intensity fluctuations took place at the WHOI VLA due to horizontal refraction (focusing/defocusing in horizontal plane), at the NRL VLA there is no such fluctuations. Warping Transform is used for mode filtering for all separate hydrophones of VLAs for different time periods, temporal dependencies of modal amplitudes are compared with the same amplitudes obtained using VLA decompositions. Dependence of modal amplitudes fluctuations on frequency and mode number is studied, the correspondence between mentioned methodologies is verified.

Excitation and propagation of whispering gallery waves in shallow water waveguide in the vicinity of a curvilinear isobath

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Sound propagation in an area of a shallow water waveguide with curvilinear isobaths and the bottom depth increasing towards their curvature center is considered. Such area could be part of a bay, lagoon or a lake with bowl-like bottom relief, it consists of deep and shallow areas, and a transition segment between them having curvilinear shape (cylindrically-symmetric in the simplest case). The possibility of the formation of waves localized in the deeper part in the vicinity of curvilinear isobaths is studied, the modal structure of such waves is investigated, and criteria for their existence and excitation by the source placed both in the deep and the shallow part are derived. In the latter case they are excited via a mechanism that can be described as tunnelling of waves penetrating into the deeper part from the outer source.

The modal structure of whispering gallery waves propagating along a curvilinear isobath is studied using horizontal rays and within the framework of parabolic equation theory.

Hydroelastic waves in ice channel caused by moving load

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The ice responses to loads moving along an unbounded ice cover are well studied for water of both infinite and constant depth by using the linear theory of hydroelasticity [1]. It is known that the ice response strongly depends on the speed of the load. If the speed is below a certain critical value, the ice deflection is localized near the load and quickly decays with the distance from the load. For higher speeds of the load, outgoing waves are formed in the far field if viscous damping is not included in the model. At the critical speed, the linear theory of hydroelasticity without damping predicts unbounded ice response. To obtain estimates of the ice response for this speed, either non linear effects or viscous damping, or both are included in the ice model.

We are investigate waves in ice channel with rectangular cross-section, caused by a load, moving along the channel. The presence of walls near the moving loads complicates the problem. In contrast to the ice sheet of infinite extent, there are many critical speeds for a frozen channel [2]. Each critical speed for an ice cover in a channel corresponds to a mode of hydroelastic wave propagating along the channel with a certain profile across the channel and its own dispersion relation.

There are several approaches which were developed to investigate the problem of moving loads in ice channel. We studied linear model with and without damping, steady-state problem, the problem when the load starts to move instantly, and the asymptotic problem for large time. The self-consistent theory of hydroelastic waves in frozen channel will be present.

Large time asymptotic solution of three-dimensional problem of hydroelasticity for a channel covered with ice sheet has been obtain analytically. This solution is valid for long time after the load started its motion and provides the structure of the ice deflection near and far from the load [4]. The large-time ice response consists of the symmetric part localised near the load and several wave systems modulated by smooth cut-off functions. The number of waves system behind and in front of the load depend of the value of the load speed with respect to the critical speeds. The amplitudes of the waves in the far field were obtained analytically, which simplifies the calculations of ice deflection and strain distributions at a distance from the load.

The obtained results without account for any damping are in good agreement with the results obtained earlier within the Kelvin–Voight model of visco-elastic ice [3]. The strains computed without account for dissipation effects are larger than actual strains. They can be used to estimate conditions of safe transportation along frozen channels.

References

[1] V. Squire, R. Hosking, A. Kerr, P. Langhorne, *Moving Loads on Ice Plates*, Kluwer Academic Publishers (1996).

[2] A. Korobkin, T. Khabakhpasheva, A. Papin, Waves propagating along a channel with ice cover, *European Journal of Mechanics – B/Fluids*, **47**, 166–175 (2014).

[3] K. Shishmarev, T. Khabakhpasheva, A. Korobkin, The response of ice cover to a load moving along a frozen channel, *Applied Ocean Research*, **59**, 313–326 (2016).

[4] T. Khabakhpasheva, K. Shishmarev, A. Korobkin, Large-time response of ice cover to a load moving along a frozen channel, *Applied Ocean Research*, **86**, 154–165 (2019).

Excitons in the Kronig–Penney model

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A minimal model for excitons in frame of time-dependent density-functional theory (TDDFT) is considered. For the simplicity the Kronig–Penney potential plays role of the Kohn–Sham potential. Also the two-band approximation is used. Two different xc kernels is studied — short-ranged contact xc and long-ranged (soft-Coulomb) xc kernel. Optical properties are discussed. As a result, one or two bound excitons are produced, depending on the parameters of system.

References

[1] Z.-H. Yang, Y. Li, C. A. Ullrich, A minimal model for excitons within time-dependent density-functional theory, *The Journal of Chemical Physics*, **137**, 014513 (2012).

[2] V. Turkowski, A. Leonardo, C. A. Ullrich, Time-dependent density-functional approach for exciton binding energies, *Phys. Rev. B*, **79**, 233201 (2009).

Inverse problems for diffusion equations

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We consider the inverse problem of determining uniquely some terms appearing in a, linear or non-linear, diffusion equation from measurements of its solutions at the boundary of the domain. In the linear case, our equation can be seen as a convection-diffusion equation describing the transfer of different physical quantities (particles, energy, ...) inside a physical system. Our inverse problem consists in determining the velocity field associated with the moving quantity as well as information about the density of the medium. We state our problem in a general setting where the quantity that we want to determine are associated with non-smooth time-dependent coefficients. In the non-linear case, we treat the determination of a more general quasi-linear term appearing in the equation. This is a joint work with Pedro Caro.

Creeping waves in the shadow region with the Neumann conditions

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We consider a short wave diffraction problem for the plane wave on the strictly convex smooth prolate body of revolution. We call this body the scatterer. The wave field satisfies the Helmholtz equation outside of the scatterer, the Dirichlet and Neumann boundary conditions on the surface

and also the limiting absorption principle far from the scatterer. The field in the illuminated area is described by the ray formulae.

The ray method fails near the light-shadow border, where the incident rays are tangential to the body. We have constructed an asymptotic solution in the Fock's boundary (surface) layer by Leotovich–Fock parabolic equation method [1–4]. The constructed solution has been described only in a small neighbourhood of the point of contact of the incident ray and the scatterer, which belongs to the light-shadow border. The solution can be continued from this neighbourhood into both the deep shadow zone and into the vicinity of the limiting ray. The neighbourhood of this point serves as a birthplace of waves of rather disparate natures. Here creeping waves are generated which travel from this point into the shadow zone, as well as a field having the character of Friedkander–Keller waves [5], a wave of Fresnel type in the illuminated part of the penumbra, and a penumbral wave in the shaded region of the penumbra.

The latter creeping waves are under our investigation in this report where we assume the surface of the scatterer satisfies the Neumann boundary conditions. We investigate the latter creeping waves and assume the Neumann boundary conditions on the surface of the scatterer. The results for the case of the Dirichlet conditions were obtained in [3].

References

[1] V. A. Fock, *Electromagnetic Diffraction and Propagation Problems*, International Series of Monograph on Elecromagnetic Waves, **1**, Pergamon Press (1965).

[2] V. M. Babich, N. Ya. Kirpichnikova, *The Boundary-Layer Method in Diffraction Problems*, Springer-Verlag (1979).

[3] N. Ya. Kirpichnikova, M. M. Popov, Leontovich–Fock parabolic equation method in the problems of short-wave diffraction by prolate bodies, *Zap. Nauchn. Sem. POMI*, **409**(42), 55–79 (2012).

[4] A. S. Kirpichnikova, N. Ya. Kirpichnikova, The Leontovich–Fock parabolic equation method in the Neumann diffraction problem on a prolate body of revolution, *Journal of Mathematical Sciences*, **238**(5), 148–173 (2019).

[5] F. G. Friedlander, J. B. Keller, Asymptotic expansion of solution of $(\nabla^2 + k^2)u = 0$, *Comm. Pure Appl. Math.*, **8**(3), 378–394 (1955).

Asymptotic "bouncing ball" type eigenfunctions of the two-dimensional Schrödinger operator with symmetric potential

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In [1] the problem of the asymptotics for $k \to \infty$ of the spectrum of the Laplace operator: $-\Delta u = k^2 u$, defined in a two-dimensional domain bounded by an ellipse with zero Dirichlet condition on the boundary was considered. This problem corresponds to a billiard dynamical system with reflections from the boundary of the domain. The closed trajectories of this dynamical system, which are stable in the linear approximation, correspond to a sequence (series) of asymptotic eigenfunctions (quasimodes). In this case, such a trajectory is the minor axis of the ellipse. The corresponding quasimodes are called asymptotic eigenfunctions of the bouncing ball type.

A related problem is the problem of the asymptotics for $h \to 0$ of the spectrum of the Schrödinger operator: $-h^2 \Delta \psi + V\psi = E\psi$. (The limiting case of a potential V equal to zero inside the domain and equal to infinity outside corresponds to the Laplace operator with zero Dirichlet condition on the boundary of the domain.)

This problem corresponds to a natural Hamiltonian dynamical system. According to [2], the closed trajectories of this dynamical system, which are stable in the linear approximation, correspond to a series of quasimodes. If the closed trajectories of the Hamiltonian system define the librations, then the corresponding quasimodes are analogues of bouncing ball type eigenfunctions of the Laplace operator. The difference is that the reflection points in this situation are turning or focal points on the trajectory. Thus it is necessary to use the integral representation for the quasimodes in the neighborhood of reflection points (see e.g. [3]). We consider the case when the potential is symmetric in one variable and using the ideas similar to [4–6] show that in this case the subsequence of quasimodes associated with corresponding librations could be globally presented in a form of Airy functions and Gaussian exponents.

References

[1] V. M. Babich, V. S. Buldyrev, *Asymptotic Methods in Short Wave Diffraction Problems*, Nauka, Moscow (1972).

[2] V. P. Maslov, *The Complex WKB Method for Nonlinear Equations I: Linear Theory*, Birkhäuser, Basel (1994).

[3] A. Yu. Anikin, S. Yu. Dobrokhotov, A. I. Klevin, B. Tirozzi, Gaussian packets and beams with focal points in vector problems of plasma physics, *Theoretical and Mathematical Physics*, **196**(1), 1059–1081 (2018).

[4] S. Yu. Slavyanov, *Asymptotics of Solutions of the One-Dimensional Schrödinger Equation*, American Mathematical Society, Providence, RI (1996).

[5] S. Yu. Dobrokhotov, D. S. Minenkov, S. B. Shlosman, Asymptotics of wave functions of the stationary Schrödinger equation in the Weyl chamber, *Theoretical and Mathematical Physics*, **197**(2), 1626–1634 (2018).

[6] S. Yu. Dobrokhotov, A. V. Tsvetkova, Lagrangian manifolds related to the asymptotics of Hermite polynomials, *Mathematical Notes*, **104**(6), 835–850 (2018).

Effect of forward-backward wave coupling on Kerr frequency comb generation in optical microresonators

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In recent years, optical whispering gallery modes (WGM) microresonators have found wide application in various fields of science and technology. Due to high quality factor they represent the most promising platform for the creation of miniature, energy-efficient components for photonics and radiophotonics. A significant breakthrough was the discovery of Kerr frequency combs and dissipative Kerr solitons (DKS) generation in microresonators [1]. Besides, the coupling of high-Q microresonator with a laser provides laser stabilization and its linewidth reduction via the self-injection locking effect [2, 3]. Recently the generation of DKS was demonstrated by the multi-frequency laser locked by high-Q microresonator [4]. Interestingly, the single-soliton regime was obtained without any additional efforts. However there is no full theory of this phenomenon. The self-injection locking effect appears due to Rayleigh scattering inside the microresonator and subsequent backward wave provides resonant feedback [2]. However, this backward wave also interacts nonlinearly with the forward wave and influence frequency comb dynamics. It was shown that such coupling may negate DKS generation [5].

In our work we study the backward wave influence on Kerr frequency comb generation that may help to understand nonlinear processes in microresonator in the self-injection locking regime. To model the microresonator comb dynamics we used the coupled mode equation system (CMES) approach [1]. We derived the modified CMES from the first principles starting from Maxwell equations. Equivalent system of coupled Lugiato–Lefever equations (LLE) was also obtained. We showed that up to some critical finesse value the system dynamics becomes more complex but for higher values it becomes independent of this parameter. For the anomalous dispersion regime forward-backward wave coupling may affect the DKS generation dynamics. Such results are similar to those got with LLE approach in [5]. We found that in normal dispersion regime the backward wave influence may induce instabilities providing a novel mechanism of comb generation. In this case some Turing pattern-like structures and chaotic regimes may be observed. It may be proposed that such mechanism may be responsible for platicon generation [6].

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References

[1] T. Herr, et al., Temporal solitons in optical microresonators, *Nature Photonics*, **8**, 145 (2014).

[2] N. M. Kondratiev, et al., Self-injection locking of a laser diode to a high-Q WGM microresonator, *Opt. Express*, **25**, 28167 (2017).

[3] R. R. Galiev, et al., Spectrum collapse, narrow linewidth, and Bogatov effect in diode lasers locked to high-Q optical microresonators, *Opt. Express*, **26**, 30509 (2018).

[4] N. G. Pavlov, et al., Narrow-linewidth lasing and soliton Kerr microcombs with ordinary laser diodes, *Nat. Photon.*, **12**, 694 (2018).

[5] Shun Fujii, et al., Effect on Kerr comb generation in a clockwise and counter-clockwise mode coupled microcavity, *Opt. Express*, **25**, 28969 (2017).

[6] V. E. Lobanov, et al., Frequency combs and platicons in optical microresonators with normal GVD, *Opt. Express*, **23**, 7713 (2015).

Water exit and entry

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"Don't believe what your eyes are telling you. All they show is limitation. Look with your understanding. Find out what you already know and you will see the way to fly." By Richard Bach from "Jonathan Livingston Seagull".

The problems of a body lifted from a water surface and entering the water surface are studied by theoretical, numerical and experimental means. It is explained how experimental results and computations may help in development of theoretical models. Vice versa, theoretical models guide computations and experiments on water entry and exit. The simplest models of water entry and exit neglect all effects except of fluid inertia. The models are generalized to account for gravity, non-linear effects and elasticity of the body by using the Computational Fluid Dynamics simulations and experimental results without significant increasing complexity of the models. Some results are counterintuitive but explained by the theoretical models.

Unsteady axisymmetric problem of a circular disc, which initially touches the flat water surface and is lifted then suddenly is studied experimentally and theoretically. The disc drags the liquid behind it. Both the external force applied at the centre of the disc and the disc acceleration at the same place are measured during an initial transient stage of the process. The external force and the disc acceleration are well related initially by the added mass approach. Later on, the measured acceleration decays even the applied force continues to increase. The disc displacement and the radius of the wetted area of the disc are recorded by high-speed cameras. The records reveal that the wetted area of the disc does not change before the disc acceleration reaches its maximum value. It is argued that these phenomena are caused by the elastic deflection of the disc during the initial transient stage. The linearised model of water exit is generalised to account for elastic behaviour of the lifted body. The results obtained with the generalised exit model well agree with the experimental results, and they were additionally confirmed by dedicated experiments with circular discs of different rigidity.

The work was done in collaboration with: T. Khabakhpasheva (UEA), K. Maki (UoM), J. Rodriguez (UC3M), P. Vega-Martinez (UC3M), S. Seng (BV).

Non-stationary mathematical model of oxygen transport in brain

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Mathematical modeling of oxygen transport in brain is an important tool to predict dangerous situations caused by impaired cerebral circulation. Following to conventional models (cf. [1, 2]), the brain material is considered as a two-compartment (blood and tissue) structure so that the mathematical model consists of coupled equations describing convective and diffusive transports processes of oxygen and its consumption in tissue. Though such a promising approach allows for conducting diverse numerical simulations, its theoretical analysis is very poor. A preliminary mathematical study of a steady-state oxygen transport model was conducted in [2]. However, currently, there is not exhaustive theoretical results on evolutionary models of oxygen transport. The current work intends to cover the lack of accurate mathematical analysis of evolutionary models.

To describe the propagation of oxygen in tissue and blood, a homogenization approach, along with the assumption that the tissue and blood fractions occupy the same spatial region $\Omega \subset \mathbb{R}^3$, is used. The resulting evolutionary continuum model of oxygen transport has the following form:

$$\partial\varphi/\partial t - \alpha\Delta\varphi + \mathbf{v}\cdot\nabla\varphi = G, \quad \partial\theta/\partial t - \beta\Delta\theta = -\gamma G - \mu, \quad x \in \Omega, \quad t \in (0,T). \tag{1}$$

Here, θ is the tissue oxygen concentration, φ the blood oxygen concentration, μ the tissue oxygen metabolic (consumption) rate associated with brain function, G the local exchange at blood-tissue interface, $\gamma = \sigma(1-\sigma)^{-1}$ with σ being the volume fraction of vessels, $\mathbf{v}$ a given velocity field (in the vessel network), α and β are diffusion coefficients for blood and tissue. The tissue oxygen metabolic rate, μ, depends on the tissue oxygen concentration (Michaelis–Menten equation) as follows: $\mu = \mu(\theta) := \mu_0\theta/(\theta+\theta_0)$, where μ_0 means the maximum value of the oxygen metabolic rate, θ_0 the value of the tissue oxygen concentration when $\mu = 0.5\mu_0$. The exchange at the blood-tissue interface is given by the formulas $G = A(\theta - \psi)$, $\psi = f^{-1}(\varphi)$ with $f := \psi + B\psi^r/(\psi^r + C)$. Here, A, B, C, and r are positive constants.

The following initial and boundary, on $\Gamma := \partial\Omega = \Gamma_1 \cup \Gamma_2$, conditions are imposed:

$$\varphi(x,0) = \varphi_0, \quad \theta(x,0) = \theta_0, \tag{2}$$

$$\varphi|_{\Gamma_1} = \varphi_b, \quad \theta|_{\Gamma_1} = \theta_b, \quad \partial_n\varphi = \partial_n\theta|_{\Gamma_2} = 0. \tag{3}$$

Here, the boundary functions φ_b and θ_b are fixed, and the symbol ∂_n denotes the normal derivative.

In the current work, the unique solvability of the problem (1)–(3) is proven. The convergence of an iterative algorithm and stabilization of solutions are proven. The theoretical analysis is illustrated by numerical examples.

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References

[1] S.-W. Su, S. J. Payne, A two phase model of oxygen transport in cerebral tissue, *Proc. 31st Ann. Int. Conf. IEEE Eng. Med. Bio. Soc. (EMBC 2009)*, 4921–4924 (2009).

[2] A. E. Kovtanyuk, A. Yu. Chebotarev, A. A. Dekalchuk, N. D. Botkin, R. Lampe, Analysis of a mathematical model of oxygen transport in brain, *Proc. Int. Conf. Days on Diffraction 2018*, 187–191 (2018).

Examples of test calculations by the acoustic mode parabolic equation with the mode interaction and the elastic bottom

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Mode parabolic equations appeared as a convenient tool for solving 3D problems of ocean acoustics. In the previous work [1] we have derived by the multi-scale expansions method an adiabatic mode parabolic equation taking into account the effects of a weak elasticity on the acoustic field. As a small parameter ϵ a ratio of a typical wavelength to a typical size of horizontal inhomogeneities has been chosen. The shear modulus was of the order of the small parameter. In this work we extend the elastic mode parabolic equation (MPE) method to the case of interacting modes [2].

As a result we have derived system of parabolic wave equations for $l = M, \ldots, N$

$$2\mathrm{i}k_l A_{l,X} + \mathrm{i}k_{l,X}A + A_{l,YY} + \sum_{j=M}^{N} \beta_{lj} A_j \exp(\theta_{lj}) = 0\,, \tag{1}$$

where $\theta_{lj} = \mathrm{i}(\theta_l - \theta_j)/\epsilon$, $P = \sum A_l \phi_l \exp(\mathrm{i}\theta_l/\epsilon)$, $k_l = \theta_{l,X}$, and β_{lj} is given by formula

$$\beta_{lj} = \alpha_{lj} - \frac{k_j^2}{\omega^2}\int_{-H}^{0} \left[2(\gamma_0^2\mu_{1z})_z + \gamma_{0z}^2\mu_1\right]\varphi_l\varphi_j\,dz - \frac{2k_j^2}{\omega^2}\left\{\mu_1\gamma_0^2(\varphi_l\varphi_j)_z + \gamma_0(\gamma_0\mu_1)_z\varphi_l\varphi_j\right\}\Big|_{z=-h_0-0}. \tag{2}$$

Here α_{lj} is the same as in the work [2]. Lame coefficient $\mu_1 = C_s^2/\gamma_0$.

To investigate the efficiency of equation (1), we performed a series of test calculations devoted to comparisons of solutions for the ASA wedge benchmark obtained by the source images method with the ones obtained by the elastic MPE. We observed excellent coincidence of the curves up to the share waves velocity $C_s \approx 300$ m/s and rather good one up to ≈ 400 m/s.

The derived equation can be effectively used in seismoacoustics wave propagation problems, when the influence of the elastic effects is essential.

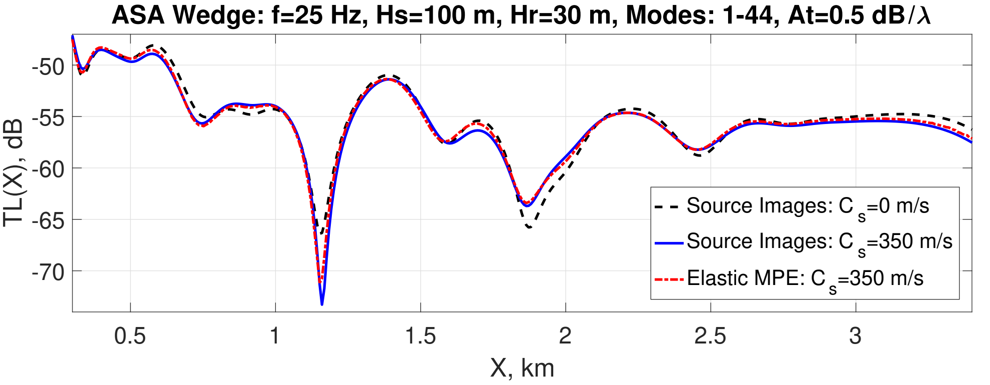

Fig. 1: Comparison of numerical solutions, obtained by equation (1) and by the source images method for $C_s = 350$ m/s. Mean square error is 0.4 dB.

References

[1] M. Yu. Trofimov, S. B. Kozitskiy, A. D. Zakharenko, Weak shear modulus in the acoustic mode parabolic equation, *Proc. Int. Conf. Days on Diffraction 2016*, IEEE, 416–421 (2016).

[2] M. Yu. Trofimov, S. B. Kozitskiy, A. D. Zakharenko, A mode parabolic equation method in the case of the resonant mode interaction, *Wave Motion*, **58**, 42–52 (2015).

Two-dimensional solitary waves with a near-bottom stagnation

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In the case of zero vorticity the first proof of existence of solitary waves in two dimensional channel of finite depth was given by Lavrentiev 1944 and then another more direct proof was suggested by Friedrichs and Hyers 1954. In the case of non-zero vorticity the existence of solitary waves on unidirectional flows is given by M. Groves, E. Wahlen, and independently by V. Hur in 2008. Here I will talk about existence of solitary waves with a near-bottom stagnation.

