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# ON THE SURFACES REPRESENTABLE AS DIFFERENCE OF CONVEX FUNCTIONS

A.D.ALEKSANDROV

Abstract. This is a translation into English of the classical paper of A.D. Aleksandrov (1912–1999) about DC surfaces.

**Keywords:** DC surface, tangent cone, second differential, intrinsic metric.

## 1. Introduction

Under a surface representable as difference of convex functions, or a "DC surface" for short, we understand the surface that is defined in the Cartesian coordinates by the equation $z = f(x, y)$, where $f$ is the difference of convex functions defined on some plane domain:

$$z = f(x, y) = h_1(x, y) - h_2(x, y), \tag{1}$$

with $h_1$ and $h_2$ such that the equations $z = h_1(x, y)$ and $z = h_2(x, y)$ define convex surfaces. We impose no additional regularity conditions on the surfaces under consideration, so that $h_1$ and $h_2$ are only subject to the convexity condition.

The convexity condition means that the straight line segment between two arbitrary points of $z = h_1(x, y)$ $\big($or $z = h_2(x, y)\big)$ lies above this surface (every point on it has the value of $z$ at least that of the point on the surface with the same values of $x$ and $y$).

We can, of course, regard as convex the surfaces lying below the chords. This is utterly immaterial since if $z = h_1(x, y)$ is convex in the first sense then $z = -h_1(x, y)$ is convex in the second sense, and conversely.

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We will understand convexity in the sense of the first definition.

In general, by a DC surface we can understand a surface whose every point has a neighborhood admitting representation in some coordinates as difference of convex functions. But we avoid using this generalized concept, restricting this exposition to the surfaces determined by equations of the form (1).

Assume that the coordinates axes $x$, $y$, and $z$ are fixed once and for all. Therefore, the projection along the $z$-axis and similar operations are assumed defined without specifying the choice of coordinate systems.

The DC surfaces are of interest for the following reasons: Once as I managed to develop the intrinsic geometry of arbitrary convex surfaces (1), the question arose naturally whether it is possible to construct a general theory of the surfaces restricted neither by the convexity requirement nor by regularity requirements in the spirit of classical differential geometry. Together with that, the surfaces to be studied must be regular enough, so that we could count on obtaining for them at least as meaningful a system of concepts and statements as that available now for general convex surfaces. Consequently, we aim at indicating an appropriate sufficiently wide class of surfaces. Of course, in this form the problem is certainly indeterminate in a strict mathematical sense, but we do not mean to pose it as such, rather we only wish to explain the reasons for studying DC surfaces. Namely, it turns out that the class of these surfaces meets our general requirements to a sufficient degree. In the framework of this article we are unable to establish this in a proper volume, but restrict exposition only to some simple propositions that are the starting point for studying DC surfaces deeper. I already presented some of these results, like the general reasons just indicated, in the last section of my *Intrinsic Geometry of Convex Surfaces* [1].

## 2. On the Volume of a Class of DC Surfaces

In this section we verify that the class of DC surfaces includes all twice continuously differentiable surfaces, as well as all polyhedra. In addition, it trivially includes all convex surfaces. Consequently, it covers all surfaces which have so far been studied in detail in geometry.

**Theorem 1.** *Every function whose first partial derivatives satisfy the Cauchy–Lipschitz condition is the difference of convex functions.*

*Namely, if $f(x,y)$ is such that*

$$\begin{aligned} \big|f_x(x+\Delta x, y+\Delta y) - f_x(x,y)\big| &\le M\big(|\Delta x + \Delta y|\big), \\ \big|f_y(x+\Delta x, y+\Delta y) - f_y(x,y)\big| &\le M\big(|\Delta x + \Delta y|\big), \end{aligned} \tag{2}$$

*then the function*

$$g(x,y) = M(x^2+y^2) - f(x,y) \tag{3}$$

*is convex, so that $f(x,y)$ is the difference of the convex functions $M(x^2+y^2)$ and $g(x,y)$.*

Proof. Obviously, it suffices to show that on every line

$$x = a + \alpha s, \quad y = b + \beta s \tag{4}$$

the derivative $\frac{dg}{ds}$ is a monotone nondecreasing function of $s$. Therefore, $g(a+\alpha s, b+\beta s)$ turns out a convex function of $s$; and since the line (4) is arbitrary, this means precisely that the function $g(x,y)$ itself is convex.