This is a joint work with N. Kuznetsov (Russia) and E. Lokharu (Sweden).

Transmutation operator method for efficient solution of forward and inverse spectral problems

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The transmutation (transformation) operators are one of the main theoretical tools of the spectral theory [1–3]. In the talk a new approach is presented for solving the classical forward and inverse Sturm–Liouville problems on finite and infinite intervals. It is based on the Gel'fand–Levitan–Marchenko integral equations and recent results on the functional series representations for the transmutation (transformation) operator kernels [4–8]. New representations of solutions to the Sturm–Liouville equation are obtained admitting the following feature important for practical applications. Partial sums of the series admit estimates independent of the real part of the square root of the spectral parameter which makes them especially convenient for approximate solution of spectral problems. Numerical methods based on the proposed approach for solving forward problems allow one to compute large sets of eigendata with a nondeteriorating accuracy. Solution of the inverse problems reduces directly to a system of linear algebraic equations. In the talk some numerical illustrations will be presented.

References

[1] B. M. Levitan, *Inverse Sturm–Liouville problems*, VSP, Zeist (1987).

[2] V. A. Marchenko, *Sturm–Liouville Operators and Applications*, Birkhäuser, Basel (1986).

[3] V. A. Yurko, *Introduction to the Theory of Inverse Spectral Problems*, Moscow, Fizmatlit (2007) (in Russian).

[4] V. V. Kravchenko, L. J. Navarro, S. M. Torba, Representation of solutions to the one-dimensional Schrödinger equation in terms of Neumann series of Bessel functions, *Applied Mathematics and Computation*, **314**, 173–192 (2017).

[5] V. V. Kravchenko, Construction of a transmutation for the one-dimensional Schrödinger operator and a representation for solutions, *Applied Mathematics and Computation*, **328**, 75–81 (2018).

[6] V. V. Kravchenko, On a method for solving the inverse scattering problem on the line, *Mathematical Methods in the Applied Sciences*, **42**, 1321–1327 (2019).

[7] V. V. Kravchenko, On a method for solving the inverse Sturm–Liouville problem, *Journal of Inverse and Ill-Posed Problems*, to appear.

[8] B. B. Delgado, K. V. Khmelnytskaya, V. V. Kravchenko, Fourier–Laguerre series expansion of Levin's transmutation kernel and an efficient method for solving the spectral problem on the half line, submitted.

Momentum ray transforms

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Momentum ray transforms are certain weighted transforms that integrates symmetric tensor fields over lines in Euclidean space with weights that are powers of the integration parameter. This is a generalization of the standard longitudinal ray transforms which have attracted significant attention due to its many tomographic applications. We present an algorithm recovering the whole symmetric tensor field of rank m from its first m+1 momentum ray transforms. We then derive certain isometry relations between the tensor field and the momentum ray transforms, and use it to derive stability estimates. Finally, we also consider inversion of certain attenuated momentum ray transforms on Riemann surfaces.

The talk is based on joint work with Ramesh Manna (TIFR CAM, India), Suman Kumar Sahoo (TIFR CAM, India) and Vladimir Sharafutdinov (Sobolev Institute of Mathematics, Russia), and another work with Rohit Kumar Mishra (University of California, Santa Cruz) and François Monard (University of California, Santa Cruz).

References

[1] S. K. Sahoo, R. Manna, V. P. Krishnan, V. Sharafutdinov, Momentum ray transforms, to appear in *Inverse Problems and Imaging*, (2019).

[2] V. P. Krishnan, R. K. Mishra, F. Monard, On solenoidal-injective and injective ray transforms of tensor fields on surfaces, to appear in *Journal of Inverse and Ill-Posed Problems*, (2019).

Radiation from nonsymmetric sources in a magnetoplasma

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Excitation of electromagnetic waves by sources immersed in a magnetoplasma has been an important research topic for a long time. The theoretical approaches used for the analysis of the wave excitation in a homogeneous magnetoplasma usually employ either dyadic Green's functions or the Fourier transform technique (see, e.g., [1, 2] and references therein). As is known, use of the dyadic Green's functions meets certain difficulties related to the fact that in the case of a magnetoplasma, these functions cannot be expressed in closed form, and are represented by improper integrals in the wave number space [1]. In view of the above, the Fourier transform technique is more frequently used for obtaining the fields excited by electromagnetic sources in a magnetoplasma [2]. However, the procedure of evaluating the corresponding Fourier integrals should be performed anew for each particular source, which is often not convenient, especially when trying to optimize the source parameters for ensuring the required radiation characteristics.

The above-mentioned circumstances make the development of methods employing eigenfunction expansions of the source-excited fields very topical, since such methods can yield more elegant representations of the fields. However, as far as the authors are aware, there exists only one published paper in which such a method was used for the analysis of the radiation from a simple axisymmetric source in a homogeneous magnetoplasma [3]. It is the purpose of the present work to develop an eigenfunction expansion method for representing the fields of spatially localized, arbitrarily distributed given electric and magnetic currents immersed in a cold unbounded homogeneous magnetoplasma. The source-excited fields are expanded in terms of vector azimuthal harmonics, which are then represented by the integrals over the continuous-spectrum eigenfunctions of two kinds corresponding to the ordinary and extraordinary normal modes of a magnetoplasma. To find the expansion coefficients, the obtained eigenfunctions are orthogonalized using the "transposed" formulation of the Lorentz theorem for gyrotropic media [4]. In applying the resultant formulation, the radiation from given nonsymmetric currents has been analyzed, with the emphasis placed on the sources that are capable of exciting waves with nonzero orbital angular momentum in a magnetized plasma medium. In particular, the radiation in the whistler range from a rotating magnetic-field source, which has the form of two circular current loops with the orthogonal symmetry axes and quadrature currents, will be discussed in detail.

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References

[1] L. B. Felsen, N. Marcuvitz, *Radiation and Scattering of Waves*, Prentice-Hall, Englewood Cliffs (1973).

[2] I. N. Toptygin, G. D. Fleishman, Eigenmode generation by a given current in anisotropic and gyrotropic media, *Phys. Usp.*, **51**, 363–374 (2008).

[3] A. V. Kudrin, N. M. Shmeleva, O. E. Ferencz, T. M. Zaboronkova, Excitation of electromagnetic waves by a pulsed loop antenna in a magnetoplasma, *Phys. Plasmas*, **19**, 063301 (2012).

[4] I. G. Kondrat'ev, A. V. Kudrin, T. M. Zaboronkova, *Electrodynamics of Density Ducts in Magnetized Plasmas*, Gordon and Breach, Amsterdam (1999).

Numerical method for electromagnetic non-polarized symmetric hybrid wave propagation problem in a non-homogeneous media with arbitrary nonlinear saturation

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Let $\Sigma = \{(x,y,z) : 0 \leqslant x \leqslant h, (y,z) \in \mathbb{R}^2\}$, where $h > 0$ be a waveguide that is located in $\mathbb{R}^3$ equipped with Cartesian coordinates $Oxyz$. At the boundary $x = 0$ the waveguide Σ has a perfectly conducted wall. The half-space $x > h$ is filled with isotropic homogeneous medium with constant permittivity $\varepsilon = \varepsilon_0\varepsilon_1$, where $\varepsilon_0 > 0$ is the permittivity of vacuum. The waveguide Σ is filled with isotropic non-homogeneous medium with saturated nonlinearity. Permittivity ε in the whole space is equal to $\widetilde{\varepsilon}\varepsilon_0$, where

$$\widetilde{\varepsilon} = \begin{cases} \varepsilon_1, & x > h, \\ \varepsilon_2(x) + f\big(|\mathbf{E}|^2\big), & 0 \leqslant x \leqslant h, \end{cases}$$

$\varepsilon_2(x) \in C^1[0,h]$, f is a monotonically increasing bounded function ($f(0) = 0$), $\mathbf{E}$ is the electric component of an electromagnetic field. We assume that $\varepsilon_2 > \varepsilon_1 \geqslant \varepsilon_0$. There are no sources in the entire space and everywhere $\mu = \mu_0$, where $\mu_0 > 0$ is the permeability of vacuum.

We consider propagation of an electromagnetic wave $(\mathbf{E},\mathbf{H})e^{-i\omega t}$ along the waveguide Σ, where $\mathbf{E}$, $\mathbf{H}$ are complex amplitudes [1], ω is a circular frequency. The complex amplitudes have the form

$$\mathbf{E} = \big(\mathrm{E}_x(x), \mathrm{E}_y(x), \mathrm{E}_z(x)\big)^\top e^{i\gamma z}, \quad \mathbf{H} = \big(\mathrm{H}_x(x), \mathrm{H}_y(x), \mathrm{H}_z(x)\big)^\top e^{i\gamma z}, \tag{1}$$

where $(\,\cdot\,)^\top$ is the transposition operation; γ is an unknown real parameter; E_x, E_y, E_z, H_x, H_y, H_z are unknown functions. The field (1) is called non-polarized symmetric hybrid wave [2, 3].

Complex amplitudes (1) satisfy Maxwell's equations

$$\operatorname{rot}\mathbf{H} = -i\omega\varepsilon\mathbf{E}, \quad \operatorname{rot}\mathbf{E} = i\omega\mu_0\mathbf{H};$$

the continuity condition for the tangential field components at the boundary $x = h$; tangential components of $\mathbf{E}$ vanish on the perfectly conducted wall ($x = 0$); the radiation condition at infinity, where electromagnetic field exponentially decays as $x \to \infty$ in the half-space $x > h$.

We look for that values of γ for which there exists nontrivial field (1) satisfied the above listed conditions. Values of γ that solve the stated problem are called propagation constants. Here in contrast to [4] we consider a monochromatic field with scalar propagation constant.

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References

[1] P. N. Eleonskii, L. G. Oganes'yants, V. P. Silin, Cylindrical nonlinear waveguides, *Soviet Physics JETP*, **35**, 44–47 (1972).

[2] Yu. Smirnov, E. Smolkin, V. Kurseeva, Electromagnetic non-polarized symmetric hybrid wave propagation in a plane waveguide with nonlinear anisotropic permittivity, *Lobachevskii Journal of Mathematics*, **39**, 1075–1089 (2018).

[3] V. Yu. Kurseeva, Electromagnetic non-polarized symmetric hybrid wave propagation in nonlinear media with saturation, *Proceedings of the International Conference DAYS on DIFFRACTION 2018*, 8553516, 198–203 (2018).

[4] Y. G. Smirnov, D. V. Valovik, Coupled electromagnetic TE-TM wave propagation in a layer with Kerr nonlinearity, *Journal of Mathematical Physics*, **53**, 123530 (2012).

Gaussian beams for Dirac equation in electromagnetic field

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Wave functions of fermions in graphene immersed in electromagnetic field satisfy the Dirac equation with two spatial coordinates. We obtained solutions of the stationary Dirac equation in semiclassical approach (using ray method), assuming that the field changes slowly. Special cases for the magnetic field being in the plane of graphene or orthogonal to one are considered. The semiclassical solutions of the Dirac equation in (3+1) were found in [1]. The case for the field being orthogonal to graphene's plane was obtained in [2]. The same subject is discussed in [3].

The semiclassical solutions can be regarded as a starting point for constructing localized solutions — the Gaussian beams. We got solutions localized near the rays by analogy with Gaussian beams for the Helmholtz equation.

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References

[1] V. P. Maslov, M. V. Fedoryuk, *Quasi-Classical Approximation for Quantum Mechanics Equations*, Nauka, Moscow (1976).

[2] V. V. Zalipaev, Complex WKB-approximations in graphene electron-hole waveguides in magnetic Field, *Graphene-Synthesis, Characterization, Properties and Applications*, IntechOpen (2011).

[3] P. Carmier, D. Ullmo, Berry phase in graphene: Semiclassical perspective, *Physical Review B*, **77**(24), 245413 (2008).

Direct and inverse spectral problems for sloshing of a two-layer fluid

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Direct and inverse eigenvalue problems are considered for a pair of harmonic functions with a spectral parameter in one of the boundary conditions and the coupling condition on the interface separating two fluids of different densities. The direct problem describes sloshing frequencies of free oscillations taking place in a two-layer fluid occupying an open container. The upper layer is bounded above by a free surface and below by a layer of fluid of greater density. Both fluids are assumed to be inviscid, incompressible and heavy, whereas their motion is supposed to be of small amplitude. The free surface and the interface between fluids are supposed to be bounded two-dimensional domains.

The aim of the so-called inverse problem is to recover some physical parameters from known spectral data. These parameters are the depth of the interface between the two layers and the density ratio that characterises stratification.

Spectral Huygens filter for pulsed broadband terahertz radiation

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Nowadays, modulating and encoding THz waves are hot topics in the field of THz communication [1]. In this paper, we consider a Huygens filter for a specified frequency lying within a broad spectrum of a pulsed THz radiation which represents simplest two zones plate. As shown in Fig. 1(a), an infinite plane wave (Case A) and a Gaussian beam (Case B) are separately used as the initial source and a mask with a circular aperture is used as a filter to modulate the THz spectrum $\nu_0 = 0.5$ THz is specified as the cutting off frequency to comply with the real experimental conditions. Fig. 1(b–d) show the numerical results (see [2, 3] for the technique description) under a plane wave input. Fig. 1(e) and (f) show the evolution of the normalized field intensity of the Gaussian beam at the intersection between the focal plan $z = z_0$ and the plane $y = 0$ for 0.5 THz. The relation between the incident energy and the radiation energy of the aperture is analyzed. An experimental validation of the technology is performed.

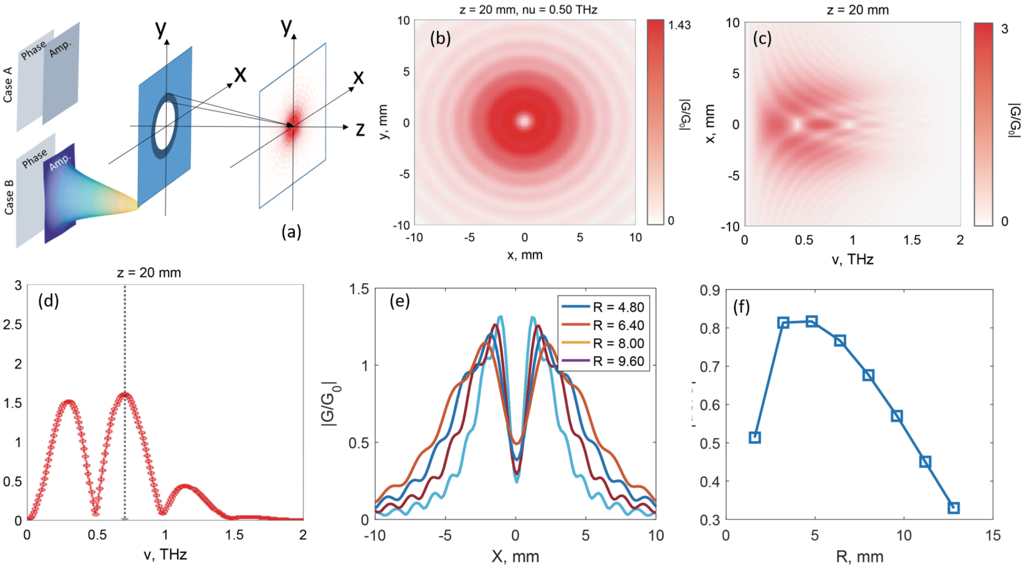

Fig. 1: (a) Working principle diagram. (b) Amplitude distribution of 0.5 THz of plane wave. (c) Spatial-spectral distribution of plane wave. (d) Spectral distribution of plane wave. (e) Amplitude distribution of 0.5 THz at $y = 0$ position of Gaussian beam diffraction. (f) Normalized amplitude gap as a function of the aperture radius.

References

[1] Y. V. Grachev, X. Liu, S. E. Putilin, A. N. Tsypkin, V. G. Bespalov, S. A. Kozlov, X. C. Zhang, Wireless data transmission method using pulsed THz sliced spectral supercontinuum, *IEEE Photonic. Tech. L.*, **30**, 103–106 (2018).

[2] M. Kulya, V. Semenova, A. Gorodetsky, V. G. Bespalov, N. V. Petrov, Spatio-temporal and spatiospectral metrology of terahertz broadband uniformly topologically charged vortex beams, *Appl. Opt.*, **58**, A90–A100 (2019).

[3] N. V. Petrov, M. S. Kulya, A. N. Tsypkin, V. G. Bespalov, A. Gorodetsky, Application of terahertz pulse time-domain holography for phase imaging, *IEEE Trans. Terahertz Sci. Technol.*, **6**, 464–472 (2016).

Modeling the interference pattern of bottom reverberation in the presence of intense internal waves on an ocean shelf

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Wideband coherent reverberation modeling is carried out for a shallow water waveguide in the presence of a solitary internal wave train. Numerical model is based on a normal mode approach. Horizontal refraction of acoustic waves or mode coupling is taken into account depending on the scenario. A low-frequency monopole sound source and a receiving system are deployed in the monostatic geometry. This system is assumed to be able to extract bottom reverberation coming from different directions in a horizontal plane. Environmental data collected during Shallow Water ’06 experiment provides the input for the proposed model. Numerical simulations demonstrate a strong variability of the interference pattern of bottom reverberation coming from the area near the internal wave front. By analyzing the time-angular dependence of the interference pattern fluctuations one can obtain the shape of the front and its speed. Note that the average intensity of bottom reverberation at any range to the scattering patch is not sensitive to the hydrodynamic perturbations. The work was supported by RFBR, 16-32-60194.

The asymptotics of the wave field scattered by an impedance sector

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This report is a natural continuation of our recent study [1] devoted to the scattering of a plane incident wave by a semi-infinite impedance sector. We develop an approach that enables us to compute different components in the far-field asymptotics. The method is based on the Sommerfeld integral representation of the scattered wave field, on the careful study of singularities of the integrand and on the asymptotic evaluation of the integral by means of the saddle point technique. In this way, we describe the waves reflected from the sector, diffracted by its edges or scattered by the vertex as well as the surface waves. Discussion of the far-field in the so-called singular directions (or in the transition zones) is also addressed.

References

[1] M. A. Lyalinov, Acoustic scattering by a semi-infinite angular sector with impedance boundary conditions, *IMA Journal on Applied Mathematics*, **83**, 53–91 (2018).

An overview of the methods for linear and nonlinear diffraction of flexural gravity waves with vertical circular cylinder

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Interaction of gravity waves with different obstacles received much attention in the past both in the context of water waves as well as in the context of flexural gravity waves. The potential flow assumptions are adopted and the problem is formulated in frequency domain for the linear and higher order interactions. Unlike the problem of water waves where the semi-analytical solution for the vertical circular cylinder, is well mastered and agreed within the community, the solution for the problem of flexural gravity waves is proposed in different forms by the different authors.

In the present work an overview of the different methods is given and it is shown that the final semi-analytical solutions remain the same whatever the method which is employed. The basic configuration together with the corresponding Boundary Value Problem (BVP) for the generic velocity potential φ induced by the cylinder, is shown in Fig. 1, where the operator κ is known horizontal operator and the constant α and the functions $v(z,\theta)$ and $Q(r,\theta)$ are specified for each particular problem and here below we define them in the context of the second order wave body interaction theory. In addition, in the case of the flexural gravity waves, the additional boundary conditions at the plate ends should be specified.

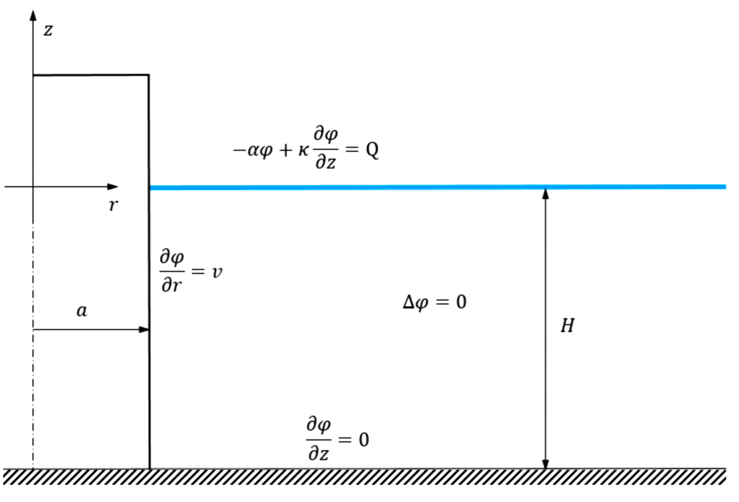

Fig. 1: Basic configuration and definitions.

It can be shown that the velocity potential at the cylinder surface can be expressed in the following form:

$$\varphi_{mn}(a) = \frac{2\mathcal{C}_n}{\mu_n H'_m(\mu_n a)} \left\{ H_m(\mu_n a) \int_{-H}^{0} f_n(z) v_m(a,z) dz \right.$$
$$\left. - \frac{\mu_n \tanh \mu_n H}{\alpha} \left[\frac{1}{a} \int_a^\infty H_m(\mu_n \rho) Q_m(\varrho) d\rho - \mathcal{D} H_m(\mu_n a) \left(\mu_n^2 \frac{\partial^2 \varphi_m}{\partial r \partial z} - \frac{\partial^4 \varphi_m}{\partial r \partial z^3} \right)_{\substack{r=a \\ z=0}} \right] \right\}$$

The quantities $\partial^2\varphi_m/\partial r\partial z$ and $\partial^4\varphi_m/\partial r\partial z^3$ at the connecting line $(a,0)$ are unknown coefficients and their values can be deduced from the appropriate edge conditions.

The above expression can be obtained using the different approaches (direct eigenfunction expansion, boundary integral equations method with appropriate Green function, Weber transforms, ...). The purpose of the present work is to compare the advantages and disadvantages of the different methods.

The condition of non-conventional synchronyzation existence in the chains of weakly coupled autogenerators

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The phenomenon of synchronization in the system of two autogenerators was first observed by Huygens. It was a stationary synchronization, or in modern terminology the synchronization on Nonlinear Normal Mode (NNM) [1]. Synchronization of this type (conventional synchronization) was intensively studied, especially during last decades. We have revealed an alternative, non-conventional synchronization or synchronization on the Limiting Phase trajectory, which corresponds to beatings as attractor [2, 3]. To prove the existence of LPT-synchronization we used the group Lie theory which allowed to find additional symmetry (to the temporal shift) and a corresponding integral of motion. It was shown numerically that one can obscure the LPT synchronization even out of the symmetry conditions. We propose now a new topological criterion of LPT synchronization existence in quasilinear chains of self-sustained oscillators. It is based on the analysis of the stationary states stability in the slow time-scale. Absence of the stationary attractors provides existence of LPT synchronization. The discussed method does not require closeness of the amplitudes of the oscillators, while such limit is supposed in the Kuramoto approximation. Our approach is illustrated on several dynamical problems having important applications.

Aknowledgement. This work was supported by Russian Foundation for Basic Research according to the research project no. 18-03-00716.

References

[1] A. Pikovsky, M. J. Rosenblum, A. Kurths, *Synchronization. A Universal Concept in Nonlinear Sciences*, Cambridge University Press, Cambridge (2001).

[2] L. I. Manevitch, M. A. Kovaleva, V. N. Pilipchuk, Non-conventional synchronization of weakly coupled active oscillators, *EPL – Europhys. Lett.*, **101**, 50002 (2013).