Calculate the derivative

$$g' = 2M(\alpha x + \beta y) - (\alpha f_x + \beta f_y).$$

Calculate the difference $\Delta g'$ for $\Delta x = \alpha \Delta s$ and $\Delta y = \beta \Delta s$:

$$\Delta g' = 2M(\alpha^2 + \beta^2)\Delta s - (\alpha \Delta f_x + \beta \Delta f_y). \tag{5}$$

By (2) we have

$$|\Delta f_x| \le M\big(|\Delta x| + |\Delta y|\big) = M\big(|\alpha| + |\beta|\big)|\Delta s|, \tag{6}$$

and similarly for $\Delta f_y$.

Therefore, (5) implies for $\Delta s > 0$ that

$$\frac{\Delta g'}{\Delta s} \ge 2M(\alpha^2 + \beta^2) - M\big(|\alpha| + |\beta|\big)^2 \ge 0. \tag{7}$$

and this means that $g'$ is a nondecreasing function of $s$. The proof of the theorem is complete.

Theorem 1 obviously implies the next theorem.

**Theorem 1a.** *Every twice continuously differentiable surface, and in general every surface with the bounded curvature of normal sections, is a DC surface.*

Thus, the class of DC surfaces includes all surfaces that are studied in the classical differential geometry.

**Theorem 2.** *Every polyhedron is a DC surface. In other words, every piecewise linear function $f(x, y)$ is the difference of two convex andalsopiecewiselinear functions.*

Proof. Consider a polyhedral surface $F$ with an equation $z = f(x, y)$. Assume that the domain of $f$ $\big($the projection of the polyhedron onto the plane $(x, y)\big)$ is convex. This is not restriction of generality since we can, if need be, extend this polyhedron $F$ so that the projection becomes convex. The dihedral angle at every edge opens either upwards or downwards: it includes an infinite part of the semiaxis $z > 0$ or the semiaxis $z < 0$.

A dihedral angle opening upwards is determined by a convex function (in the sense that it lies below the chord). Naturally, a function $v_i$ of this type, determining the dihedral angle $V_i$ of the surface $F$, is defined on the entire plane $(x, y)$, and we assume that its domain is the same as that of the function $f(x, y)$ determining the polyhedron $F$.

Define now the function $g(x, y)$ as the sum of all functions $v_i(x, y)$ corresponding to all angles of $F$ opening upwards:

$$g(x, y) = \sum_i v_i(x, y). \tag{8}$$

This function is convex and piecewise linear as the sum of convex piecewise linear functions $v_i$. In other words, the equation $z = g(x, y)$ determines a convex polyhedron; denote the latter by $G$. This $G$ is in a sense the sum of dihedral angles $V_i$. Each of its edges lies below the corresponding edge of the polyhedron $F$: the projection of every edge of $G$ includes the projection of an edge of $F$ with an angle $V_i$ opening upwards. Conversely, each edge of $F$ with an angle opening upwards lies below the corresponding edge of $G$.

Verify that the difference $h(x, y) = g(x, y) - f(x, y)$ is a convex function as well.

Since $g$ and $f$ are piecewise linear functions, so is $h = g - f$; thus, the equation $z = h(x, y)$ represents a polyhedron which we denote by $H$.