[3] M. A. Kovaleva, V. N. Pilipchuk, L. I. Manevitch, Nonconventional synchronization and energy localization in weakly coupled autogenerators, *Phys. Rev. E*, **94**, 032223 (2016).

Resonant state completeness for a ball with attached wires in magnetic field

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Nanostructures like quantum dots with connected wires are widely discussed (see e.g. [1–3]). Investigation of system's transmission coefficient is interesting for physical applications and from theoretical point of view. The transmission coefficient dependence on energy has a resonant nature and the completeness of resonant states for 3D quantum dot with two 1D wires attached is proved in [4]. In current paper we consider the resonant state completeness problem for similar (to described in [4]) system being under influence of the magnetic field. In this case the initial operator can be defined as follows:

$$H = H^1 \oplus H^B \oplus H^2,$$

where H^j, $(j = 1, 2)$ acts as second derivative (since it is defined on the wire); H^B is the Schrödinger operator defined on the ball taking into account the effects of a magnetic field. The model Hamiltonian is described in the framework of the theory of self-adjoint extensions of symmetric operators via switching on the connection between parts of the system (see e.g. [5, 6]). The resonance state completeness for such system is examined.

References

[1] A. Mikhailova, B. Pavlov, I. Popov, A. Yafyasov, Scattering on a compact domain with few semi-infinite wires attached: resonance case, *Mathematische Nachrichten*, **235**(1), 101 (2002).

[2] V. A. Geyler, V. A. Margulis, M. A. Pyataev, Resonant tunneling through a two-dimensional nanostructure with connecting leads, *J. Exper. Theor. Phys.*, **97**(4), 100 (2003).

[3] E. N. Grishanov, D. A. Eremin, D. A. Ivanov, I. Y. Popov, Dirac operator on the sphere with attached wires, *Chin. Phys. B*, **25**(4), 047303/1–4 (2016).

[4] I. Y. Popov, A. I. Popov, Quantum dot with attached wires: resonant states completeness, *Reports On Mathematical Physics*, **80**, 1–10 (2017).

[5] B. S. Pavlov, Extensions theory and explicitly solvable models, *Uspekhi Mat. Nauk*, **42**(6), 99–131 (1987).

[6] S. Albeverio, P. Kurasov, *Singular Perturbations of Different Operators. Solvable Schrödinger Type Operators*, Cambridge University Press, Cambridge (2000).

On homogenization of periodic parabolic systems

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The talk is devoted to homogenization of periodic differential operators. In $L_2(\mathbb{R}^d;\mathbb{C}^n)$, we consider matrix elliptic second order differential operator B_ε. Coefficients of the operator depend on $\mathbf{x}/\varepsilon$. The principal part of B_ε is given in a factorized form $A_\varepsilon = b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$. It is assumed that the matrix-valued function $g(\mathbf{x})$ is bounded and uniformly positive definite. Next, $b(\mathbf{D})$ is a first order differential operator with constant coefficients. The symbol $b(\boldsymbol{\xi})$ is subject to some condition which ensures strong ellipticity of the operator B_ε. The coefficients of the lower order terms belong to some $L_p(\Omega)$-spaces. It is assumed that $B_\varepsilon > 0$.

For the semigroup $e^{-B_\varepsilon t}$, $t > 0$, we obtain approximation in the L_2-operator norm with the error estimates of the order $O(\varepsilon)$ and $O(\varepsilon^2)$ for fixed time:

$$\|e^{-B_\varepsilon t} - e^{-B^0 t}\|_{L_2(\mathbb{R}^d)\to L_2(\mathbb{R}^d)} \leqslant C_1\varepsilon(t+\varepsilon^2)^{-1/2}e^{-C_2 t}, \quad t \geqslant 0; \tag{1}$$

$$\|e^{-B_\varepsilon t} - e^{-B^0 t} - \varepsilon K(\varepsilon;t)\|_{L_2(\mathbb{R}^d)\to L_2(\mathbb{R}^d)} \leqslant C_3\varepsilon^2 t^{-1}e^{-C_2 t}, \quad t \geqslant \varepsilon^2. \tag{2}$$

Here B^0 is the effective operator with constant coefficients and $K(\varepsilon;t)$ is the corrector.

For the operator A_ε, an analogue of estimate (1) was proven in [1], and an analogue of (2) was obtained in [2]. So, our results (see [3, 4]) are generalisation of results by T. A. Suslina and E. S. Vasilevskaya.

The method of investigation is based on the scaling transformation, the Floquet–Bloch theory, and the analytic perturbation theory.

References

[1] T. A. Suslina, On homogenization of periodic parabolic systems, *Funktsional. Anal. i Prilozhen.*, **38**(4), 86–90 (2004).

[2] E. S. Vasilevskaya, Homogenization with a corrector for a parabolic Cauchy problem with periodic coefficients, *Algebra i Analiz*, **21**(1), 3–60 (2009).

[3] Yu. M. Meshkova, Homogenization of the Cauchy problem for parabolic systems with periodic coefficients, *Algebra i Analiz*, **25**(6), 125–177 (2013).

[4] Yu. M. Meshkova, Homogenization with the corrector for periodic parabolic systems in the $L_2(\mathbb{R}^d)$-norm, *Algebra i Analiz*, to appear.

Homogenization of elliptic and parabolic equations in a bounded domain with the Neumann boundary condition

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Let $\Gamma \subset \mathbb{R}^d$ be a lattice. Let $0 < \varepsilon \leqslant 1$ be a small parameter. Suppose that $\mathcal{O} \subset \mathbb{R}^d$ is a bounded domain of class $C^{1,1}$. In $L_2(\mathcal{O}; \mathbb{C}^n)$, we consider a selfadjoint matrix second order differential operator B_ε with the Neumann boundary condition. Formally, the operator is given by the differential expression

$$B_\varepsilon = b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D}) + \sum_{j=1}^{d} \big(a_j(\mathbf{x}/\varepsilon) D_j + D_j a_j(\mathbf{x}/\varepsilon)^* \big) + Q(\mathbf{x}/\varepsilon).$$

Here $b(\mathbf{D}) = \sum_{l=1}^{d} b_l D_l$ is the $(m \times n)$-matrix first order differential operator. It is assumed that $m \geqslant n$ and that the symbol $b(\boldsymbol{\xi}) = \sum_{l=1}^{d} b_l \xi_l$ has maximal rank for any $0 \neq \boldsymbol{\xi} \in \mathbb{C}^d$. A matrix-valued function $g(\mathbf{x})$ of size $m \times m$ is assumed to be Γ-periodic and such that $g(\mathbf{x}) > 0$; $g, g^{-1} \in L_\infty$. The matrix-valued Γ-periodic functions $a_j(\mathbf{x})$ and $Q(\mathbf{x}) = Q^*(\mathbf{x})$ of size $n \times n$ are, in general, unbounded (they belong to suitable $L_{p,\mathrm{loc}}$-classes). The precise definition of B_ε is given in terms of the quadratic form defined on the Sobolev space $H^1(\mathcal{O}; \mathbb{C}^n)$. It is assumed that B_ε is positive definite.

We study the behavior of the generalized resolvent $(B_\varepsilon - \zeta Q_0^\varepsilon)^{-1}$, where $\zeta \in \mathbb{C} \setminus \mathbb{R}_+$ is a parameter, $Q_0^\varepsilon(\mathbf{x}) = Q_0(\mathbf{x}/\varepsilon)$ and Q_0 is a bounded and positive definite periodic matrix-valued function. We show that this resolvent converges in the L_2-operator norm to the generalized resolvent $(B^0 - \zeta \overline{Q_0})^{-1}$ of the effective operator B^0 with constant effective coefficients. Here $\overline{Q_0}$ is the mean value of Q_0. An error estimate depending on two parameters, ε and ζ, is found. For fixed ζ, this estimate is of order $O(\varepsilon)$ (which is sharp). Also, approximation for the operator $(B_\varepsilon - \zeta Q_0^\varepsilon)^{-1}$ in the norm of operators acting from $L_2(\mathcal{O}; \mathbb{C}^n)$ to $H^1(\mathcal{O}; \mathbb{C}^n)$ is found. For fixed ζ, the corresponding error estimate is of order $O(\varepsilon^{1/2})$.

The method is based on the results for a similar problem in $\mathbb{R}^d$, introduction of the boundary layer correction term, and a careful analysis of this term.

The results are applied to study homogenization of the initial boundary value problem for a parabolic equation

$$Q_0^\varepsilon(\mathbf{x}) \partial_t \mathbf{u}_\varepsilon(\mathbf{x}, t) = -(B_\varepsilon \mathbf{u}_\varepsilon)(\mathbf{x}, t),$$

in a cylinder $\mathcal{O} \times (0, T)$ (where $0 < T \leqslant \infty$) with the initial condition $\mathbf{u}_\varepsilon(\mathbf{x}, 0) = \boldsymbol{\varphi}(\mathbf{x})$ and the natural condition on $\partial\mathcal{O} \times (0, T)$. Approximations for the solution $\mathbf{u}_\varepsilon(\cdot, t)$ in $L_2(\mathcal{O}; \mathbb{C}^n)$ and in $H^1(\mathcal{O}; \mathbb{C}^n)$ are obtained. This application is based on the representation of the operator exponential via the contour integral of the resolvent.

The similar problem with the Dirichlet boundary condition was studied in [1, 2]. The results on the Neumann problem will appear in [3].

References

[1] Yu. M. Meshkova, T. A. Suslina, Homogenization of the Dirichlet problem for elliptic and parabolic systems with periodic coefficients, *Funct. Anal. Appl.*, **51**(3), 230–235 (2017).

[2] Yu. M. Meshkova, T. A. Suslina, Homogenization of the Dirichlet problem for elliptic systems: Two-parametric error estimates, *arXiv*: 1702.00550v4 (2017).

[3] Yu. M. Meshkova, T. A. Suslina, Homogenization of elliptic and parabolic equations in a bounded domain with the Neumann boundary condition, *St. Petersburg Math. J.*, to appear.

Krein strings with nonsmooth mass density. Forward and inverse problems

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We consider forward and inverse dynamic problems for Krein strings [1–3] with stepwise mass density. We solve the dynamic and spectral inverse problems and also consider the effect of the appearing of a finite speed of wave propagation in this dynamical system for a smooth density.

References

[1] A. S. Mikhaylov, V. S. Mikhaylov, Boundary Control method and de Branges spaces. Schrödinger equation, Dirac system and Discrete Schrödinger operator, *Journal of Mathematical Analysis and Applications*, **460**(2), 927–953 (2018).

[2] A. S. Mikhaylov, V. S. Mikhaylov, Inverse dynamic problems for canonical systems and de Branges spaces, *Nanosystems: Physics, Chemistry, Mathematics*, **9**(2), 215–224 (2018).

[3] A. S. Mikhaylov, V. S. Mikhaylov, Dynamic inverse problem for Jacobi matrices, *Inverse Problems and Imaging*, **13**(3), 431–447 (2019).

Asymptotics of the stationary Schrödinger equation in the Weyl chamber

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The Schrödinger equation with a monotonic potential U in a polyhedral angle (Weyl chamber) with the Dirichlet boundary condition is studied. The potential has the form $U(x) = \sum_{j=1}^{n} V(x_j)$, $x \in \mathbb{R}^n$ with $V'(\xi) > 0$. Semiclassical asymptotic formulas for eigenfunctions can be constructed using the Slater determinant composed of Airy functions with arguments depending nonlinearly on x_j. Eigenvalues are found from corresponding quantization conditions. Such asymptotics are valid uniformly for small and large wave numbers.

These results are obtained together with S. Yu. Dobrokhotov and S. B. Shlosman and are supported by the Russian Foundation for Basic Research – CNRS (Grant No. 17-51-150006).

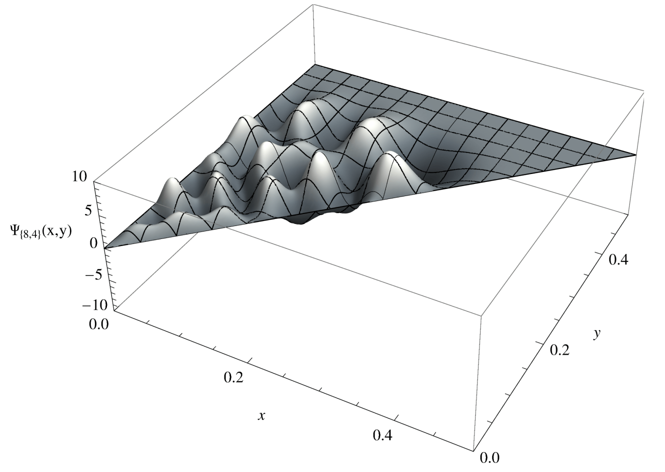

Fig. 1: Wave function asymptotics in 2D Weyl chamber $0 \le x \le y$ with wave numbers 8 and 4.

References

[1] S. Yu. Dobrokhotov, D. S. Minenkov, S. B. Shlosman, Asymptotics of wave functions of the stationary Schrödinger equation in the Weyl chamber, *Theoretical and Mathematical Physics*, **197**(2), 1626–1634 (2018).

Dislocation problem for the Dirac operator

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We consider the dislocation problem for the Dirac operator with periodic potential on the real line. The dislocation is parameterized by the real parameter t. For each parameter value the absolutely continuous spectrum has band structure and there are open gaps between spectral bands. In each open gap there are eigenvalue at some parameters. We show that in each open gap there exist two unique "states" (an eigenvalue or resonance) of the dislocation operator, such that they runs clockwise around the gap. These states are separated from each other by the Dirichlet eigenvalue and they make half as many revolutions as the Dirichlet does in unit time. We find asymptotic of this motion for the cases when states are near the boundary of gaps and when they collide with Dirichlet eigenvalues. This talk is based on joint work with E. Korotyaev.

Non-uniqueness in the problem of forward motion of bodies in a two-layer fluid

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Appearing in the framework of the surface wave theory, the classical linear problem of ship waves describes forward motion of rigid bodies with a constant speed in an unbounded heavy fluid having a free surface (see e.g. [1] and references therein). The model was mostly studied for the homogeneous fluid, but cases of fluid's stratification have also attracted interest in view of phenomena related to internal waves. In this work we consider the case when the fluid consists of two layers of different density and the contours of bodies are totally submerged in one of the layers. The two-dimensional statement of the corresponding boundary value problem is studied. For an arbitrary given geometry of bodies, it is known [2] that the problem is uniquely solvable almost everywhere in the quarter plane constituted by positive values of speed and a parameter characterizing stratification. The technique of proof describes only basic properties of the (possibly empty) set of exceptional values and the question of their existence was open — in this work we give a positive answer. We construct examples of non-uniqueness developing the ideas suggested by [3, 4] for the case of homogeneous fluid. The approach is based on boundary integral equations of the potential theory, introduction of two compact self-adjoint operators and investigation of some functionals on their eigenfunctions. The existence of non-uniqueness examples at isolated values of forward velocity (depending on the geometry and the ratio of densities) is discovered numerically in both regimes of motion — subcritical (with superposition of surface and internal waves at infinity downstream) and supercritical (with surface waves only). For the considered bodies, numerical computation of the wave resistance is performed for intervals of speed and some features of resistance's behavior are shown.

References

[1] N. Kuznetsov, V. Maz'ya, B. Vainberg, *Linear Water Waves: a Mathematical Approach*, Cambridge University Press (2002).

[2] O. V. Motygin, Solvability of boundary integral equations for the problem of a body moving in a two-layer fluid, *Comput. Math. Math. Phys.*, **43**(2), 268–275 (2003).

[3] O. V. Motygin, P. McIver, On uniqueness and solvability in the linear problem of ship waves, *Quarterly J. Mech. Appl. Math.*, **62**, 495–523 (2009).

[4] O. V. Motygin, P. McIver, Non-uniqueness in the plane problem of steady forward motion of bodies, *Proc. of 25th Workshop on Water Waves and Floating Bodies*, Harbin, China, 117–120 (2010).

Limiting bulk acoustic waves for the basal plane of cubic crystals

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The limiting bulk wave is an important notion in the theory of surface and interface acoustic waves, as well as bulk-acoustic-wave reflections in crystals. The limiting wave is the slowest bulk wave along the boundary relative to the selected sagittal plane or, in a more general definition, all bulk waves with energy flux directed along the intersection of the boundary plane and the sagittal plane are included in this notion. The existence regions for surface/interface waves and the ranges of incidence angles with qualitatively different patterns of bulk-wave reflections are determined by limiting bulk waves. The purpose of this presentation is to study the evolution of the threshold velocities, given by the limiting waves, with the sagittal plane is rotated. Both pure-elastic and piezoelectric cases are considered. The conditions for finding limiting waves are derived applying the same procedure as suggested previously for acoustic axes in crystals [1]. Using these conditions, the angular dependencies of the limiting-wave velocities are calculated for the basal plane of cubic crystals. These velocity curves explain a jump-like change of the angular existence region of interface waves found numerically in twisted silicon bicrystals [2].

References

[1] V. G. Mozhaev, F. Bosia, M. Weihnacht, Oblique acoustic axes in trigonal crystals, *J. Comput. Acoust.*, **9**, 1147–1161 (2001).

[2] V. G. Mozhaev, S. P. Tokmakova, M. Weihnacht, Interface acoustic modes of twisted Si(001) wafers, *J. Appl. Phys.*, **83**, 3057–3060 (1998).

The polarization matrix for a junction of elastic rods

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The dimension reduction procedure on asymptotic analysis of thin bodies associates a two- or three-dimensional boundary value problem on a junction of thin domains with a family of ordinary differential equations on the graph (the junction “skeleton”) and the transmission conditions connecting the equations and imposed at the graph vertices corresponding to the junction nodes. The classical Kirchhoff transmission conditions are obtained by analyzing in detail the boundary layer phenomenon near nodes. At the same time, for many goals it is not sufficient to possess only the information obtained on the basis of the primary one-dimensional model which can be derived by a simple limit passage from the original multi-dimensional problem. By constructing of two- or multi-term asymptotic expansions the Kirchhoff transmission conditions are replaced with the Robin–Steklov conditions involving a certain energy characteristic of the junction zone namely, the polarization matrix naturally arising in the problem about a boundary layer near a node.

The polarization matrix M is introduced for the problem of elasticity theory governing deformations of a three-dimensional body composed of half-cylinders at the node Θ bounded by a piecewise smooth contour. Such problems describe the boundary layer phenomenon near nodes of a junction of thin elastic rods.

Properties of the polarization matrix are studied. We prove the symmetry of M and the positive definiteness of M which is proved by "extending" the domain Θ in an appropriate way, i.e., by shifting the origin of the local coordinates along the axes of the half-strips.

Moutard type transformations for generalized analytic functions and for the conductivity equation

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The transformations of the Darboux–Moutard type go back to the publication [1]. Recently, we have constructed and studied Moutard type transformations for generalized analytic functions (that is for the Carleman system or Bers–Vekua system) and further for the conductivity equation. This talk is based, in particular, on works [2–4].

References

[1] T. F. Moutard, *J. Ecole Polytechnique*, **45**, 1–11 (1878).
[2] P. G. Grinevich, R. G. Novikov, *J. Geometric Analysis*, **26**, 2984–2995 (2016).
[3] R. G. Novikov, I. A. Taimanov, *Russian Math. Surveys*, **71**, 970–972 (2016).
[4] P. G. Grinevich, R. G. Novikov, https://hal.archives-ouvertes.fr/hal-01673778 (2019).

Numerical modeling of active microcavities with piercing holes using the Muller boundary integral equations and the Galerkin method

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Following [1, 2], we reduce the original problem for eigenmodes of active microcavities with piercing holes formulated on the plane to a convenient for numerical solution eigenvalue problem for the system of Muller boundary integral equations. All the boundary integral operators in this system of second kind are weakly singular or have smooth kernels and nonlinearly depend on desired parameters [3]. Using the Galerkin method with the trigonometric basis we get a nonlinear algebraic eigenvalue problem.

For circular microcavities with non-concentric circular holes we obtain explicit expressions for the matrix elements. On the basis of presented numerical approach we investigate spectra and thresholds of laser modes of such microcavities. The computational experiments demonstrate that under shift of holes and increase of their radiuses there exist modes that preserve low thresholds. Our numerical results coincide well with exact solutions previously obtained by the method of separation of variables in [4] (see also [5]) for circular microcavities with concentric circular holes.

References

[1] A. O. Spiridonov, et al., *J. Opt. Soc. Am. B*, **34**, 1435–1443 (2017).

[2] A. O. Spiridonov, et al., *Optics Communications*, **439**, 112–117 (2019).

[3] A. O. Spiridonov, et al., *Lobachevskii Journal of Mathematics*, **39**(8), 1148–1157 (2018).

[4] A. S. Zolotukhina, et al., *Opt. Quant. Electron.*, **47**(12), 3883–3891 (2015).

[5] A. S. Zolotukhina, et al., *Appl. Phys. B*, **123**, art. no. 32 (2017).

Optical engineering of vector pulsed beams with parabolic and elliptic cross-sections and their propagation through planar interface

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Beam profile engineering, where a desired optical intensity distribution can be generated by an array of phase shifting (or amplitude changing) elements is a promising approach in laser material processing [1]. For example, a spatial light modulator (SLM) is a dynamic diffractive optical element allowing for experimental implementations of controllable beam profile. Scalar Mathieu beams have elliptical intensity distribution perceivable as “optical knives” in the transverse plane. On the other hand scalar Weber beams have a parabolic cross-section, which enables us to call them “optical showels”. Both families exhibit non-diffracting properties similar to Bessel beams, where a relatively long focal depth retains unchanging intensity distribution, which makes them a promising approach in laser processing. Here, we introduce vector versions of those beams with controllable polarization and investigate numerically their spatial spectra. We use vector Mathieu and Weber beams as a basis to construct controllable on-axis phase and amplitude distributions with polarization control. We investigate here aberrations, which occure due to the propagation through a planar interface between two materials. Furthermore, generate components of vector Mathieu and Weber beams experimentally using diffractive optics elements. We report on our achievements in the control over the beam shape and dimensions along the propagation axis. Lastly, we discuss generation of femtosecond pulsed “knive” and “shovel” type beams using geometric phase elements, made with laser induced nanograting ripples in fused silica and produced by “Workshop of Photonics”.

References

[1] S. Orlov, V. Vosylius, P. Gotovski, A. Grabusovas, J. Baltrukonis, T. Gertus, Vector beams with parabolic and elliptic cross-sections for laser material processing applications, *Journal of Laser Micro/Nanoengineering*, **13**, 280–286 (2018).

Homogenization of multivalued monotone operators under nonstandard growth conditions

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In a bounded Lipschitz domain in $\mathbb{R}^d$, we consider the Dirichlet boundary value problem for an elliptic multivalued maximal monotone operator A_ε for which growth estimates of power type with a variable exponent are satisfied. This variable exponent $p_\varepsilon(x)$ and also the symbol of the operator A_ε oscillate with a small period ε with respect to the space variable x. We prove a homogenization result for this problem, thus, obtaining, in the limit as $\varepsilon \to 0$, a multivalued maximal monotone operator A, much simpler than the original one. Namely, A does not depend on the space variable and satisfies new type growth estimates which are formulated in terms of a convex function $f(\xi)$ instead of the power function $|\xi|^{p_\varepsilon(x)}$ used in growth conditions for A_ε. The function $f(\xi)$ as well as the symbol of A are found via auxiliary problems stated on the unit cell of periodicity. The Dirichlet problem for the operator A_ε is posed in the variable order Sobolev space $W_0^{1,p_\varepsilon(\cdot)}(\Omega)$, while the limit problem with the operator A is posed in the Sobolev space $W_0^f(\Omega)$ with generalized Orlicz integrability defined by the function $f(\xi)$. We extend here the homogenization result [1] obtained previously by V. Zhikov and S. Pastukhova in the case of single-valued strictly monotone operators.

This result is proved jointly with V. Chiadò Piat.

References

[1] V. V. Zhikov, S. E. Pastukhova, Homogenization of monotone operators under conditions of coercitivity and growth of variable order, *Math. Notes*, **90**(1-2), 48–63 (2011).

Approximate controllability of the wave equation with mixed boundary conditions

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We consider initial boundary-value problem for acoustic equation in the time space cylinder $\Omega \times [0, 2T]$ with unknown variable speed of sound, zero initial data, and mixed boundary conditions. We assume that Neumann controls are located at some part $\Sigma \times [0, 2T]$, $\Sigma \subset \partial\Omega$ of the lateral surface of the cylinder. The domain of observation is $\Sigma \times [0, 2T]$ and the pressure at another part $(\Omega \setminus \Sigma) \times [0, 2T]$ is assumed to be zero for any control. We prove the approximate boundary controllability for functions from the subspace $V \subset H^1(\Omega)$ whose traces have vanished on Σ provided that the observation time is $2T$ more than two acoustic radii of the domain Ω. We give an explicit procedure for solving Boundary Control Problem (BCP) for smooth harmonic functions from V (i.e., we are looking for a boundary control f which generates a wave u^f such that $u^f(\cdot, T)$ approximates any prescribed harmonic function from V). Moreover, using the Friedrichs–Poincare inequality, we obtain a conditional estimate for this BCP. Note that, for solving BCP for these harmonic functions, we do not need the knowledge of the speed of sound.

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On some numerical method for solving integral equations of the theory of diffraction

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The purpose of this report is to present an effective numerical method for solving integral equations of potential theory, allowing one to find approximate solutions to a wide range of boundary problems in the theory of diffraction. The effectiveness of the method is achieved using some information regarding the solution, established from formal asymptotic expansions. The method is illustrated by a scalar 2D example, although it is easily generalised to a higher dimensional scalar and vector problems.

Let us assume that we have the Fredholm integral equation of the first kind for current $\psi(x)$ on some boundary S of body:

$$f(x) = \int_S K(x,x')\psi(x')dS, \quad x \in S, \tag{1}$$

where $f(x)$ is the function defined by the incident field, the kernel $K(x,x')$ is expressed through the free-space Green's function. It is known that the kernel and the current can have singularities. The integral equation may also contain large parameters (for example, a large wave number), whereby the solution is a rapidly oscillating function. So, there is a need to use high-dimensional arrays. All the singularities entails a shrinking of the sphere by using usual numerical methods and the need to find individual approaches.

Using some information about a solution obtained from the asymptotical expansion allows some difficulties to be relieved and hence allows to expand the area of applicability of numerical method. The unknown function $\psi(x)$ can be represented as the sum of two terms:

$$\psi(x) = \psi_0(x) + \psi_1(x),$$

where $\psi_0(x)$ is the geometric-optical part (determined by the incident field), $\psi_1(x)$ is the diffracted part. We can separate, using asymptotic expansion, the singularities and the rapidly oscillating parts from the current. Assume that function $\theta(x)$ will contain this singularities. Then function

$$\tilde{\psi}(x) = \psi_1(x)\theta^{-1}(x)$$

will be bounded, smooth and slowly varying.

So, the kernel and the integrated term in equation (1) will be following:

$$\tilde{K}(x,x') = K(x,x')\theta(x'), \qquad \tilde{f}(x) = f(x) - \int_S K(x,x')\psi_0(x')dS.$$

We divide the entire integration domain into n parts and replace function $\tilde{\psi}(x)$ by its linear approximation in each part of the domain:

$$\psi_i(x) = y_i + (y_{i+1} - y_i)(x - x_i)/h,$$

where $y_i = \tilde{\psi}(x_i)$.

So, we obtain the following algebraic system of equations:

$$a_{i,j}y_j = b_i, \quad i,j = 1,\ldots,n,$$

where

$$a_{i,j} = h\int_0^1 \tilde{K}(x_i, x_j + ht)(1-t)dt, \qquad b_i = \tilde{f}(x_i).$$

We show that for a wide class of problems, one need not choose n to be too large in order to obtain a solution with a high accuracy. The effectiveness of the method is illustrated by the example of parabolic antennas.

Generalized quadratic Helmholtz–Gauss beams

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We consider solutions of the parabolic equation

$$U_{xx} + U_{yy} + 2ikU_z = 0\,, \tag{1}$$

which generalise quadratic Bessel–Gauss beams obtained in [1]:

$$U_m(z, r, \varphi) = \frac{C}{\sqrt{q_1(z)q_2(z)}} \exp\left\{iku(z)r^2\right\} J_{|m|/2}(kv(z)r^2) \exp(im\varphi)\,, \tag{2}$$

where r and φ are polar coordinates in the xy–plane, J is a Bessel function, C is a complex constant, $q_j(z) = z - z_j - ib_j$, $j = 1, 2$, and z_j and $b_j > 0$ are real constants,

$$u(z) = \left(q_1^{-1}(z) + q_2^{-1}(z)\right)/4\,, \quad v(z) = \left(q_1^{-1}(z) - q_2^{-1}(z)\right)/4\,.$$

Following the approach developed in [2], we seek generalizations of (2) in a form

$$U(z, r, \varphi) = \frac{C}{\sqrt{q_1(z)q_2(z)}} \exp\left\{iku(z)r^2\right\} H(Z, R, \Phi)\,, \tag{3}$$

where the amplitude function $H = H(Z, R, \Phi)$ depends on the variables Z, R, and Φ defined by $Z = \ln(q_1(z)/q_2(z))$, $R = v(z)r^2$, $\Phi = 2\varphi$, we come to a secondary parabolic equation for H having a form

$$R(\widehat{\Delta}H + k^2H) + 2ikH_Z = 0\,, \tag{4}$$

with periodic conditions $H(Z, R, \Phi + 4\pi) = H(Z, R, \Phi)$, where the operator

$$\widehat{\Delta} = \frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial}{\partial R} + \frac{1}{R^2}\frac{\partial^2}{\partial \Phi^2}$$

is the Laplacian on a two-sheet complex surface. Taking any solution of the Helmholtz equation

$$\widehat{\Delta}H + k^2H = 0\,, \tag{5}$$

we arrive at the amplitude function independent of Z. We call functions (3) which are localized with respect to transverse variables, generalized quadratic Helmholtz–Gauss beams. Functions (2) are special cases of such beams with $H = J_{|m|/2}(kR)\exp(im\Phi/2)$. Another obvious solution of (5) is a plane wave $H = \exp(ikR\cos\Phi)$ which corresponds to the simple astigmatic Gaussian beam. Also we present other nontrivial solutions of (5), giving novel beam-like solutions of (1) in the 3D physical space.

We are indebted to prof. A. P. Kiselev who initialized this research.

References

[1] C. F. R. Caron, R. M. Potvliege, Bessel–modulated Gaussian beams with quadratic radial dependence, *Opt. Commun.*, **164**, 83–93 (1999).

[2] A. P. Kiselev, New structures in paraxial Gaussian beams, *Opt. Spectrosc.*, **96**, 479–481 (2004).

Digital holographic imaging of inhomogeneities at the optical fiber soldering area

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The digital holography has opened an access to modeling and synthesizing the intensity and phase of the electromagnetic field wave [1, 2], which serves as an alternative to analog methods. The article presents a result of reconstruction digital hologram carrying information about refractivity of the sample for qualitative and more accurate analysis of the properties of objects during deformation and also determining the shape of objects, visualizing phase distributions with high spatial resolution.

Numerical reconstruction of a digitally recorded hologram is carried out in accordance with the scalar diffraction theory in the Fresnel approximation for the Rayleigh–Sommerfeld diffraction integral. Restored diffracted field $Q(\xi, \eta)$ in the image plane (ξ, η) at a distance d from the hologram plane can be represented in the paraxial approximation as follows:

$$Q(\xi,\eta) = \frac{1}{i\lambda d} \exp\left(i\frac{2\pi}{\lambda}d\right) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} o_2(x,y) I(x,y) \exp\left[i\frac{\pi}{\lambda d}\left[(\xi-x)^2 + (\eta-y)^2\right]\right] dxdy. \qquad (1)$$

From equation (1) it can be seen that the resulting field is determined by a two-dimensional Fourier transform of the product of intensity distribution in the hologram $I(x, y)$, complex amplitude of the reference wave $o_2(x, y)$ and quadratic phase function of spatial wave propagation:

$$w(x,y) = \exp\left[i\frac{\pi}{\lambda d}(x^2+y^2)\right]. \qquad (2)$$

Note that integral (1) is a convolution integral for the first two factors and function (2). Intensity $I(x, y; d)$ and phase $\varphi(x, y; d)$ of reconstructed images can be obtained from the complex field $Q(\xi, \eta)$ calculated at a distance d using the following relations:

$$\begin{aligned} I(x,y;d) &= |Q(x,y)|^2, \\ \varphi(x,y;d) &= \arctan\left(\frac{\mathrm{Im}\,(Q(x,y))}{\mathrm{Re}\,(Q(x,y))}\right) = \arg\left(Q(x,y)\right). \end{aligned} \qquad (3)$$

Phase values $\varphi(x, y; d)$, which was obtained by this formula correspond to a discontinuous function with a region of variation within the interval $[-\pi, \pi]$. One of the well-known phase unwarping algorithms can be applied to restore a continuous unfolded image of phase values. This method was used to determine defects in the welded joint of two of the same type fibers. Special fiber-optic blanks were made, on one of which an obvious displacement of the welded joint was created, and the other was created at normal conditions. Defects of the welded joint is also observed with digital hologram for case when displacement of optical fibers is not visible with optical microscope.

References

[1] U. Schnars, *Digital Holography*, Springer-Verlag, Berlin, Heidelberg, New York (2005).

[2] K. K. Myung, Principles and techniques of digital holographic microscopy, *SPIE Reviews*, **1**(1), 018005–7 (2010).

Nonlinear waves of the inner medium due to dynamic loading of bi-continuum

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The phenomenon of materials destruction is one of the most studied one, as it is of fundamental importance for any equipment and structures. However, there are many mechanisms that can be considered as the main cause of destruction. These mechanisms have different physical nature: fluctuations and statistics as in Zhurkov's model, cf. [1], as well as the surface effects in the Griffith model, cf. [2].

At the same time, there are a number of facts that show that the external environment in which the destruction occurs is of fundamental importance for the destruction modeling. Usually, this external environment is not always taken into account when calculating the strength. However, in vacuum, as a rule, the strength of material is higher than that in air, and the placement of the tested samples in different liquid media can change the strength, both upwards and downwards.

One of the first who detected this phenomenon was A. F. Ioffe while carrying out experiments with saline columns. But so far, this effect has no general unambiguous explanation.

A number of studies show that the fracture is accompanied by that the surface of samples is covered with micropores and microcracks. The sharp edges of these cracks cause the dissociation of water and other gases with the formation of highly active radicals [3]. Hydrogen is the most dangerous among them [3].

The surface influence can be described by means of boundary conditions however as shown by experimental studies, the destruction begins with small defects and pores on the surface. That is, we have singular boundary conditions.

Secondly, layer-by-layer experimental scanning shows that the hydrogen is unevenly distributed throughout the body of the sample. There is a thin surface layer in which its concentration is about 100 times higher than in the inner region of the metal.

The description of the interaction of hydrogen with the metal matrix is given in the framework of the bi-continuum model in which the inner medium models the diffusion mobile hydrogen.

Diffusion process is slow. The destruction process requires describing the behavior of hydrogen under the action of external dynamic loads. For this, a wave approach was used to analyze the equations of a mathematical model. The report provides an assessment of the possible influence of nonlinear effects on the propagation of a solitary wave of the concentration of diffusion mobile hydrogen.

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References

[1] S. N. Zhurkov, The problem of solid strength, *Vesnik AN SSSR*, **11**, 78–82 (1957) (in Russian).

[2] C. B. Griffith, The phenomena of rupture and flow in solids, *Philosophical Transactions of the Royal Society of London A*, **221**(582-593), 163–198 (1921).

[3] Y. A. Khrustalev, Hydrogen saturation became as a result of its destruction, *Vestnik Tambovskogo Universiteta*, **5**(2-3), 234–236 (2000) (in Russian).

Asymmetric modified Fabry–Perot cavities for increase the sensitivity of gravitational wave detectors

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The high sensitivity of modern laser detectors of gravitational waves requires a high value of circulating power (about 1 MW) in the optical cavities of the detector to increase the probability of observation of gravitational waves from the merger of double black holes or neutron star systems. But the high power of energy in these experiments, as well as the presence of a large number of high-Q optical modes, can lead to an undesirable effect of parametric oscillatory instability, which limits the light power circulating in the shoulders of the interferometer (standard Fabry–Perot cavities with spherical mirrors) [1, 2]. In addition beam splitter in optical system of gravitational detectors has non-sufficient size, it makes to use non-symmetric light beams in arms (light spot on input mirror is slightly smaller than on end mirror).

To solve these two problems, it is proposed, firstly, to use an optical cavity with non-spherical mirrors [3, 4], which will lead to a sufficiently effective suppression of high-order optical modes and, in turn, increase the threshold of parametric instability. Secondly, it is proposed to use an asymmetric cavity with different mirrors, which will reduce the size of the laser spot at the output of the cavity with a slight increase in the diffraction losses of the main optical mode.

Numerical calculations of asymmetric Fabry–Perot cavities with non-spherical mirrors with a rarer spectrum and higher losses of high-order optical modes were carried out, which can be obtained by optimizing the surface profile of mirrors, for example, described by the function:

$$A(r) = x_0 \cdot \exp\bigl(-\eta\bigl(1 + \alpha \cdot \eta + \beta \cdot \eta^2\bigr)\bigr), \text{ where } \eta = \frac{r^2}{2 \cdot R_c \cdot x_0}, \tag{1}$$

(at $\alpha = \beta = 0$ and $x_0 \to \infty$, it passes into the profile of the spherical mirror $\frac{r^2}{2 \cdot R_c}$ with the radius of curvature R_c) and the subsequent shift of the cavity eigenmodes along the optical axis by some optimal distance, which leads not only to a decrease in the light spot on one mirror, but also to a further increase in the diffraction losses of high-order optical modes inside such a cavity. Numerical simulations are carried out for the standard characteristics of Fabry–Perot cavities in the Advanced LIGO detectors: length $L = 4000$ m, the radius of curvature of the mirrors $R_c = 2076$ m, the wavelength is $\lambda = 1.064$ µm [5].

References

[1] V. B. Braginsky, et al., Parametric oscillatory instability in Fabry–Perot (FP) interferometer, *Physics Letters A*, **287**(5-6), 331–338 (2001).

[2] V. B. Braginsky, et al., Analysis of parametric oscillatory instability in power recycled LIGO interferometer, *Physics Letters A*, **305**(3-4), 111–124 (2002).

[3] F. Ferdous, et al., Microcavity morphology optimization, *Physical Review A*, **90**, 033826 (2014).

[4] A. B. Matsko, et al., Mitigating parametric instability in optical gravitational wave detectors, *Physical Review D*, **93**(8), 083010 (2016).

[5] M. V. Poplavskiy, et al., On fundamental diffraction limitation of finesse of a Fabry–Perot cavity, *Journal of Optics*, **20**(7), 075609 (2018).

On theory of surface wave propagation on smooth strictly convex surfaces embedded in $\mathbb{R}^3$

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Theory of surface waves such as whispering gallery and the creeping waves in the short-wave approximation is well developed in 2D cases when they propagate along the smooth strictly convex curves in plane, see e.g. [1, 2] and bibliography therein. In the case of surfaces embedded in 3D those waves propagate along geodesic lines where they have torsion and form caustics of different structures, that gives rises to problems in the theory of surface wave propagation.

We propose one approach to surface wave propagation theory which allows overcoming above mention problems. The approach is based on extension of the locality principle well known in the short-wave propagation and diffraction theory and it reassembles Gaussian beams summation method [3].

The proposed theory includes the following basic steps:

1. Construction of the field of geodesic lines associated with the generated wave field under consideration: whispering waves on concave side of the smooth strictly convex surface (Σ) or creeping waves on convex side of Σ.
2. For each geodesic line we construct a short-wave asymptotic solution of Helmholtz equation localized in small neighborhood of this geodesic line in the tangent bundle of Σ respectively. It is important that this asymptotic solutions have no singularities at focal points of the geodesic lines.
3. The total wave field on the Σ we present as the superposition (or integral) of those local solutions.

Thus to calculate the contribution of the surface wave at any observation point we need to cover same vicinity of the point by a fan of geodesic lines. Then to sum up at this point contributions of all asymptotic solutions related to the fan of geodesic lines, compare with Gaussian beam summation method [3].

References

[1] V. M. Babich, N. Ya. Kirpichnikova, *The Boundary-Layer Method in Diffraction Problems*, Springer-Verlag (1979).

[2] V. M. Babich, V. S. Buldyrev, *Asymptotic Methods in Short-Wavelength Diffraction Theory*, Alpha Science International (2007).

[3] M. M. Popov, A new method of computation of wave fields using Gaussian beams, *Wave Motion*, **4**, 85–97 (1982).

Microlocal analysis of a Compton tomography problem

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Here we present a novel microlocal analysis of a new toric section transform which describes a two dimensional image reconstruction problem in Compton scattering tomography and airport baggage screening. By an analysis of two separate limited data problems for the circle transform and using microlocal analysis, we show that the canonical relation of the toric section transform is 2–1. This implies that there are image artefacts in the reconstruction. We provide explicit expressions for the expected artefacts and demonstrate these by simulated reconstructions.

This is joint work with James Webber (ECE, Tufts University).

Fully symmetric guided electromagnetic waves in a shielded slab filled with anisotropic dielectric

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Let us consider the slab $\Sigma := \{(x, y, z) : 0 \leqslant x \leqslant h, (y, z) \in \mathbb{R}^2\}$ located in the Cartesian coordinates $Oxyz$, which has infinitely conducting walls σ_0 and σ_h at $x = 0$ and $x = h$, respectively.

We study propagation of a monochromatic electromagnetic wave $(\mathbf{E}, \mathbf{H})e^{-i\omega t}$ inside Σ, where ω is a circular frequency. Vector-fields $\mathbf{E}$, $\mathbf{H}$ have the form

$$\mathbf{E} = \big(e_x(x), e_y(x), e_z(x)\big)^\top \cdot e^{i(\gamma_y y+\gamma_z z)}, \quad \mathbf{H} = \big(h_x(x), h_y(x), h_z(x)\big)^\top \cdot e^{i(\gamma_y y+\gamma_z z)}, \tag{1}$$

where γ_y, γ_z are unknown real parameters, $(\,\cdot\,)^\top$ is a transposition operation.

Guided waves of the form (1) in the slab Σ we call the *fully symmetric guided waves* (FSGWs). This type of waves includes transverse-electric and transverse-magnetic guided waves and can be considered as their generalisation. In addition, idea of FSGWs in some sense is similar to the concept of hybrid guided waves in a circle cylindrical waveguide [1].

The slab Σ is filled with non-magnetic anisotropic homogeneous medium characterised by the following permittivity tensor

$$\boldsymbol{\epsilon} = \epsilon_0 \begin{pmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{pmatrix}, \tag{2}$$

where the diagonal entries are real and, in general, pairwise different [2, 3]. We assume that $\mu = \mu_0 > 0$ is the permeability of free space. Without loss of generality $\epsilon_x \geqslant \max\{\epsilon_y, \epsilon_z\}$. Indeed, making appropriate coordinate rotation, any of the diagonal entries in (2) can be put into the ϵ_x-position. In the general case, we impose the constraints $\epsilon_x > \max\{\epsilon_y, \epsilon_z\}$, $\epsilon_y \neq \epsilon_z$ and $\gamma_y \neq 0$, $\gamma_z \neq 0$.

In the linear case, parameters ϵ_x, ϵ_y, and ϵ_x as well as γ_y, γ_z can be complex numbers.

Tensor (2) is chosen as, one the one hand, it is the simplest case in which the FSGWs arise, on the other hand, it models many real materials: in [2] there are presented a number of real cases in which the diagonal tensor (2) can be used.

The main problem, which we call the **problem** P, is to determine pairs (γ_y, γ_z) for which there exists a nontrivial field $(\mathbf{E}, \mathbf{H})e^{-i\omega t}$ such that the field (1) satisfies Maxwell's equations

$$\operatorname{rot}\mathbf{E} = i\omega\mu\mathbf{H}, \quad \operatorname{rot}\mathbf{H} = -i\omega\boldsymbol{\epsilon}\mathbf{E}, \tag{3}$$

where tangential components of the electric field vanish at the walls σ_0, σ_h.

Any pair (γ_y, γ_z) that solves the problem P is called the *coupled propagation constants* and denoted by $(\widetilde{\gamma}_y, \widetilde{\gamma}_z)$.

The dispersion equation with respect to γ_y and γ_z is derived and studied. Dispersion curves and eigenfunctions are plotted.

The work was supported by the Russian Science Foundation under grant 18-71-10015.

References

[1] D. V. Raschetova, D. V. Valovik, Fully symmetric guided electromagnetic waves in a shielded plane dielectric slab, *Lobachevskii Journal of Mathematics*, submitted.

[2] L. D. Landau, E. M. Lifshitz, L. P. Pitaevskii, *Course of Theoretical Physics. Electrodynamics of Continuous Media*, Butterworth-Heinemann, Oxford (1993).

[3] E. A. Pamyatnykh, E. A. Turov, *Fundamentals of Electrodynamics of Material Media in Variable and Inhomogeneous Fields*, Nauka, Moscow (2000) (in Russian).

Numerical simulation of sound scattering on a partly rough pressure-release surface via boundary element method

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The problem of sound scattering on the surface of water is an old problem that is successfully solved using a statistical approach and approximate methods. However, finding a solution to the wave equation with an arbitrary certain realization of the boundary still remains a difficult task that cannot be solved by analytical methods. This task becomes relevant in connection with the improvement of experimental and computational techniques for finding the shape of the surface of the water.

We discuss how the scattering of sound at the infinite boundary can be calculated by the model with a finite number of boundary elements. Geometry of our problem contains an infinite plane boundary with a finite area of deformation. We use Helmholtz integral equation on closed surface. Part of the surface is an artificial boundary so the boundary condition there is obtained from decomposition of pressure field in the exterior domain. The boundary condition contains integration over finite surface only, thus the problem can be solved via boundary element method.

The computational scheme is checked on tasks for which the answer is known, its operation is demonstrated in cases with shading and multiple reflection phenomena.

References

[1] F. G. Bass, I. M. Fuks, *Wave Scattering from Statistically Rough Surfaces*, Pergamon (1979).

[2] F. Ochmann, H. Brick, Acoustical radiation and scattering above an impedance plane, *Computational Acoustics of Noise Propagation in Fluids – Finite and Boundary Element Methods*, Springer, Berlin, 459–494 (2008).