The projection of each edge of $H$ onto the plane $(x, y)$ coincides necessarily with the projection of an edge of one of the polyhedra $F$ and $G$ since everywhere on $F$ and $G$ away from the edges both functions $f$ and $g$ are linear, and so is their difference. Verify that the dihedral angles at all edges of $H$ open upwards. To this end, consider the three possible cases of the location of the projection of an edge $p$ of $H$ onto the plane $(x, y)$:

(1) this projection coincides with the projection of an edge of $F$, but not $G$;

(2) it coincides with the projection of an edge of $G$, but not $F$;

(3) it coincides with the projection of some edges of both $F$ and $G$.

In the first case the dihedral angle at the corresponding edge $q$ of $F$ opens downwards. Indeed, every edge with the angle opening upwards corresponds to an edge of $G$, but in this case that is assumed excluded.

Thus, in a neighborhood of the projection of an edge $p$ of $H$ the function $g$ determining the polyhedron $G$ is linear, which corresponds to a facet of $G$; but the function $f$ determining the polyhedron $F$ is "concave" since the angle at $q$ opens downwards. In result, the difference $h = g - f$ is convex, which means that the angle at $p$ opens upwards.

In the second case an edge of $G$, but not of $F$, corresponds to $p$. Therefore, the function $f$ is linear in a neighborhood of the projection of $p$, while $g$ is everywhere convex. Consequently, in this case $h = g - f$ is convex and the angle at $p$ opens upwards.

In the third case some edges $q$ and $r$ of $F$ and $G$ correspond to $p$. Then by the main property of the polyhedron $G$ the angle $V_k$ at the edge $q$ of $F$ opens upwards, and by (8) the function $v_k(x, y)$ in the equation $z = v(x, y)$ representing this angle appears as a summand in $g(x, y)$. Obviously, $v_k(x, y)$ near $q$ coincides with $f(x, y)$.

Therefore, in a neighborhood of $p$ we have

$$h(x, y) = g(x, y) - f(x, y) = \sum_i v_i(x, y) - v_k(x, y).$$

The right-hand side amounts to the sum of the convex functions $v_i$, with the exception of $v_k$; thus, it is a convex function. Consequently, $h(x, y)$ in a neighborhood of the projection of $p$ is convex, and the angle at the edge opens upwards.

Thus, we have proved that all dihedral angles of the polyhedron $H$ open upwards, so that in a neighborhood of each edge it is convex downwards. It is known that a locally convex polyhedron of this type is globally convex.[1] Therefore, the proof of the theorem is complete.

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[1]This claim holds since we assume that the projection of $F$, and so of $H$ as well, onto the plane $(x, y)$ is convex. Obviously we can assume that it represents a polygon. To prove the convexity of $H$, take the plane $P$ parallel to the plane $(x, y)$ so that $H$ lies entirely below it (i.e., in the domain of smaller $z$). Connecting all points of $H$ to this plane by perpendicular line segments, we obtain a solid polyhedron $K$ bounded below by $H$, above by its projection onto $P$, and on the sides by a prismatic surface. Since all angles of $H$ are convex downwards, all dihedral angles of $K$ are less than $\pi$.

If all dihedral angles of a polyhedral angle are less than $\pi$, then it is convex. Therefore, all vertex figures of $K$ are convex. Hence, it is now easy to conclude that $K$ itself is convex, and so is $H$.

## 3. Derivatives of Differences of Convex Functions and the Corresponding Properties of DC Surfaces

Since the differential of the difference of two functions is the difference of their differentials, many properties of the differentials of convex functions carry over easily to differences of convex functions. In this section we indicate the main properties of the derivatives of differences of convex functions, obtained in this way from the properties of the derivatives of convex functions. These properties translate directly into equivalent geometric properties of DC surfaces.

The contingency of $M$ at a point $A$ is the figure formed by the limits of all possible sequences of rays going from $A$ into a variable point $X$ of $M$ as $X \to A$. If the contingency is a surface, then it is obviously a cone with apex $A$, and then we speak of the tangent cone. Dihedral angles and planes also belong here; if the tangent cone reduces to a plane then it is nothing but the tangent plane.