[3] B. M. Salin, M. B. Salin, Combined method for measuring 3d wave spectra. I. Algorithms to transform the optical-brightness field into the wave-height distribution, *Radiophysics and Quantum Electronics*, **58**(2), 123–133 (2015).

[4] G. A. Baidakov, A. S. Dosaev,D. D. Razumov, M. B. Salin, Estimation of broadening of the spectra of short surface waves in the presence of long waves, *Radiophysics and Quantum Electronics*, **61**(5), 332–341 (2018).

[5] E. L. Shenderov, *Radiation and Scattering of Sound*, Sudostroenie, Leningrad (1989).

Spectrum of Schrödinger operators with guided potentials on periodic graphs

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We consider discrete Schrödinger operators with periodic potentials on periodic graphs perturbed by guided positive potentials, which are periodic in some directions and finitely supported in other ones. The spectrum of the unperturbed operator is a union of a finite number of non-degenerate bands and eigenvalues of infinite multiplicity. We show that the spectrum of the perturbed operator consists of the "unperturbed" one and the additional guided spectrum, which is a union of a finite number of bands. We estimate the number and the positions of the guided bands in gaps of the unperturbed operator in terms of eigenvalues of Schrödinger operators on some finite graphs. We also determine sufficient conditions for the guided potentials under which the guided bands do not appear in gaps of the unperturbed problem.

On homogenization for a locally periodic elliptic Dirichlet problem

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Let Ω be a domain in $\mathbb{R}^d$ with sufficiently smooth boundary, and let $\mathcal{A}^\varepsilon$ be a matrix strongly elliptic operator on Ω given by the expression $\mathcal{A}^\varepsilon = -\operatorname{div} A(x, x/\varepsilon)\nabla$ with Dirichlet boundary condition. The function A is Lipschitz in the first argument and periodic in the second one, and therefore the coefficients of $\mathcal{A}^\varepsilon$ rapidly oscillate if ε is small. It is well known that the resolvent $(\mathcal{A}^\varepsilon - \mu)^{-1}$ converges, in some sense, as $\varepsilon \to 0$. We present the first terms of approximations in the uniform operator topology on $L_2(\Omega)^n$ (which is the strongest type of operator topology) for $(\mathcal{A}^\varepsilon - \mu)^{-1}$ and $\nabla(\mathcal{A}^\varepsilon - \mu)^{-1}$. Particular attention will be paid to the rates of approximation.

Asymptotic solution of the explicit difference scheme for the wave equation with localized initial data

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We consider the Cauchy problem for the wave equation with localized data

$$u_{tt} = c^2(x)u_{xx}, \quad u|_{t=0} = V(x/\mu), \quad u_t|_{t=0} = 0, \quad x \in \mathbb{R},$$

where $V(y)$ is a smooth fast-decaying function with derivations and the parameter $\mu \ll 1$ is the parameter of the localization, function $c(x)$ is smooth and bounded. The asymptotic solution of this problem can be constructed with the help of the Maslov's canonical operator.

On the other hands, the solution of this problem can be obtained via the numerical methods due to solving the difference scheme. The difference scheme can be written as a pseudodifferential operator due to shifting operator $Tu = e^{h\frac{\partial}{\partial x}}u$, where $h \ll 1$ is a step of discretisation in the scheme.

Thus the difference scheme can be also studied with the help of the Maslov's theory and the asymptotic solution of the difference scheme can be studied. In such approach we have two small parameters: h and μ and the asymptotic solution depends on the ratio h/μ.

In papers [1, 2] the asymptotic solution for such equations were obtained for wide class of the initial conditions. The asymptotic solution is presented in the combined form: asymptotic with respect to small parameters and with respect to the smoothness [3, 4].

Recently in [5] another approach to the asymptotic solution for wave equation was presented. It is based on the vertical Lagrangian manifolds and it was shown that the the leading term of the wave can be described in terms of the caustic points of the manifold, which are is the leading wave front.

In the present speech we investigate these two methods for the building the main part of the asymptotic solution of the difference scheme in the vicinity of the leading wave front. It is shown that on the lattice of the difference scheme these two methods provide the same answers.

We compare the numerical solution and asymptotic solution of the difference scheme. Even the explicit scheme is unstable one can provide interesting results of such comparison depending the ratio between the step of the difference scheme h and parameter of localization μ.

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References

[1] V. G. Danilov, V. P. Maslov, Pontryagin's duality principle for calculation of an effect of Cherenkov's type in crystals and difference schemes. I, *Proceedings of the Steklov Institute of Mathematics*, **166**, 143–177 (1986).

[2] V. G. Danilov, V. P. Maslov, Pontryagin's duality principle for calculation of an effect of Cherenkov's type in crystals and difference schemes. II, *Proceedings of the Steklov Institute of Mathematics*, **167**, 103–116 (1986).

[3] V. G. Danilov, P. N. Zhevandrov, On Maslov's method for constructing combined asymptotics for h-pseudodifferential equations, *Mathematics of the USSR – Izvestiya*, **34**(2), 425–439 (1990).

[4] V. P. Maslov, *Operator Methods (Operatornye metody)*, Nauka, Moscow (1973), (in Russian).

[5] S. Yu. Dobrokhotov, V. E. Nazakinskii, Punctured Lagrangian manifolds and asymptotic solutions of the linear water wave equations with localized initial conditions, *Mathematical Notes*, **101**(5-6), 1053–1060 (2017).

On the simulation of pulse acoustic signals in the shallow water with slope of the bottom

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We pose the problem of the propagation of the pulse acoustic signals in the waveguide in the shallow water with slope of the bottom. We assume that either the sound speed is uniform in the area or the presence of the thermocline.

We present asymptotic solution to this problem based on the ray theory and Maslov's canonical operator. We analyze the 3D geometry of the rays and their projection onto the horizontal plane in both cases.

It is known that in the case of the constant sound speed the envelope of the rays is hyperbola. In the case of the of the thermocline it is shown that such hyperbolas are stretched.

Asymptotic analysis of harmonic waves propagation in a viscoelastic layer in the cases of fractional Voigt and Maxwell models

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The problem of propagation of harmonic waves in an infinite layer is thoroughly investigated on the basis of elastodynamics theory, which do not take into account the dissipation of the energy because of the internal friction. To describe the latter, a model of a viscoelastic material can be used. Since the different types of internal friction depending on the properties of the material, temperature and other factors are known, there are different types of viscoelastic models. In paper [1] the asymptotics of the roots of dispersion equations for viscoelastic layer described by Rabotnov's model [2] were obtained. Nowadays, various dynamic problems for viscoelastic bodies are considered with the use of fractional calculus. In paper [3] it is shown that Rabotnov's model is equivalent to the rheological model which is referred to as Koeller's model. There are the other types of fractional viscoelastic models [4], for example fractional Voigt model and fractional Maxwell model.

In this work, the method developed in [1] is applied to 3D fractional Voigt and Maxwell models, which are constructed as the generalization of one dimensional models [4]

$$\sigma(t) = m\,\varepsilon(t) + b\frac{d^{\gamma}\varepsilon(t)}{dt^{\gamma}}$$

and

$$\sigma(t) + a\frac{d^\gamma \sigma(t)}{dt^\gamma} = b\frac{d^\gamma \varepsilon(t)}{dt^\gamma},$$

respectively. Here σ is the stress, ε is the strain, m, b, a, γ are parameters of the material.

Asymptotics of the dispersion equation roots for small and large frequencies are obtained. Comparative analysis of the asymptotic behavior of the dispersion equation roots for these models with Rabotnov's model is presented.

The results were obtained within the framework of the state assignments of the Ministry of Education and Science of Russia No. 9.8570.2017 / 8.9.

References

[1] N. V. Sergeeva, M. V. Wilde, Asymptotic study of dispersion relations for viscoelastic waveguides of canonical shapes in the case of Rabotnov's rheological model, *Proceedings of the International Conference Days on Diffraction 2017*, 294–299 (2017).

[2] Yu. N. Rabotnov, *Elements of Hereditary Solid Mechanics*, Nauka, Moscow (1977) (Engl. transl. by Mir Publishers, Moscow, 1980).

[3] Yu. A. Rossikhin, M. V. Shitikova, Comparative analysis of viscoelastic models involving fractional derivatives of different orders, *Fractional Calculus and Applied Analysis*, **10**(2), 111–121 (2007).

[4] F. Mainardi, *Fractional Calculus and Waves in Linear Viscoelastisity. An Introduction to Mathematical Models*, Imperial College Press, London (2010).

Matrix Klein–Gordon equations for waveguides

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A useful tool for studying wave processes in complicated waveguides is the "waveguide finite element method" (WFEM) [1]. An arbitrary waveguide is modeled within this method by the equation

$$\left[\mathrm{D}_2\frac{\partial^2}{\partial x^2} + \mathrm{D}_1\frac{\partial}{\partial x} + \mathrm{D}_0 - \mathrm{M}\frac{\partial^2}{\partial t^2}\right]\mathrm{U}(t,x) = \mathrm{F}(t,x), \tag{1}$$

where t is the time, x is the longitudinal coordinate directed along the waveguide, $\mathrm{U}(t,x)$ is the vector of unknowns of dimension $N \times 1$, D_j and M are coefficient matrices, $\mathrm{F}(t,x)$ is the vector excitation function. This equation can be called a matrix Klein–Gordon equation (MKGE), taking into account an analogy with a well-known scalar Klein–Gordon equation

$$\left[d_2\frac{\partial^2}{\partial x^2} + d_0 - m\frac{\partial^2}{\partial t^2}\right]u(t,x) = f(t,x). \tag{2}$$

An MKGE can mimic behavior of various physical system. In the talk we demonstrate how such equations can be built for a liquid planar layer or an elastic plate. Respectively, dispersion diagrams of MKGEs can be rather complicated.

To simplify the analysis, an MKGE can be restricted onto some linear subspace of smaller dimension. We demonstrate examples of such a restriction stressing the physical meaning of it.

After simplification, the system becomes locally represented by an MKGE of a small dimension N. A very important case is $N = 2$. An MKGE of dimension 2 can be used for modeling interaction between subsystems of a larger system. In particular, avoiding–crossings of a dispersion diagram can be described by such an MKGE. We propose a classification of MKGEs having dimension 2.

References

[1] H. Liu, S. Finnveden, Waveguide finite element method, In: *"MID-FREQUENCY" — CAE Methodologies for Mid-Frequency Analysis in Vibration and Acoustics*, KU Leuven, Eds.: W. Desmet, B. Pluymers, O. Atak, 263–308 (2012).

Diffraction of TM polarized EM waves by nonlinear inhomogeneous Goubau line

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The electromagnetic wave diffraction by homogeneous (see [1] where early and classical results for circular geometries are summarized) or inhomogeneous (see e.g. [2]) cylindrical dielectric bodies filled with linear medium has been a subject of intense studies since the late 1940s. However, the case of nonlinear inhomogeneous filling still constitutes to a big extent an open problem, both in view of mathematical justification and creation of efficient numerical techniques. A progress here is associated with recently developed techniques [3–5] for the analysis of nonlinear boundary value problems for the Maxwell and Helmholtz equations.

In this work, we consider the diffraction of TM waves by an open waveguide, a Goubau line (perfectly conducting cylinder covered by a dielectric layer), with a nonlinear dielectric cover. Numerical experiments are carried out for the nonlinearity with saturation and Kerr nonlinearity. The physical problem is reduced to solving a nonlinear boundary value problem for a system of ordinary differential equations. Numerical results are obtained using a modification of the shooting method which makes it possible to determine and plot the amplitude of the reflected field with respect to the amplitude of the incident field. Comparison between the nonlinear problem and the corresponding linear problem is performed. These simulation results describe the essential relationships between linear and nonlinear problems. Namely, the nonlinear reflected field can be predicted from that obtained from the linear problem using the perturbation theory method (for small value of nonlinearity coefficient).

References

[1] R. W. P. King, T. T. Wu, *The Scattering and Diffraction of Waves*, Harvard University Press (1959).

[2] Y. Miyazaki, Scattering and diffraction of electromagnetic waves by inhomogeneous dielectric cylinder, *Radio Science*, **16**, 1009–1014 (1981).

[3] E. Smolkin, M. Snegur, Yu. Shestopalov, Diffraction of TM polarized EM waves by a nonlinear inhomogeneous dielectric cylinder, *Progress in Electromagnetics Research Symposium*, 8598028, 66–70 (2018).

[4] E. Smolkin, M. Snegur, Yu. Shestopalov, TE-polarized wave diffraction by a cylinder covered with nonlinear dielectric layer, *Proceedings of the 2018 IEEE 7th Asia-Pacific Conference on Antennas and Propagation*, APCAP, 8538309, 321–322 (2018).

[5] Y. Smirnov, E. Smolkin, Yu. Shestopalov, Diffraction of a TE-polarized wave by a nonlinear Goubau line, *Radio Science*, **54**(1), 151–157 (2019).

Forced oscillation of a system with time-varying parameters possessing a single trapped mode: a resonant case

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We consider a forced oscillations of an infinite-length mechanical system, with time-varying parameters, possessing a single trapped mode characterized by frequency $\Omega_0(\epsilon t)$. The system is a string, lying on the Winkler foundation, and equipped with a discrete linear mass-spring oscillator of time-varying stiffness. The discrete oscillator is subjected to harmonic external force with constant frequency $\hat{\Omega}$. In the case of the passage through the resonance, we obtain the principal term of the asymptotic expansion describing the motion of the discrete inclusion. To do this we use the combination of two asymptotic approaches. The first one was suggested in [1] and used in our recent study [2] to describe the free localized oscillation in the system under consideration. The second one was used in [3, 4] to describe the passage through the resonance in a single degree of freedom system. The obtained result was verified by independent numerical calculations based on solution of the corresponding PDE by means of the method of finite differences. The comparison demonstrates a good mutual agreement.

References

[1] S. N. Gavrilov, D. A. Indeitsev, *PMM Journal of Applied Mathematics and Mechanics*, **66**, 825–833 (2002).
[2] S. N. Gavrilov, E. V. Shishkina, Yu. A. Mochalova, In: H. Altenbach, et al. (Eds.), *Dynamical Processes in Generalized Continua and Structures, Advanced Structured Materials 103*, Springer (2019).
[3] J. Kevorkian, *SIAM Journal on Applied Mathematics*, **20**, 364–373 (1971).
[4] J. Kevorkian, *SIAM Journal on Applied Mathematics*, **26**, 686–686 (1974).

Homogenization of high order elliptic operators with correction terms

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Our talk is devoted to the homogenization problem for high order periodic elliptic operators with rapidly oscillating coefficients. We consider matrix strongly elliptic operator $\mathcal{A}_\varepsilon$ in $L_2(\mathbf{R}^d)$:

$$\mathcal{A}_\varepsilon = b^*(D)g(x/\varepsilon)b(D), \quad \varepsilon > 0.$$

Here $g(x)$ is periodic bounded positive-defined matrix-valued function; $b(D)$ is homogeneous matrix differential operator of order $p \geq 2$. Its symbol $b(\xi)$ has maximal rank.

The main approximation term of the operator resolvent $\mathcal{A}_\varepsilon$ was established in [1]. Precisely, the following estimate holds

$$\left\|(\mathcal{A}_\varepsilon + I)^{-1} - (\mathcal{A}_0 + I)^{-1}\right\|_{L_2(\mathbf{R}^d)\to L_2(\mathbf{R}^d)} = O(\varepsilon), \quad \varepsilon \to 0,$$

for some effective operator $\mathcal{A}_0$ with constant coefficients. We find a more accurate approximation of the resolvent $(\mathcal{A}_\varepsilon + I)^{-1}$, taking the so-called correctors into account.

For simplicity, we consider the 4th order operator $\mathcal{A}_\varepsilon$ (the most important case for applications). Our main result is the following approximation

$$(\mathcal{A}_\varepsilon + I)^{-1} = (\mathcal{A}_0 + I)^{-1} + \varepsilon\mathcal{K}_1(\varepsilon) + \varepsilon^2\mathcal{K}_2(\varepsilon) + O\big(\varepsilon^3\big), \quad \varepsilon \to 0.$$

The principal term of approximation for the resolvent of $\mathcal{A}_\varepsilon$ was found in [1]. The correctors $\mathcal{K}_1(\varepsilon)$ and $\mathcal{K}_2(\varepsilon)$ are of order $O(1)$, $\varepsilon \to 0$, and can be described in terms of the solution of the auxiliary cell problems.

References

[1] A. A. Kukushkin, T. A. Suslina, Homogenization of high order elliptic operators with periodic coefficients, *St. Petersburg Math. J.*, **28**(1), 65–108 (2017).

Covering of elliptic curves and Fuchsian equations

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It is well known that there is a hypergeometric equation that has a solution in the form of a linear combination of complete elliptic integrals of the first kind (see. for example [1]). In particular, the integral

$$U(\alpha,\beta|\ell) = \int_\ell \frac{dt}{s},$$

where ℓ is arbitrary closed path on the elliptic curve

$$\Gamma_0: \ s^2 = t(t-\alpha)(t-\beta),$$

satisfies hypergeometric equation

$$U_{\alpha\alpha} + \left(\frac{1}{\alpha} + \frac{1}{\alpha-\beta}\right) U_\alpha + \frac{1}{4\alpha(\alpha-\beta)} U = 0.$$

Let there be a mapping σ of the some hyperelliptic curve Γ to elliptic curve Γ_0. Then there exist mappings of integrals and cycles:

$$\sigma^*: \frac{dt}{s} \to d\mathcal{U}, \quad \hat{\sigma}: \mathcal{L} \to \ell,$$

and then the following equation holds

$$\int_{\mathcal{L}} d\mathcal{U} = \int_{\mathcal{L}} \sigma^* \frac{dt}{s} = \int_{\hat{\sigma}\mathcal{L}} \frac{dt}{s} = \int_\ell \frac{dt}{s}.$$

Therefore the hyperelliptic integral $\mathcal{U}(\alpha,\beta|\mathcal{L})$ satisfies the same hypergeometric equation. This statement allows us to find the differential $d\mathcal{U}$.

If the hyperelliptic curve Γ has a genus $g = 2$, there exists another mapping to the another elliptic curve, and in this case the second hyperelliptic integral satisfies new Fuchsian equation. In particular, the integral

$$\mathcal{U}(\alpha,\beta,\gamma|\mathcal{L}) = \int_{\mathcal{L}} \frac{d\lambda}{\sqrt{(\lambda^2+\gamma)(\lambda^2+\gamma-\alpha)(\lambda^2+\gamma-\beta)}}$$

satisfies the following Heun equation

$$U_{\alpha\alpha} + \left(\frac{1}{\alpha} + \frac{1}{\alpha-\beta} + \frac{1}{\alpha-\gamma}\right) U_\alpha + \frac{3\alpha-\beta-\gamma}{4\alpha(\alpha-\beta)(\alpha-\gamma)} U = 0.$$

Finally, let us remark that there exist hyperelliptic integrals of second kind which also satisfy some Fuchsian equations.

References

[1] M. Abramowitz, I. A. Stegun (Eds.), *Handbook of Mathematical Functions with Formulae, Graphs and Mathematical Tables*, Wiley-Interscience, New York (1972).

On the diffraction of whispering gallery waves by boundary inflection points

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The problem of interest is that of a whispering gallery high-frequency asymptotic mode propagating along a concave part of a boundary and approaching a boundary inflection point. Like Airy ODE and associated Airy function are fundamental for describing transition from oscillatory to exponentially decaying asymptotic behaviors and so e.g. transition from light to shadow near caustics at high frequencies, the boundary inflection problem leads to an arguably equally fundamental canonical boundary-value problem for a special PDE, describing transition from a "modal" to a "scattered" high-frequency asymptotic behaviour. The latter problem was first formulated and analysed by M. M. Popov starting from 1970-s [1]. The associated solutions have asymptotic behaviors of a modal type (hence with a discrete spectrum) at one end and of a scattering type (with a continuous spectrum) at the other end. Of central interest is to find the map connecting the above two asymptotic regimes. The problem however lacks separation of variables, except in the asymptotical sense at both of the above ends, and has over the years proved not to be explicitly solvable, see [2, 3] for some latest efforts in this direction and further references therein.

Nevertheless, we argue that certain further progress can be made by the problem's further asymptotic analysis as well as subsequent numerical analysis. In particular, following and developing some old ideas of [4] together with those of M. M. Popov [1], a perturbation analysis of the problem at the continuous spectrum end can be performed, ultimately describing the desired map connecting the two asymptotic spectral representations. Further, the fact that the solutions remain (exponentially) decaying sufficiently far away from the boundary should allow to deform the integrals over the continuous spectrum into the complex plane. The values of the analytic continuation of the limit spectral function at certain points appear to describe the intensities of creeping waves emerging behind the inflection point. The ultimate evaluation of the actual "scattering matrix" may be quite a delicate numerical problem, and we will also briefly discuss certain ways in that direction including re-formulation of the problem as a one-dimensional boundary integral equation (cf. e.g. [5]) and its further regularization.

References

[1] M. M. Popov, The problem of whispering gallery waves in a neighbourhood of a simple zero of the effective curvature of the boundary, *Journal of Soviet Mathematics* (now J. Math. Sci. (NY)), **11**, 791–797 (1979).

[2] A. Ya. Kazakov, "Separation of variables" in model problems in diffraction theory. Formal scheme, *Zap. Nauchn. Sem. St. Peterburg. Otdel. Mat. Inst. Steklov. (POMI)*, **471**, 124–139 (2018) (in Russian; English translation to appear at *J. Math. Sci.*).

[3] D. P. Hewett, J. R. Ockendon, V. P. Smyshlyaev, Contour integral solutions of the parabolic wave equation, *Wave Motion*, **84**, 90–109 (2019).

[4] V. M. Babich, V. P. Smyshlyaev, The scattering problem for the Schrödinger equation with a potential linear in time and in space. II: Correctness, smoothness, behavior of the solution at infinity, *Journal of Soviet Mathematics*, **38**, 1562–1576 (1987).

[5] M. M. Popov, Whispering-gallery waves in a neighborhood of an inflection point of the boundary. Asymptotics of the wave field as $t \to \infty$, *Journal of Soviet Mathematics*, **32**, 205–214 (1986).

Nonintegrability of the energy density of "complex source" wavefields

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Over the last few decades, "complex source" wavefields [1, 2] have gained a great attention as exact solutions of the wave equation having Gaussian localization properties. Both time-harmonic and non-stationary cases were studied (see, e.g., a fresh review [3]). Such solutions satisfy the inhomogeneous wave equation (or, in time-harmonic case, the Helmholtz equation) with a source function supported by a certain antenna with a circular edge. Here, we address energy density of the "complex source" wavefield in the vicinity of the edge of antenna and show that it is nonintegrable.

References

[1] A. A. Izmest'ev, One parameter wave beams in free space, *Radiophys. Quant. Electron*, **13**(9), 1062–1068 (1970).

[2] G. A. Deschamps, Gaussian beam as a bundle of complex rays, *Electron. Lett.*, **7**(23), 684–685 (1971).

[3] A. M. Tagirdzhanov, A. P. Kiselev, Complexified spherical waves and their sources. A review, *Opt. Spect.*, **119**(2), 257–267 (2015).

The regularised weighted mean curvature flow equation for a coupled physics inverse conductivity problem

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The subject of our consideration is the weighted Mean Curvature Flow (MCF) problem

$$\begin{aligned} &u_t = |\nabla u| \nabla \cdot \left(a(x) \frac{\nabla u}{|\nabla u|} \right) \quad \text{in} \quad Q_T = \Omega \times (0, T], \\ &u = f(x) \quad \text{on} \quad \partial\Omega \times (0, T], \\ &u = u_0(x) \quad \text{in} \quad \overline{\Omega} \times \{0\}, \end{aligned}$$

where $a = \sigma|\nabla u|$ is a positive bounded away from zero function and Ω is a bounded domain whose boundary $\partial\Omega$ has a positive mean curvature. This problem is introduced as an alternative to the weighted least gradient problem currently used in Current Density Impedance Imaging (CDII). The latter is one of the nowadays medical imaging modalities based on the coupled physics (hybrid) inverse problems, in which the interactions between the different physical fields are exploited to enhance both the contrast and spatial resolutions in the entire domain. A coupled physics inverse problem consists of recovering the pair (u, σ) from the function f imposed on the boundary $\partial\Omega$ and the interior measurements of a. However, due to the singularity and elliptic degeneracy of the differential operator in the right-hand side of the MCF equation, this problem is ill-posed in the sense of Hadamard. Following Evans and Uraltseva–Oliker, the problem is regularised, and a combination of the Rothe and second-order accuracy finite difference methods is applied to reduce it to a family of systems of linear equations corresponding to all temporal layers. The regularised successive approximations are constructed to provide the regularised solutions to the weighted MCF problem. It is demonstrated in numerical experiments that for a sufficiently large time the regularized solutions are in proximity of the equilibrium solution, i.e., the function of the Dirichlet weighted least gradient. However, the numerical convergence is no more than linear. Therefore, further studies, analytical and numerical, need to be done to establish the global existence, uniqueness and convergence result for the MCF problem and to improve the rate of convergence. The work is supported by the National Science Foundation grant DMS-181882.

On quantitative acoustic imaging via boundary control method

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An initial boundary value problem

$$\begin{aligned}
&\rho(x)\nabla\cdot\big(\rho^{-1}(x)\nabla p\big)-c^{-2}(x)p_{tt}=0 \text{ in } R^n_+\times(0,T],\\
&\nabla p\cdot\nu=f(x,t) \text{ on } \Gamma\subset\partial R^n_+\times[0,T],\\
&p(x,0)=p_t(x,0)=0 \text{ in } R^n_+,
\end{aligned}$$

where p is the acoustic pressure, ρ, c are the density and sound speed distributions that are supposed to be positive and essentially bounded, and ρ is sufficiently smooth, ν is the outer normal vector, and f is the source function, is introduced as a forward model of quantitative acoustic imaging. Well describing the propagation and scattering of acoustic waves in fluid media, such as seawater, porous sediments filled with oil, soft biological tissues, etc., this model may be exploited in underwater acoustics, geophysical exploration of oil deposits offshore, and ultrasound tomography of female breast, human skin, and eyes. Considering the forward model as a dynamical system, and the function f — as a Neumann boundary control, a dynamical inverse problem can be formulated as follows. Let p satisfies the problem indicated above. Given the scattered acoustic wave field on Γ for every $f(x,t)$, $x\in\Gamma$, $t\in(0,2T]$, determine the pair (ρ,c) or ρ if c is known, or c if ρ is known, on a certain sub-domain of a region spanned by rays emitted at points belonging to Γ in the directions orthogonal to Γ. In terms of the dynamical theory, the given data is equivalent to knowledge of the partial response map $R^{2T}: f\to p_\Gamma$, where p_Γ is the trace of the solution to the forward model problem on the lateral hypersurface $\Gamma\times[0,2T]$. To solve numerically this inverse problem, a certain modification of the Boundary Control Method (BCM) is applied. In particular, the amplitude formula, which is the core of BCM, is slightly changed, and the regularisation of the ill-conditioned Gram system is performed on a spectrum of the Gram matrix. Some preliminary results of numerical experiments with a test problem, in which both the density ρ and sound speed c are supposed to be constant, are shown and discussed in order to demonstrate the computational feasibility of the approach proposed.

Regularisation of a weighted least gradient Robin problem for conductivity imaging

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We consider a coupled physics inverse conductivity problem of recovering an isotropic electrical conductivity σ in a bounded domain Ω from the magnitude of one current density field in Ω. Under certain conditions, this problem is reduced to the weighted 1-Laplacian in a conformal Euclidean metric

$$\nabla \cdot \left(a(x) \frac{\nabla u}{|\nabla u|} \right) = 0 \quad \text{in} \quad \Omega$$

subject to the Robin boundary condition

$$(\sigma \cdot \nu) + \varphi u = f \quad \text{on} \quad \partial\Omega.$$

Here, $a = \sigma|\nabla u|$ is the magnitude of the current density, which is supposed to be positive, and ν is the outer normal vector. The inverse problem consists of determining the pair (σ, u) in Ω given (a, φ, f). Since the weighted 1-Laplacian is singular and elliptic degenerate, this inverse problem is ill-posed in the sense of Hadamard. The variational approach is applied to regularise the problem. Specifically, a regularised energy functional is introduced as

$$G_{\alpha,\varepsilon}(v; a) = \int_\Omega a|\nabla u|dx + \frac{1}{2}\int_{\partial\Omega} \varphi_\varepsilon (v - h_\varepsilon)^2 ds + \alpha \int_\Omega |\nabla(v - h_\varepsilon)|dx,$$

where $\varphi_\varepsilon, f_\varepsilon$ are smooth approximations of φ, f, $h_\varepsilon = f_\varepsilon/\varphi_\varepsilon$ is a harmonic extension from $\partial\Omega$ into Ω, and $\alpha > 0, \varepsilon > 0$. The minimizing sequences for this functional are constructed, the compactness of which is established. An iterative algorithm is proposed to recover a minimum residual conductivity $\sigma_{\alpha,\varepsilon}$. The computational effectiveness of this algorithm is demonstrated in numerical experiments. The work of the first author is supported in part by the National Science Foundation grant DMS-181882.

Preliminary 2D modelling of the screw pinch plasma in PROTO-SPHERA

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This work deals with the properties of plasmas produced in the PROTO-SPHERA experiment [1]. The plasma appears as a linear discharge with variable diameter for a length of about 1 m and takes the shape of a mushroom in front of two annular electrodes (Fig. 1), in agreement with magnetic field lines shaping by external coils. Previous analysis in 1D [2], revealed that neutral gas dynamics is relevant in a narrow edge layer, whereas most of the plasma is fully ionized. On this basis, a simplified 2D description is explored neglecting neutrals dynamics. The main 2D feature is a factor of four variation of plasma radius along the axis. The equatorial section S_1 has a radius of about $R_1 = 0.25$ m while the narrow sections S_2 at the end of the linear region have both $R_2 = 0.065$ m. Since total axial current (10 kA typical) is constant, the pinch force at S_2 is much larger than the one at S_1. The question is how this large imbalance reflects on pressure distribution and on plasma flows. The point is that different cross-sections are connected by magnetic field lines, and any parallel pressure gradient would have to be balanced either by electrostatic forces (not considered in this work), or by convective flows.

Larger pinch force at smaller cross-sections will likely produce inwards flow, and vice versa. The resulting convective cell is outlined in Fig. 1. The assumed radial flows in turn tend to equalize net radial forces, in fact radial velocity V_r across axial field B_z produces azimuthal current $j_\phi = -B_z V_r/\eta$, where η is resistivity; the resulting contribution to $\mathbf{j} \times \mathbf{B}$ force is inwards if velocity is outwards, and vice versa.

Plasma configurations and flow patterns will be evaluated assuming radial force equalization, in order to provide suitable trial solutions for full 2-D calculations.

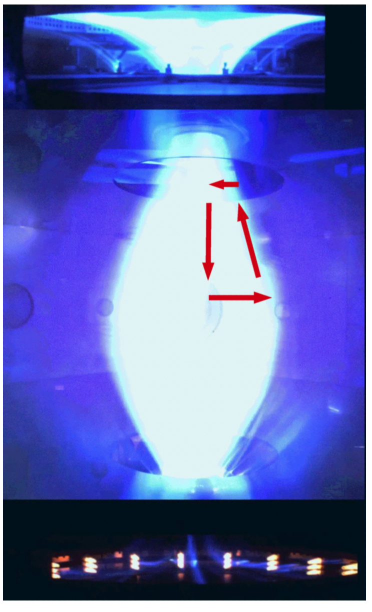

Fig. 1: Image of PROTO-SPHERA plasma from three cameras, one for the anodic region (top), one for the nearly cylindrical region (middle) and one for the cathode (bottom). View at transitions from linear to mushroom-shaped regions is impeded by coils. The mushroom-shaped plasma is clearly visible in the anodic region at the top. The cathode region is obscured by hot electron emitters (visible as brilliant spots). Arrows outline the flow pattern that would compensate the larger confining pinch force at smaller plasma cross-sections.

References

[1] F. Alladio, et al., *Nucl. Fusion*, **46**, S613–S624 (2006).

[2] P. Buratti, B. Tirozzi, F. Alladio, P. Micozzi, *Fluids*, **4**, 42 (2019).

Consideration of reflected waves in the parabolic equation method of arbitrarily high accuracy

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The parabolic equations of arbitrarily great accuracy were offered in [1]. Specifically, we understand this as a system

$$2i\frac{1}{\rho}n_0A_{j,X} + \left(\frac{1}{\rho}A_{j,Z}\right)_Z + \left[i\left(\frac{1}{\rho}n_0\right)_X + \frac{1}{\rho}\nu\right]A_j + \left(\frac{1}{\rho}A_{j-1,X}\right)_X = 0, \tag{1}$$

which approximates the acoustic Helmholtz equation

$$\left(\frac{1}{\rho}u_x\right)_x + \left(\frac{1}{\rho}u_z\right)_z + \frac{1}{\rho}\kappa^2u = 0, \tag{2}$$

where $u = A_0\exp(i\theta) + \epsilon A_1\exp(i\theta) + \ldots$, $X = \epsilon x$, $Z = \epsilon z$, ϵ is a small parameter, $\rho = \rho(X,Z)$, $\kappa^2 = n^2 + \epsilon\nu(X,Z) + \ldots$, θ is a phase variable. Note that $\theta = \theta(X)$. Hereafter A_{-1} is assumed to be zero. The derivation was done by the generalized multiple scale method.

Now let Γ be the vertical boundary, on which the index of refraction n and the density ρ are discontinuous. Then we can use the ray expansions of our field on Γ to obtain the following formulas [3]

$$A_j^{\mathrm{ref}} = \Psi A_j + \frac{1}{1+Y}\delta(j-1), \quad A_j^{\mathrm{rfd}} = DA_j + \frac{1}{1+Y}\delta(j-1),$$

where A_j^{ref} and A_j^{rfd} are respectively the reflected and refracted waves and

$$\Psi = \frac{1-Y}{1+Y}, \quad D = \frac{1}{1+Y}, \quad Y = \frac{\rho_2 n_2}{\rho_1 n_1}, \quad \delta_{j-1} = \frac{1}{n_1}\left(\frac{\rho_1}{\rho_2}\frac{\partial A^{\text{rfd}}}{\partial X} - \frac{\partial A}{\partial X} - \frac{\partial A^{\text{ref}}}{\partial X}\right).$$

We describe now how to solve the problem. Let A be the initial condition at $X=0$. We first solve the problem for the $j=0$ to Γ and then we calculate the A_0^{ref} and A_0^{rfd}. We continue calculate from Γ to some end point $X = X_{\text{end}}$ using the A^{rfd} as initial conditions. We then calculate from the X_{end} to 0 using 0 as the initial conditions and the A^{ref} as the A^{rfd} in the first case. Received fields are added together. Subsequent computations are different from the first using zero initial conditions at $X=0$.

References

[1] M. Yu. Trofimov, P. S. Petrov, A. D. Zakharenko, A multiple-scale approach to the parabolic equation method, *Wave Motion*, **50**, 85–95 (2013).

[2] Yu. A. Kravtzov, Yu. I. Orlov, *Geometric Optics of Inhomogeneous Media*, Nauka, Moscow (1980) (in Russian).

Mathematical modeling of pulse laser therapy using gold nanoparticles

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Mathematical modeling of passing the pulse radiation through biological tissue is important for estimating the effectiveness of laser thermotherapy of tumors. It is required to ensure a temperature field that is sufficient to damage the tumor and is safe for the surrounding tissue. Direct effect of laser radiation on tumors can be ensured, for example, by placing gold nanoparticles in the tumor [1].

The radiative-conductive heat transfer model describing the passage of radiation through an isotropic medium filling the domain G has the following form:

$$\frac{1}{c}\frac{\partial I(r,\omega,t)}{\partial t} + \omega\cdot\nabla_r I(r,\omega,t) + \mu I(r,\omega,t) = \frac{\mu_s}{4\pi}\int_\Omega I(r,\omega',t)d\omega', \tag{1}$$

$$\rho c_p \frac{\partial T(r,t)}{\partial t} - k\Delta T(r,t) = \frac{\mu_a}{4\pi}\int_\Omega I(r,\omega,t)d\omega. \tag{2}$$

Here, $I(r,\omega,t)$ is the intensity of radiation at the point $r\in G$ in the direction $\omega\in\Omega$, where Ω is the unit sphere, and in time moment $t\in(0,\tau)$, $T(r,t)$ is the temperature, c is the speed of light in the medium, μ is the attenuation factor, μ_s is the scattering coefficient, μ_a is the absorption coefficient, k is the thermal conductivity, ρ is the density, c_p is the specific heat capacity.

At the boundary Γ of the domain G, we set the following boundary conditions:

$$I|_\Gamma = I_b(r,\omega,t), \quad \omega\cdot n<0, \quad k\partial_n T + \gamma(T_b - T)|_\Gamma = 0, \tag{3}$$

where n is the outward normal, ∂_n is the outward normal derivative, γ, I_b, and T_b are given functions. Particularly, the function I_b on the part of boundary describes the incoming pulse laser radiation.

The initial conditions are given by

$$I(r,\omega,0) = I_0, \quad T(r,0) = T_0. \tag{4}$$

The mathematical model requires the calculation of radiation characteristics for gold nanoparticles placed in the skin tissue. The absorption and scattering coefficients for a gold nanoparticle has been calculated on the base approximation formulas from [2].

In the current work, an iterative algorithm to find a solution of the initial-boundary value problem (1)–(4) is constructed and implemented. The convergence of the iterative algorithm, the stabilization of the temperature field, and the influence of gold nanoparticles on the temperature distribution are studied. The effectiveness of the use of gold nanoparticles in laser therapy of tumors is demonstrated.

References

[1] A. N. Yakunin, Y. A. Avetisyan, V. V. Tuchin, Quantification of laser local hyperthermia induced by gold plasmonic nanoparticles, *J. Biomed. Opt.*, **20**(5), 051030 (2015).

[2] H. C. van de Hulst, *Light Scattering by Small Particles*, Dover Publications, New York (1981).

Structure of invariant subspaces of the rotation group image under the Jordan mapping

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We studied structure of invariant subspaces of effective hamiltonian of phase light modulation process [1]. Hamiltonian generators satisfy the commutational relationships of SU(2) rotation group [2], thus their bose-operator representation are derived using Jordan mapping technique. However, due to dimensionality of Fock space and additional symmetries of bose-operators, tasks of obtaining eigenbasis, as well as complete classification of invariant subspaces, become nontrivial. In this work, we obtain the relations between the bases of invariant subspaces and derive a classification of the invariant spaces.

References

[1] G. P. Miroshnichenko, et al., Algebraic approach to electro-optic modulation of light: exactly solvable multimode quantum model, *JOSA B*, **34**(6), 061177–14 (2017).

[2] I. M. Gelfand, *Representations of the Rotation and Lorentz Groups and Their Applications*, Courier Dover Publications, 384 (1963).

Wide-angle mode parabolic equation with transparent boundary conditions and its applications in shallow water acoustics

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Modeling of sound propagation in the shallow-water environments with the three-dimensional inhomogeneous bottom is one of the main research branches of underwater acoustics. Sound field $P\,(x, y, z)$ produced by a time-harmonic point source in a 3D waveguide is described by the three-dimensional Helmholtz equation. Its solution can be expressed in the form of modal decomposition

$$P\,(x, y, z) \approx \sum_{j=1}^{N} A_j\,(x, y)\,\varphi_j\,(z, x, y)\,, \tag{1}$$

where $\varphi_j\,(z, x, y)$ are the mode functions and $A_j\,(x, y)$ are the modal amplitudes (where z is the depth, and x, y are horizontal coordinates). Mode functions $\varphi_j\,(z, x, y)$ and the respective horizontal wavenumbers k_j can be obtained from acoustical spectral problem [1], and the modal amplitudes $A_j(x, y)$ satisfy the so-called horizontal refraction equation

$$\frac{\partial^2 A_j}{\partial x^2} + \frac{\partial^2 A_j}{\partial y^2} + k_j^2 A_j = -\varphi_j(z_s)\delta(x)\delta(y). \tag{2}$$

One-way solution of the latter equation can be approximated the solution of wide-angle mode parabolic equation [1]

$$\frac{\partial A_j}{\partial x} = ik_{0j}\left(\frac{a+bL}{1+cL} - 1\right)A_j, \tag{3}$$

where k_{0j} is the reference wavenumber and $k_{0j}^2 L = \partial_{zz} + k_j^2 - k_0^2$ (a, b, c are certain constants chosen as explained in [1]). Such equation can be solved using the standard Crank–Nicolson method and discrete transparent boundary conditions [2].

In this study we consider various examples of sound propagation problems for shallow-water waveguides with bottom inhomogeneities, including sloping bottom and an underwater canyon. Obtained results are compared with analytical solutions of these problems.

References

[1] F. A. Jensen, W. Kuperman, M. Porter, H. Schmidt, *Computational Ocean Acoustics*, Springer, New York (2011).

[2] A. Arnold, M. Ehrhardt, Discrete transparent boundary conditions for wide angle parabolic equations in underwater acoustics, *J. Comp. Phys*, **145**, 611–638 (1998).

Propagation of waves in three-dimensional periodic media

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Propagation of waves governed by the Helmholtz equation in a medium containing periodic lattice of spherical or cylindrical inclusions will be considered. Assuming that the transmission boundary conditions on inclusions interface is imposed and the radius of inclusion is small, we suggest a novel method for determining the dispersion relations of the Floquet–Bloch waves. The complete asymptotic expansion of the dispersion relations will be obtained with the rigorous estimate of the remainder. The method is based on a reduction of the original singular perturbed problem to a regular one. (These are joint results with Yu. Godin.)

Centralized networks

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We consider continuous time Hopfield-like recurrent networks as dynamical models for gene regulation and neural networks. We are interested in networks that contain n high-degree nodes preferably connected to a large number of N_s weakly connected satellites, a property that we call n/N_s-centrality. If the hub dynamics is slow, we obtain that the large time network dynamics is completely defined by the hub dynamics. Moreover, such networks are maximally flexible and switchable, in the sense that they can switch from a globally attractive rest state to any structurally stable dynamics when the response time of a special controller hub is changed. In particular, we show that a decrease of the controller hub response time can lead to a sharp variation in the network attractor structure: we can obtain a set of new local attractors, whose number can increase exponentially with N. These new attractors can be periodic or even chaotic. We provide an algorithm, which allows us to design networks with the desired switching properties, or to learn them from time series, by adjusting the interactions between hubs and satellites. Such switchable networks could be used as models for context dependent adaptation in functional genetics or as models for cognitive functions in neuroscience [2].

We describe applications of models of such kinds for genetic networks, and also to simulate human body motions [1].

References

[1] S. Vakulenko, O. Radulescu, I. Morozov, A. Weber, Centralized networks to generate human body motions, *Sensors*, **17**, 2907 (2017).

[2] S. Vakulenko, I. Morozov, O. Radulescu, Maximal switchability of centralized networks, *Nonlinearity*, **29**, 2327–2354 (2016).

Reconstruction for the coefficients of a quasilinear elliptic partial differential equation

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In this talk, we consider an inverse coefficients problem for a quasilinear elliptic equation of divergence form $\nabla \cdot \vec{C}(x, \nabla u(x)) = 0$, in a bounded smooth domain Ω. We assume that $\overrightarrow{C}(x, \vec{p}) = \gamma(x)\vec{p} + \vec{b}(x)|\vec{p}|^2 + \mathcal{O}(|\vec{p}|^3)$, by expanding $\overrightarrow{C}(x, \vec{p})$ around $\vec{p} = 0$. We give a reconstruction method for γ and $\vec{b}$ from the Dirichlet to Neumann map defined on $\partial\Omega$.

This is a joint work with Cătălin I. Cârstea (distinguished associate researcher), School of Mathematics, Sichuan University, China and Gen Nakamura (emeritus professor), Department of Mathematics, Hokkaido University, Japan.

Fluctuations of the spectrum of symmetrically deformed unitary invariant random matrix ensemble

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We consider the ensemble of $n \times n$ random matrices

$$H_n = U_n^* A_n U_n + C_n^* V_n^* B_n V_n C_n,$$

where A_n, B_n and $C_n C_n^*$ are hermitian (C_n is invertible) random (or non-random), having the limiting Normalized Counting Measure (NCM) of eigenvalues, U_n and V_n are unitary, uniformly distributed over $U(n)$, and A_n, B_n, C_n, U_n, and V_n are mutually independent. By using the technic described in [1], we establish the convergence of NCM of ensemble H_n to the non-random limit then $n \to \infty$ and find the limiting NCM via its Stieltjes transform. We also prove the Central Limit Theorem for the fluctuations of linear eigenvalue statistics of ensemble and calculate in the explicit form the variance of limiting Gaussian distribution of fluctuations in terms of the Stieltjes transforms of the limiting NCM of ensemble using the functional equations for it derived in [2] and [3].

References

[1] L. Pastur, M. Shcherbina, *Eigenvalue Distribution of Large Random Matrices*, Mathematical Surveys and Monographs 171, AMS, Providence, Rhode Island (2011).

[2] V. Vasilchuk, On the law of addition of random matrices: Covariance of traces of resolvent for random summands, *Journal of Mathematical Physics, Analysis, Geometry*, **6**(1), 96–127 (2010).

[3] V. Vasilchuk, On the fluctuations of eigenvalues of multiplicative deformed unitary invariant ensembles, *Random Matrices: Theory and Applications*, **5**(2), 1650007 (2016).

All-fiber polarization-dependent optical-vortex-controlling via acousto-optic interaction

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It is well-known that the acousto-optic interaction (AOI) can be efficiently used for versatile controlling of a light beam. Indeed, frequency shifters, wavelength tunable filters, deflectors and intensity modulators have been implemented both in the bulk [1] and fiber configurations [2]. Recently the possibility of the all-fiber stable wave-length tunable generation of OAM-bearing optical vortices (OVs) via the AOI induced by the flexural acoustic wave (FAW) of the lowest-order has been theoretically predicted [3] and demonstrated [4]. Here we report a new dynamic polarization-dependent mode conversion in a circular optical fiber endowed with the AOI, in which the sign of the topological charge of the generated OV are governed by the linear polarization direction of the incident beam. The permittivity of a fiber under consideration is given by [5]:

$$\hat{\varepsilon}(r,\varphi,z,t) = \varepsilon_0(r) + 2\Delta\varepsilon_{\rm co}u_0 f_r\cos\varphi\ \cos(Kz-\Omega t) + \varepsilon_{\rm co}^2 pKu_0\begin{pmatrix} 0 & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 0\end{pmatrix}\sin(Kz-\Omega t). \qquad (1)$$

Here the unperturbed fiber permittivity $\varepsilon_0(r) = \varepsilon_{\rm co}[1-2\Delta f(r)]$, where $\Delta = (\varepsilon_{\rm co}-\varepsilon_{\rm cl})/2\varepsilon_{\rm co}$ is the normalized index difference, $\varepsilon_{\rm co}$ and $\varepsilon_{\rm cl}$ are the core and cladding values of the permittivity, respectively, and $f(r)$ is the fiber's profile function. In the second term $f_r = df/dr$ and u_0, K, and Ω are the amplitude, wavevector and frequency of the FAW, respectively. In the last term for silica at the wavelength $\lambda = 0.63$ µm the constant of photoelasticity is $p = -0.075$. The cylindrical coordinates (r,φ,z) are implied.

Let the OV linearly polarized along x or y-axis $|\mathrm{LV}_\ell^\sigma\rangle$ at frequency ω be incident on such a fiber. Under certain conditions the incident beam will be completely transformed in the following way:

$$|\mathrm{LV}_\ell^x\rangle\ e^{-i\omega t} \longrightarrow |\mathrm{LV}_{-\ell}^x\rangle\ e^{-i\omega t}, \quad |\mathrm{LV}_\ell^y\rangle\ e^{-i\omega t} \longrightarrow |\mathrm{LP}_\ell^y\rangle\ e^{-i\omega t}.$$

This transformation describes a novel type of the optical polarization-dependent mode conversion in fiber acousto-optics. The key feature of such a conversion is that the topological charge ℓ or $-\ell$ (and OAM per photon $\hbar\ell$ or $-\hbar\ell$) of the generated field is determined by the direction of linear polarization $\sigma = x, y$ of the incident vortex beam. The described effect can be useful in developing new acousto-optic devices such, for example, as polarization-controlled optical vortex intensity modulators. Yet, this type of all-fiber wavelength-tunable optical vortex generation and controlling should be especially useful in micromechanics and simulation of quantum computing.

References

[1] A. Yariv, P. Yeh, *Optical Waves in Crystals*, John Wiley & Sons, Inc., New York (1984).

[2] B. Y. Kim, J. N. Blake, H. E. Engan, H. J. Shaw, All-fiber acousto-optic frequency shifter, *Opt. Lett.*, **11**, 389–391 (1986).

[3] M. A. Yavorsky, All-fiber polarization-dependent optical vortex beams generation via flexural acoustic wave, *Opt. Lett.*, **38**, 3151–3153 (2013).

[4] W. Zhang, L. Huang, K. Wei, et al., High-order optical vortex generation in a few- mode fiber via cascaded acoustically driven vector mode conversion, *Opt. Lett.*, **41**, 5082–5085 (2016).

[5] M. A. Yavorsky, D. V. Vikulin, E. V. Barshak, B. P. Lapin, C. N. Alexeyev, Revised model of acousto-optic interaction in optical fibers endowed with a flexural wave, *Opt. Lett.*, **44**, 598–601 (2019).

Stable components of wave fields in underwater acoustic waveguides

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The main factor limiting the possibility of theoretical prediction of sound fields in the ocean is the inevitable mismatch between the real propagation medium and its mathematical model used to solve the wave equation.

The presentation is aimed at analyzing such field components in a hydroacoustic waveguide that are weakly sensitive to variations in the waveguide parameters and therefore can be relatively accurately predicted even under conditions of uncertain environment.

In Ref. [1] it is shown that the field component low sensitive to large-scale sound speed fluctuations is formed by a narrow beam of rays. If the beam width is smaller than the fluctuation spatial scales, then the rays actually pass through the same inhomogeneities, and therefore their phases in the presence of perturbation acquire close increments. At a given observation distance, the components of the monochromatic field, formed by such a beam of rays in a perturbed and unperturbed waveguide, differ only by a constant phase factor. For pulsed signals, the perturbation only leads to an additional delay of the stable component as a whole. These simple physical considerations make it possible to expect that the stable component is formed by waves arriving in a relatively small depth interval at grazing angles from a small angular interval.

Thus, at a given observation distance, the stable field component is associated with a small region of the phase plane 'depth – arrival angle'. Two methods are proposed for the isolation of this component from the total wave field. One of them is based on using the coherent state expansion developed in quantum mechanics. Another method is based on the modal representation of the wave field. From the viewpoint of modal description, the stable component is formed by a group of constructively interfering modes.

Stable components can be used to modify the traditional method of match field processing used in underwater acoustics to solve inverse problems related to estimating the coordinates of a sound source and reconstructing unknown environmental parameters. This method is based on the comparison of the complex amplitudes of the measured and calculated wave fields at the aperture of the receiving antenna. An alternative approach discussed in the presentation is based on comparing the intensity distributions of these fields in the 'depth – arrival angle' plane [2]. The main advantage of this approach is its low sensitivity to the inevitable inaccuracy of an environmental model used in calculation.

References

[1] A. L. Virovlyansky, Stable components of sound fields in the ocean, *J. Acoust. Soc. Am.*, **141**, 1180–1189 (2017).

[2] A. L. Virovlyansky, Matched shadow processing, *JASA Express Letters. J. Acoust. Soc. Am.*, **142**, EL136–EL142 (2017).

Normal mode coupling in a waveguide with a range-dependent sound speed profile in the bottom

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Sound propagation in shallow water over the bottom with varying impedance is considered in the normal mode approach. Water depth is assumed to be constant. Bottom roughness is not taken into account. Range-dependent impedance is only associated with the spatial variability of sound speed in the bottom. The results of 3D seismic survey conducted in the Kara Sea are taken as an input data for sound propagation modeling. Numerical simulations show that the typical horizontal gradients of the bottom sound speed provide mode coupling. It has negligible effect on depth averaged acoustic intensity. However, the sound field interference pattern is rather sensitive to this type of perturbation. The work was supported by RFBR, 16-32-60194 and 19-02-00127.

Remote sensing problem in the existence of acoustic noise in the ocean

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In the single scattering approximation and point source the solution of the inverse problem (remote sensing problem) is obtained in the paper [1]. In this paper authors consider the remote sensing problem taking into account acoustical noise in the ocean.

Echo signal propagates in the medium $G \equiv \mathbb{R}^2$. Let J_0 describe a point isotropic sound source located at the origin O and emitted a pulse in the initial time. J_1 is the round distributed source located at the point r_0:

$$J_0(\mathbf{r},\mathbf{k},t) = P_0\delta(\mathbf{r})\delta(t), J_1(\mathbf{r},\mathbf{k},t) = \begin{cases} P_1, & |\mathbf{r}-r_0| < R; \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Here, δ denotes the Dirac delta-function and P_0, P_1 are the powers of the sources J_0, J_1, respectively. Authors assume that there are no sound sources in the medium up to the moment $t = 0$. The point receiver located at the point O. In the case of [1], the source was point and isotropic ($J = J_0$). Thus, the solution of the initial-boundary value problem:

$$I(O,\mathbf{k},t) = I_1(O,\mathbf{k},t) = \frac{J_1}{|\Omega|}\exp(-\mu ct)\sigma\Big(\frac{ct}{2}\mathbf{k}\Big)\frac{1}{t} =: I_1(O,\mathbf{k},t). \tag{2}$$

Redefined $\mathbf{r} = ct\mathbf{k}/2$ and received signal $I(\mathbf{r},t) := I(O,\mathbf{k},t)$ in the equation (2) authors obtain the relation for determining the coefficient of volume scattering [1]:

$$\sigma(\mathbf{r}) = \frac{|\Omega| t\exp(\mu ct)}{P_0}\overline{I}(\mathbf{r},t), \quad \mathbf{r} \in \mathbb{R}^2. \tag{3}$$

Equation (3) is a solution of the inverse problem for determining the volume scattering coefficient σ in the single scattering approximation, a point source, emitted a pulse at the initial time, and in the case of a receiver concentrated at the origin.

The signal measured by the receiver in the single scattering approximation with a combined source $J = J_0 + J_1$ takes the following form:

$$I(O,\mathbf{k},t) = I_0(O,\mathbf{k},t) + I_1(O,\mathbf{k},t) = \int_0^{ct} \exp(\mu t') J_1\Big(t'\mathbf{k}, \mathbf{k}, t - \frac{t'}{c}\Big) dt' + \frac{P_0}{|\Omega|} \exp(-\mu ct)\sigma\Big(\frac{ct}{2}\mathbf{k}\Big)\frac{1}{t}$$
$$+ \frac{1}{|\Omega|}\int_0^{ct} \exp(-\mu t')\sigma(-t'\mathbf{k}) \int_0^R \int_0^{2\pi} J_1\Big(\mathbf{x} - t'\mathbf{k}, -\frac{\mathbf{x}}{|\mathbf{x}|}, t - \frac{t' + |\mathbf{x}|}{c}\Big)\frac{\exp(-\mu|\mathbf{x}|)}{|\mathbf{x}|}\rho\, d\rho\, d\phi\, dt', \quad (4)$$

where $\mathbf{x} = r_0 + t'\mathbf{k} + \rho(\sin(\phi), \cos(\phi))$.

The goal of the computational experiment is analyzing of the influence of the additional source J_1 in the solution of the remote sensing problem which is obtained in the case of a point isotropic source. From the practical point of view, authors consider the problem of remote sensing in the ocean. By scattering coefficient σ, they simulate biological objects (plankton clusters, schools of fish). J_1 describe an active source in the ocean (acoustic buoy, autonomous unmanned vehicle or submarine).

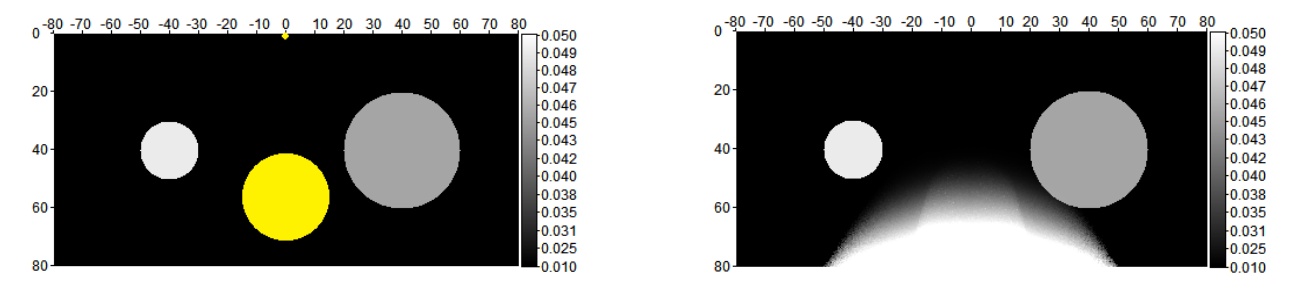

Fig. 1: Distribution of volume scattering coefficient. Left: exact solution and position of sources (yellow color). Right: Reconstruction of σ function.

References

[1] P. A. Vornovskikh, A. A. Sushchenko, Remote sensing problem with multiple scattering effect, *Proceedings of SPIE – The International Society for Optical Engineering*, **10466**, Art. no. 104661Y (2017).

Modified Z-scan method: numerical simulation and analytical solution

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Nowadays, there are sources of the broadband pulsed THz radiation of such a high intensity, that it gets possible to observe nonlinear effects in the spectral range mentioned. The most utilized technique to evaluate the nonlinear properties of the media is the Z-scan method. Recently, our research team has proposed a more sensitive modification of the method [1]. The comparison of the results obtained through the standard Z-scan method and its modification is shown in Fig. 1(a) [1].

Usually, THz pulses contain 1–1.5 electromagnetic field oscillations and, therefore, have wide spectrum. The standard Z-scan method is based on the monochromatic approach, which means it has certain limitations when employed for the THz pulses. Therefore, a new mathematical model for the Z-scan method for few-cycle pulses is needed. This work provides the analytical solution of the modified Korteweg – de Vries equation [2] for the case of the THz pulse with folowing parameters:

central wavelength of the THz radiation $\lambda_0 = 0.3$ mm, period of oscillations $T_0 = 0.3 - 10$ ps, peak intensity $I_0 = 10^7$ W/cm^2, the nonlinear refractive index coefficient $n_2 = 4 \cdot 10^{-11}$ cm^2/W [3]. ZnSe crystal with thickness $L = 0.3$ mm was used as the sample.

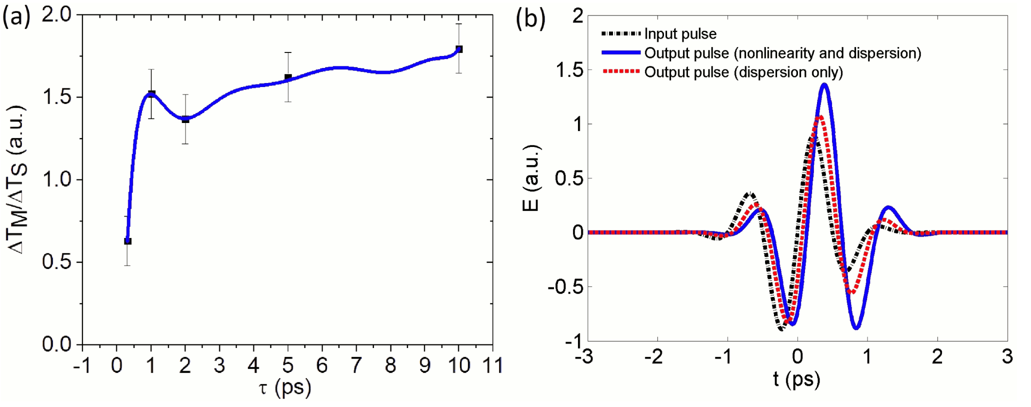

Fig. 1: (a) The ratio of the peak-to-valey values for the modified ΔT_M and standard ΔT_S Z-scan methods for the sample thickness $L = 0.3$ mm depending on the pulse duration τ. (b) Comparison of the analytical solutions for the input and output pulses with and without nonlinear effects consideration

Fig. 1(b) represents an example of the analytical solution for the parameters above considering the dispersion and diffraction effects only and together with the nonlinearity. For the chosen parameters consideration of the nonlinearity is crucial for the correct description of the pulse changes.

In this work the comparison of the numerical simulation results and the analytical solution for the cases of both the standard and modified Z-scan methods is demonstrated.

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References

[1] I. O. Vorontsova, M. V. Melnik, S. E. Putilin, A. N. Tcypkin, S. A. Kozlov, Analysis of the Z-scan method for few-cycle terahertz pulses, *Collection of Scientific Papers of the VIII International Conference on Photonics and Information Optics*, (2019) (in Russian).

[2] S. A. Kozlov, V. V. Samartsev, *Fundamentals of Femtosecond Optics*, Elsevier, Saint Petersburg (2013).

[3] A. N. Tcypkin, S. E. Putilin, M. C. Kulya, M. V. Melnik, A. A. Drozdov, V. G. Bespalov, X.-C. Zhang, R. W. Boyd, S. A. Kozlov, Experimental estimate of the nonlinear refractive index of crystalline ZnSe in the terahertz spectral range, *Bull. Russ. Acad. Sci. Phys.*, **82**, 1547–1549 (2018).

Emission illusion at the angle

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Mathematical methods of transformation optics, proposed by Ulf Leonhardt [1] and John Pendry [2, 3], have made it possible to calculate such exotic devices as invisibility cloaks, “black holes”, absorbers, “wormholes”, etc. Transformation optics based on differential geometry and tensor analysis, although its concept was determined by Fermat’s principle. The light propagation direction depends on the material refractive index. This indicates that any optical medium establishes geometry. According to this, at present, transformation optics is a unique scientific tool that allows for the combining of the mathematical mapping of desired distortions of space with the actual special distribution of refractive index in physical space to control light propagation.

This work proposes to use a fundamentally different approach for mirage creation based on angular dependence of constitutive parameters [4]. Such an approach allows creation of an emission illusion in any angular directions. We have theoretically described and numerically simulated the radar illusion device using artificial materials with the cylindrical polar angle dependent tensors of permittivity and permeability. The illusion device was simulated by the Finite Element Method in the frequency domain.To create a mirage of source emission it is necessary to consider two materials (see Fig. 1): from 0 to φ_2 is a right-handed medium and from φ_2 to Φ is a left-handed medium. The radiation source should be placed into the permissible domain (φ_1;φ_2) in the first material, otherwise the expected effect is not observed. Influence of different factors (metamaterials anisotropy, source location) on the appearance of a source illusion were considered.

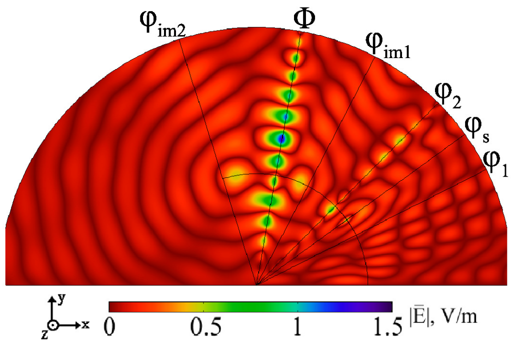

Fig. 1: The scheme of the illusion device. Electric field distribution ($|\bar{E}|$) of the point source placed at the angle φ_s. Images of the source are located at the angles φ_{im1} and φ_{im2}.

References

[1] U. Leonhardt, Optical conformal mapping, *Science*, **312**, 1777–1780 (2006).

[2] J. B. Pendry, D. Schurig, D. R. Smith, Controlling electromagnetic fields, *Science*, **312**(5781), 1780–1782 (2006).

[3] D. Schurig, J. B. Pendry, D. R. Smith, Calculation of material properties and ray tracing in transformation media, *Opt. Express*, **14**(21), 9794–9804 (2006).

[4] E. Gurvitz, S. Andronaki, A. Vozianova, M. Khodzitsky, Cloak based on the angle dependent constitutive parameters tensors, *Opt. Express*, **23**(20), 25738–25746 (2015).

On discrete WKB methods for resonance electromagnetic traps

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The WKB asymptotics are well known and widely used to obtain semiclassical estimates of eigenstates and the spectrum for the one-dimensional Schrödinger operator, that is a second-order differential operator. On the other hand, many one-dimensional quantum systems can be naturally represented in terms of difference equations. Namely, if one considers a quantum system in the spectral representation of a coordinate operator, where the coordinate is an action operator with a discrete spectrum, then the corresponding spectral equation takes the form of a difference equation. The theory of semiclassical asymptotics for a difference equation, or discrete WKB methods, can be formally obtained by considering a difference equation as a pseudodifferential equation. The rigorous WKB asymptotics for difference equations including tunneling effects are also presented [1–3] but are significantly less known than their continuous counterparts.

We consider the semiclassical asymptotics for a second-order difference equation that corresponds to resonance electromagnetic traps [4, 5]. The one-dimensional Hamiltonian is obtained by quantum averaging of small anharmonic terms in the hyperbolic resonance trap. We express the Hamiltonian in terms of integrals (symmetries) of the ideal resonance trap. These integrals form a non-Lie algebra with a creation-annihilation-neutral operator structure. Here the neutral operator is an action operator with a discrete spectrum. The one-dimensional effective Hamiltonian is a polynomial on these basis operators. Therefore, it can be presented as a difference operator on the spectral representation of neutral operator.

We discuss several basic theorems of discrete WKB methods and their application to the tunneling asymptotics for resonance electromagnetic traps Hamiltonians. This is a joint work with M.V. Karasev and E.M. Novikova. The work was supported by the Program for Fundamental Research of Higher School of Economics.

References

[1] O. Costin, R. Costin, Rigorous WKB for finite-order linear recurrence relations with smooth coefficients, *SIAM J. Math. Anal.*, **27**(1), 110–134 (1996).

[2] A. Fedotov, F. Klopp, The complex WKB method for difference equations and Airy functions, *arXiv*: 1810.04918 (2018).

[3] E. V. Vybornyi, On the WKB method for difference equations: Weyl symbol and the phase geometry, *Nanostructures. Math. Phys. and Modelling*, **2**(15), 5–20 (2016).

[4] M. Karasev, E. Novikova, E. Vybornyi, Instantons via breaking geometric symmetry in hyperbolic traps, *Math. Notes*, **102**(5–6), 776–786 (2017).

[5] M. Karasev, E. Vybornyi, Bi-orbital states in hyperbolic traps, *Russ. J. Math. Phys.*, **25**(4), 500–508 (2018).

Transition of edge waves to wedge waves in a semi-infinite elastic plate with facets

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Edge waves (EW) and wedge waves (WW) represent two examples of one-dimensionally guided waves in elastic waveguides. Usually, the term EW means the waves propagating along the end face of a structure shaped like a semi-infinite plate, whereas WW propagate along the tip of a wedge formed by two non-parallel semi-infinite planes. Until quite recently, these types of waves were studied separately (see [1, 2] for EW and [3] for WW). But there is a relation between them, that can be investigated on the basis of 3D theory of elastodynamics. So, it is shown [2, 4], that the fundamental EW become pair of WW as the frequency tends to infinity. This result is obtained for the waves propagating in the direction x_1 in a semi-infinite plate of rectangular cross-section ($-\infty < x_1 < \infty$, $|x_2| \leq h$, $x_3 \leq 0$) (see Fig. 1,a).

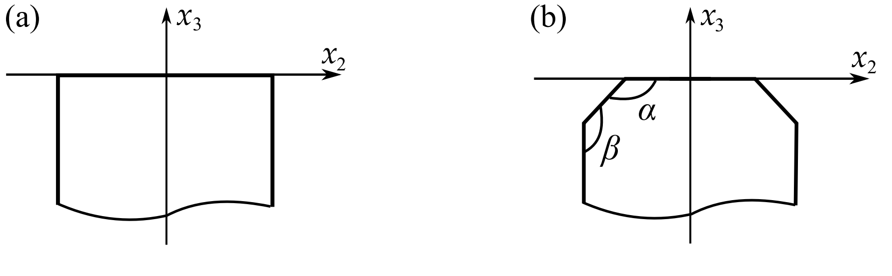

Fig. 1: Cross-section of the plate: (a) – rectangular plate, (b) – plate with two facets.

In this work, more interesting case of a plate with the cross-section shown in Fig. 1,b is considered. In this case, there are two pair of non-rectangular wedges with different aperture angles α and β, which can guide quasi-WW at high frequencies. At the low frequencies, the influence of the facets is small, consequently, the properties of EW are nearly the same as in the rectangular plate. The goal of this work is to investigate the transition of EW to quasi-WW in the plate with facets. The method of solution of corresponding problem of elastodynamics is developed, which is based on the modal expansion technique. The numerical results for EW dispersion curves are presented for various values of α, which show how they tend to WW as the frequency grows, or vanish when WW does not exist at the given aperture angle.

References

[1] J. B. Lawrie, J. D. Kaplunov, Edge waves and resonance on elastic structures: an overview, *Math. Mech. Solids*, **17**, 4–16 (2012).

[2] M. V. Wilde, J. D. Kaplunov, L. Yu. Kossovich, *Edge and Interfacial Resonance Phenomena in Elastic Bodies*, Fizmatlit, Moscow (2010) (in Russian).

[3] P. Pupyrev, A. Lomonosov, A. Nikodijevic, A. Mayer, On the existence of guided acoustic waves at rectangular anisotropic edges, *Ultrasonics*, **71**, 278–287 (2016).

[4] A. A. Krushynska, Flexural edge waves in semi-infinite elastic plates, *J. Sound Vib.*, **330**, 1964–1976 (2011).

Steering light by tailored excitation of nano-antennas and applications to nano-metrology

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The field of nano optics involves the study of light and matter interactions at nano-scale. One of the key elements is nanoantenna: nanoscaled particle that interacts resonantly with free space light [1, 2]. Using the so-called antenna effect, free space light can be focused down to extremely small point, and then reemitted in a highly directional way through the interference of different antenna modes. The reshaping of the light field by nanoantennas has propelled many intriguing applications ranging from holography to imaging devices.

Attractive ideas have been developed to use different spatial modes of light for the excitation of different antenna modes [3–8]. For example, it has been shown cylindrical vector beams such as radially polarized beam and azimuthally polarized beam can excite the electric and magnetic resonances of the nano-antenna respectively [5]. However, the true power of this selective excitation lies in designing the excitation field on purpose for better antenna directivity. This would require optimization of the excitation field in terms of polarization, amplitude and phase.

If the nanoantenna is placed in a strongly inhomogeneous field, its far-field scattering directivity can be largely changed depending on its relative position [6–9]. If the incoming field contains rapidly changing features, the displacement sensitivity of the nanonantenna within the field can be extremely large. In this talk, we will talk about our recent works on this subject. In particular, we will show that extremely high far-field scattering sensitivity for a displacement much smaller than the wavelength can be achieved using the optimized illumination and nano-antenna structures. This method could bring about important understandings of super-resolution technique involving singular optics as well as interesting applications in optical subwavelength metrology.

References

[1] P. Bharadwaj, B. Deutsch, L. Novotny, Optical antennas, *Advances in Optics and Photonics*, **1**(3), 438–483 (2009).

[2] L. Novotny, N. Van Hulst, Antennas for light, *Nature Photonics*, **5**(2), 83–90 (2011).

[3] T. Bauer, et al., Nanointerferometric amplitude and phase reconstruction of tightly focused vector beams, *Nature Photonics*, **8**(1), 23–27 (2014).

[4] J. Lin, et al., Polarization-controlled tunable directional coupling of surface plasmon polaritons, *Science*, **340**(6130), 331–334 (2013).

[5] Z. Xi, et al., Broadband active tuning of unidirectional scattering from nanoantenna using combined radially and azimuthally polarized beams, *Optics Letters*, **41**(1), 33–36 (2016).

[6] M. Neugebauer, et al., Polarization tailored light driven directional optical nanobeacon, *Nano Letters*, **14**(5), 2546–2551 (2014).

[7] M. Neugebauer, et al., Polarization-controlled directional scattering for nanoscopic position sensing, *Nature Communications*, **7**, 11286 (2016).

[8] Z. Xi, et al., Accurate feeding of nanoantenna by singular optics for nanoscale translational and rotational displacement sensing, *Physical Review Letters*, **117**, 113903 (2016).

[9] Z. Xi, H. P. Urbach, Magnetic dipole scattering from metallic nanowire for ultrasensitive deflection sensing, *Physical Review Letters*, accepted.

Tunable spin-orbit optical coupling to retrieve the shape of deep subwavelength objects from the scattered far field intensity

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The far field scattering behaviour of deep subwavelength particles can strongly depend on the properties of the particles and the illumination. An example is the configuration of two parallel gold nanoparticles with a gap of order 15 nm between them, which are illuminated by a polarised azimuthally polarised focused spot. When a wavelength is used for which the particles have a localised plasmon resonance, the scattered far field is very sensitive to the position of the particles. In fact, a scattering pattern that is symmetric as function of the azimuthal scattering angle is obtained when the centre (dark) spot is exactly inbetween the two particle, but for a very small displacement the scattering becomes strongly uni-directional [1].

Here we study a scheme to retrieve size parameters of a nano-particle on a glass substrate at a scale much smaller than the wavelength. Apart from being very sensitive to the shape parameters, the scattered far field also strongly depends on the position of the particle with respect to the illumination. We illuminate a silicon nano-particle by two interfering p-polarised plane waves at the Brewster angle of incidence, to create an illumination with locally varying elliptically polarised state. The scattered far field is detected in the glass medium. It is underlined that the scheme works not only for particles made of silicon but also for dielectric and semiconductor particles. Two polar scattering angle regions can be distinguished: the undercritical angle region (UAR) for which radiation occurs in a cone with axis equal to the normal to the glass surface, and its complement: the supercritical angle region (SAR) where the radiation is into large polar angles beyond the angle of total internal reflection in the glass. The intensity of the far field in the SAR depends exponentially on the height of the particle. Therefore, by measuring the ratio of the intensity scattered in the SAR over that in the UAR, the height of the particle can be determined.

The scattering by a nanowire with elliptical cross-section can be well described by a line current at the centre of the nanowire with polarisation parallel to that of the local incident electric field. When the line current has a particular elliptic polarisation, the field scattered in SAR is strongly asymmetric: the scattering is maximum in the direction of negative azimuthal angles of the SAR whereas it vanishes in the direction of positive azimuthal angles, or conversely (see Fig. 1c). Since the particle is much smaller than the wavelength used, the polarisability of the particle can be expressed in its shape parameters using a quasi-static model. This polarisability depends strongly on the shape of the cross-section of the nanowire and on the polarisation of the incident light. By scanning the particle with the illuminating standing wave with spatially varying polarisation states, the polarisability can be determined where the far field scattering asymmetry is maximum. From this the shape parameters can be retrieved with deep sub-nanometer precision.

The technique can be extended to retrieve the parameters of nanoparticles of ellipsoidal shape as shown in Fig. 1d. Sizes of order of 10 nm are retrieved from far field scattering measurements using two orthogonal illuminations by p-polarised standing waves (Fig. 1b and 1c). As in the case of the nanorod, the far field scattering data also give highly precise information about the position of the particle.

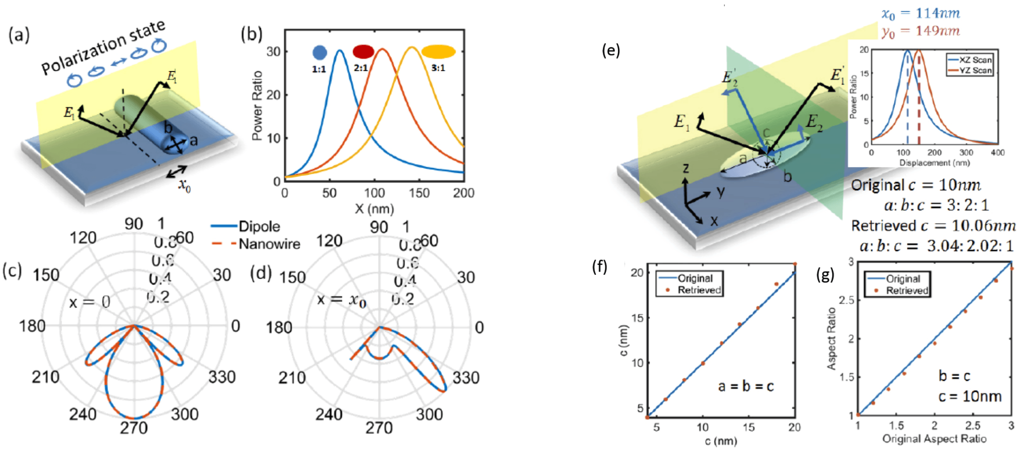

Fig. 1: (a) Nanorod illuminated by two p-polarised plane waves at the Brewster angle. (b) Power ratio of radiated intensity in the positive and negative azimuthal angle parts of the SAR, as function of the position of the illumination and for different aspect ratios of the elliptic cross-sections. Far field scattering pattern when the excited electric dipole is linear polarised (c), and when the excitation is such that maximum asymmetry in the SAR scattering occurs (d). The red curve is rigorously computed, the blue curve is obtained with the single dipole model. (e) Ellipsoidal particle illuminated by p-polarised plane waves. The inset is the power ratio similar to (b), but now for illumination along both the x-and y-directions. (f) and (g): Comparison between actual and retrieved parameters values.

References

[1] Z. Xi, H. P. Urbach, *Physical Review Letters*, **119**, 053902 (2017).

[2] Z. Xi, H. P. Urbach, *Physical Review Letters*, **90**, 34–42 (2003).

Nonstationary inhomogeneous waves near the boundary of an anisotropic elastic body

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We study the kinematics and dynamics of inhomogeneous elastic waves with a complex phase that arise at a discontinuity in an elastic medium, such as an interface between two media or a boundary surface of an elastic body. These waves form an important part of a solution to the boundary problem of the reflection / refraction of elastic waves. Here, the boundary is assumed to be smooth, and elastic media, inhomogeneous. The displacement vectors for both inhomogeneous and homogeneous (with real phases) waves arising upon reflection / refraction of an incident wave, are sought in the form of asymptotic ST (Space-Time) series [3]. The incident elastic wave is given by its ST ray expansion as a homogeneous wave with real phase $p\varphi_0$, where $p \gg 1$ is a large parameter of the problem.

The aforementioned waves belong to a class of non-stationary high-frequency oscillations modulated in both amplitude and frequency. We posit that an ST-modulated wave, at any point $M_0(\overrightarrow{r_0}, t_0)$ in 4D space, can be represented as a locally plane wave with instantaneous frequency $\omega = \frac{-\partial(\operatorname{Re}\varphi)}{\partial t}$ and local wavevector $\overrightarrow{k} = \nabla(\operatorname{Re}\varphi)$, where φ is the phase of the ST wave. As in the case of monochromatic plane waves, at a point M on the boundary, the phase velocities V of locally plane waves may be expressed as a function of the directional unit local wave vector, where velocities V are found as positive roots of the characteristic equation of a real 3-row matrix. This interpretation enables the creation of wave surfaces, such as phase velocity and refraction (inverse velocity) surfaces. The refraction curves situated on the incidence planes where the reflection/refraction waves arise, are formed by the intersection of these planes with the refraction surface [1, 2]. According to Snell's law, change to the incidence angle ψ of incident wave $\overrightarrow{U_0}$ within $[\psi_1, \psi_2)$ and $[\psi_2, \frac{\pi}{2}]$, respectively, where ψ_1 and ψ_2 denote critical angles of $\overrightarrow{U_0}$, gives rise to inhomogeneous elastic waves, one of which is quasi-longitudinal, and the other two are quasi-transverse.

For inhomogeneous waves, in zeroth approximation of asymptotics, we arrive at the problem of finding the eigenfunctions and eigenvalues of a symmetric three-row matrix operator in complex Hilbert space, with its eigenvectors formed by the principal terms of the ST ray expansions, and its complex eigenvalues given by $\lambda = \left(\frac{\partial\tau}{\partial t}\right)^2$, where τ is the phase of an inhomogeneous wave.

The eigenvalues λ are found as the roots of a third-degree algebraic equation with complex coefficients expressed via Cardano's formulas with a non-zero discriminant. In the Riemann metric related to the surface boundary, and in the vicinity of that boundary, we use these eigenvalue expressions together with the homogeneous equation for the eigenfunctions to construct the complex phases τ and polarization vectors of the inhomogeneous waves as expansions in powers n (coordinate n is the distance of point M to the boundary).

The principal term of asymptotics and higher-order coefficients of the ray series include arbitrary functions Ψ as scalar factors in their polarization vector. Use of the ST ray procedure to derive the coefficients of the ray series, combined with the boundary conditions, leads to a first-order differential equation in coordinate n for function Ψ. Thus, analytical expressions for the wave amplitudes are obtained outside the boundary. Furthermore, the requirement for phase τ to meet $\frac{\partial \operatorname{Im}\tau}{\partial n} > 0$ at $n = 0$, implies that the inhomogeneous waves are evanescent and decay exponentially away from the boundary surface.

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References

[1] F. I. Fedorov, *Theory of Elastic Waves in Crystals*, Nauka, Moscow (1965) (In Russian).

[2] G. I. Petrashen, *Propagation of Waves in Anisotropic Elastic Media*, Nauka, Leningrad (1980) (in Russian).

[3] V. M. Babich, On the space-time ray method in the theory of elastic waves, *J. Izv. Akad. Nauk, Fizika Zemli*, **2**, 3–13 (1979) (In Russian).

Acoustics@home volunteer computing project and an investigation on the accuracy of dispersion-based geoacoustic inversion method

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Volunteer computing is a type of distributed computing, in which resources of private persons are used [1]. Volunteer computing suits well to solve hard problems, which can be decomposed into a set of independent subproblems. In 2017, the volunteer computing project Acoustics@home was launched, which is aimed at solving hard problems in underwater acoustics.

It is known, that water column and bottom parameters can be reconstructed from acoustic data using geoacoustic inversion [2]. Recently it was shown that acoustical parameters of sea bottom can be estimated from a pulse signal recorded by a single hydrophone (see, e.g., [3]). The inversion procedure in this case relies on the dependence of arrival times on frequency and mode number (see details, e.g., in [3, 4]).

In this study, the accuracy of reconstruction of bottom parameters and the sound speed profile in a shallow-water waveguide by a dispersion-based geoacoustic inversion scheme is studied. The depths of the sound speed profile nodes are also considered inversion parameters. The inversion problem was transformed into a problem of black-box minimization of a mismatch function.

Several large-scale computational experiments were held in Acoustics@home and on a computing cluster. In these experiments, the performance of the inversion algorithm with three mismatch functions was investigated under various uncertainties in the information on the media and in the presence of various waveguide inhomogeneities that can potentially affect the accuracy of media parameters reconstruction from the dispersion data. In particular, it is shown that a fitness function that assigns to a point of a dispersion curve the weight associated with the respective spectrogram magnitude in most cases ensures better accuracy of media parameters inversion than a uniform fitness function.

References

[1] D. P. Anderson. BOINC: a system for public-resource computing and storage, *Proceedings of the 5th IEEE/ACM International Workshop on Grid Computing*, IEEE Computer Society, 4–10 (2004).

[2] F. B. Jensen, W. A. Kuperman, M. B. Porter, H. Schmidt, *Computational Ocean Acoustics*, Springer, New York, et al. (2011).

[3] J. Bonnel, N. R. Chapman, Geoacoustic inversion in a dispersive waveguide using warping operators, *The Journal of the Acoustical Society of America*, **130**(2), EL101–EL107 (2011).

[4] P. S. Petrov, A method for single-hydrophone geoacoustic inversion based on the modal group velocities estimation: Application to a waveguide with inhomogeneous bottom relief, *Proceedings of the International Conference Days on Diffraction*, 186–191 (2014).

Electromagnetic waves scattering from infinite periodic arrays of thin absorbing wires

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A subject of electromagnetic waves scattering from periodic structures has been very popular for the last three or more decades in acoustics, water waves, the photonic crystal optics, microwave physics, and other areas of wave propagation. One could fairly mention a study of electromagnetic waves propagating through periodic arrays of spherical scatterers where the spheres were modelled by some combination of electric and magnetic dipoles, see for example [1]. It is worth to mention an another example of guided electromagnetic waves propagating along one-dimensional arrays of dielectric spheres that was studied recently in [2] on the basis of a rigorous analysis of a wave field superposition of vector spherical wave functions. In papers [3] and [4] an excitation of the guided localized electromagnetic waves propagating along 1D finite and infinite arrays of thin metallic parallel wires in a free space and near to flat interface between two dielectric media were studied quite recently. It is apparent that this subject is very important for many applications of metamaterials physics and, in particular, for the metasurfaces.

In the paper a scattering of plane electromagnetic wave from periodic arrays of thin absorbing wires is studied semi-analytically and numerically. Two types of arrays are involved into the presented analysis. For the first type structure we study a few layers of parallel 1D periodic arrays of infinitely long thin absorbing wires with the same period and different spacing in between layers. This structure is located over perfectly conducting metallic (PEC) screen. The scattering is being considered in the case of the long wave approximation, that is, the radius of the wires are much smaller than the wavelength. For the second type the doubly periodic 2D array of parallel thin absorbing wires of finite length is illuminated by a plane electromagnetic wave. This lattice is also located over PEC screen. The length of the wires is assumed to be of the order of the wavelength whereas the radius of the wires are much smaller than the wavelength and their length either (the long wave approximation). For the fist type lattice of infinite thin wires we obtain an approximate analytical solution. For the structure of the second type with the thin wires of finite length the approach based on a well-known numerical analysis of method of Pocklington integral equation was used. In both scattering problems the quasi-periodic wave field is constructed as a superposition of wave fields generated by linear electric currents that satisfies the boundary conditions on the surfaces of the wires. For the second problem we obtain an infinite system of real linear algebraic equations. The presented analysis is based on the accurate and efficient computation of lattice sums that was necessary for computations of the linear electric currents of the thin wires. Both problems is a natural extension of the equivalent electromagnetic problems studied in [3, 4].

For both scattering problems their solutions were tested numerically using propagation power balance criterion exactly for the case when all the thin wires are PEC metallic conductors. In this case all of the incident plane wave power is totally reflected back from the lattices located over PEC screens. However, for the thin absorbing wires a part of the reflected energy transforms into Joule losses. The effect of significant decrease in the reflected energy has been studied for quite wide frequency radio bands. In this paper we report on some examples for both scattering problems with specific geometries of very sharp decrease in the reflected power for a few frequencies of resonance absorbance for both structures with thin absorbing wires located over PEC screens. This effect has various applications, especially in radiophysics. Firstly it is vital in engineering of the radio absorbing surfaces.

References

[1] R. A. Shore, A. D. Yaghjian, Traveling electromagnetic waves on linear periodic arrays of lossless penetrable spheres, *IEICE Trans. Commun.*, E88-B(6), 2346–2352 (2005).

[2] C. M. Linton, V. Zalipaev, I. Thompson, Electromagnetic guided waves on linear arrays of spheres, *Wave Motion*, **50**(1), 29–40 (2013).

[3] V. V. Zalipaev, Electromagnetic localized modes of linear periodic arrays of thin metallic wires, *Proceedings of the International Conference Days on Diffraction*, 311–317 (2018).

[4] V. V. Zalipaev, S. Yu. Kosuinikov, Guided electromagnetic waves for periodic arrays of thin metallic wires near an interface between two dielectric media, *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, **475**(2221), 20180399 (2019).

Features of the bottom seismic wave by the generation of a hydroacoustic source in a condition of the shore zone (3D numerical simulation)

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Three-dimensional numerical simulation of hydroacoustic and bottom surface seismic waves is performed based on application of finite element method. The wave excitation is carried out by a hydroacoustic source installed in the coastal water area, where the horizontal bottom is replaced by an inclined. The modes of pulsed operation and continuous harmonic pulsations of the hydroacoustic source at different depths of the dive and distances from the coastal edge of the water-land are considered. The structure and characteristics of the propagation of excited hydro and seismic surface Rayleigh waves in the case of rock bottom are studied. Wave hodographs are constructed, the analysis of which allows to select wave types. The possibility of detecting a surface bottom wave overlooking the land along the coastal slope with seismic receivers is discussed. A comparative assessment of its level relative to the hydroacoustic wave level is given. The amplitude-wave spatial distribution obtained by modeling is demonstrated in the illustrations.

The symmetric mode of an elastic solid wedge with the opening close to a flat angle

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The existence of a symmetric mode in an elastic solid wedge for all allowable values of the Poisson ratio and arbitrary openings close to π has been proven. A radically new effect — the presence of a wave localized in a vicinity of the edge of a wedge with an opening larger than a flat angle — has been found.

Giant nonlinear Goos–Hänchen effect at the reflection of light from gyrotropic liquid metacrystal

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Highly tunable and reconfigurable liquid metamaterials attract significant attention in recent years [1, 2] because of the possibilities of controlling the optical fields at micro- and nanoscales. In this context, they may become the basis of novel structural elements of photonic devices. A while ago, a new class of liquid metamaterials called liquid metacrystals (LMCs) [3] was suggested. LMCs not only extend and enhance the basic properties of liquid metamaterials, but also bring additional ones, for example, strong nonlinearity [4, 5]. LMCs are composed of micro- or nanoparticles (meta-atoms) with anisotropic polarizability suspended in a viscous liquid or gel. The external dc electric or magnetic field applied to LMC aligns meta-atoms' anisotropy axes along one direction, which makes LMC anisotropic. At the same time, LMC can become anisotropic in the response to high-frequency field, which suggests strong nonlinearity of this metamaterial.

This report is devoted to the study of resonant excitation of electromagnetic surface waves in Otto configuration by an incident beam at the interface between metal and LMC with suspended gyrotropic spherical meta-atoms. The value and sign of LMC gyrotropy can be changed by an external static magnetic field. Furthermore, in the region of evanescent field, electromagnetic radiation possesses the finite angular momentum (so called 'photonic spin'), which induces the additional ordering of magnetic momenta of meta-atoms that, in turn, defines the LMC magnetic nonlinearity. We show that under condition close to the condition of phase synchronism of incident and surface waves, the reflected beam can experience a giant lateral shift relatively to the incident one (Goos–Hänchen effect), which is comparable to the incident beam width. In the nonlinear regime, the value of the shift has bistable character demonstrating the hysteresis behavior in dependence on the incident beam intensity. One should also notice that the value of the reflected beam shift can be tuned in the wide range by means of strength and direction of external dc magnetic field.

References

[1] A. B. Golovin, O. D. Lavrentovich, *Appl. Phys. Lett.*, **95**, 254104 (2009).

[2] M. Liu, K. Fan, W. Padilla, X. Zhang, I. V. Shadrivov, *Adv. Mater.*, **28**, 1553 (2016).

[3] A. A. Zharov, A. A. Zharov, Jr., N. A. Zharova, *J. Opt. Soc. Am. B*, **31**, 559 (2014).

[4] A. A. Zharov, A. A. Zharov, Jr., N. A. Zharova, *Phys. Rev. E*, **90**, 023207 (2014).

[5] A. A. Zharov, A. A. Zharov, Jr., N. A. Zharova, *Phys. Rev. A*, **98**, 013802 (2018).

High-frequency diffraction by a contour with a Hölder discontinuity of curvature

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A high-frequency diffraction of a plane wave by a contour with a Hölder-type discontinuity in otherwise smooth curvature is addressed. The discontinuity assumed to have the form $s^{\lambda}H(s)$, where $H(s)$ is the Heaviside function, $0 < \lambda < 1$ and s is the arc-length. A systematic version of boundary layer approach is applied. The techniques is similar to that developed in [1] for diffraction by a contour having a jump of curvature, which is in a sense a limit case of current problem as $\lambda \to 0$.

According to the Keller geometrical theory of diffraction [2], total wavefield in the illuminated region is a sum of incident, geometrically reflected and diffracted waves; in transition zone, diffracted and reflected waves loose their individuality and wavefield is expressed in terms of the parabolic cylinder function $D_{-3-\lambda}$.

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References

[1] E. A. Zlobina, A. P. Kiselev, High-frequency diffraction by a contour with a jump of curvature, *Proc. Intern. Conf. "Days on Diffraction 2018"*, 325–328 (2018).

[2] J. B. Keller, Geometrical theory of diffraction, *J. Opt. Soc. Amer.*, **52**, 116–130 (1962).

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