**Lemma.** *In order for the surface $F$ with an equation $z = f(x, y)$ to have tangent cone at $A(x_0, y_0)$ also representable by an equation of the form $z = g(x, y)$, it is necessary and sufficient that $f(x, y)$ at $(x_0, y_0)$ have the derivative in every direction depending continuously on the direction and, furthermore, the convergence of the ratios of differences to the derivatives be uniform in all directions. The derivative in a given direction is the angular coefficient of the corresponding generator of the tangent cone.*

This lemma is completely obvious from the relation between the derivative and tangent line. The only part of it possibly not so obvious is that the existence of the tangent cone implies uniform convergence of the ratios of differences to the derivatives, or, which is the same, of the angular coefficients of the secant lines to those of the tangent lines generating the cone. Let us verify this claim of the lemma.

Suppose that at $A$ the surface $F$ has tangent cone $K$. Suppose that the convergence of the angular coefficients of secant lines to those of the generators of $K$ is not uniform in all directions. Then there is a sequence of secant lines $AX_n$ such that $X_n \to A$, but the angles between $AX_n$ and the generators $L_n$ of $K$ with the same directions of the projections onto the plane $(x, y)$ remain greater than some $\varepsilon > 0$. Choosing a converging sequence of directions, we may assume that the generators $L_n$ converge. Then their limit is a generator $L$ of $K$. But the limit of the secant lines $AX_n$ is distinct from $L$, and by definition it is also a generator of $K$. This is impossible since by assumption to every direction in the plane $(x, y)$ there corresponds only one generator of the tangent cone $\big($obviously, this means precisely that this cone is represented by an equation of the form $z = g(x, y)\big)$. Thus, we have established our claim.

**Theorem 3.** *Every DC surface has tangent cone at each point which is a DC surface itself.*

Proof. Each convex surface has tangent cone at every point. Using the lemma, we translate this into the language of derivatives of convex functions. For derivatives it is clear that the same result is valid for the differences of convex functions. Again by our lemma, this means that every DC surface has tangent cones. Suppose that $z = g(x, y)$ is an equation of a tangent cone of the surface $z = h_1(x, y) - h_2(x, y)$, where $h_1$ and $h_2$ are convex functions. The function $g$ is the difference of functions

determining respectively the tangent cones to the convex surfaces $z = h_1(x, y)$ and $z = h_2(x, y)$.

Consequently, each tangent cone to a DC surface is itself a DC surface.

Let us point out a series of properties of tangent cones to a DC surface, which follow directly from the corresponding properties of tangent cones to convex surfaces.

**Theorem 4.** *The tangent cones to every DC surface enjoy the following properties:*

(A) *The set of conical points, i.e., those where the tangent cone fails to reduce to a dihedral angle or to a plane, is at most countable. The set of points where the tangent cone reduces to a dihedral angle in the projection onto the plane $(x, y)$ has measure zero; moreover, it lies on an at most countable collection of rectifiable curves. Consequently, every DC surface has tangent planes almost everywhere.*

(B) *If a DC surface has tangent plane $P$ at a point $A$ then the tangent cones at $X \to A$ converge to this plane. The angle formed by the secant line $XY$ passing through arbitrary points $X$ and $Y$ of a surface with tangent plane $P$ tends to zero as $X, Y \to A$; we express this by saying that $P$ is the tangent plane in a strong sense.*

(C) *In every closed domain inside a DC surface the slopes of the tangent lines are bounded. In other words, derivatives in all directions are bounded.*

Verify claim (A). Suppose that $z = g(x, y) - h(x, y)$ is an equation for some surface $F$ and, furthermore, the equations $z = g(x, y)$ and $z = h(x, y)$ represent convex surfaces $G$ and $H$.

Take a conical point $A$ on $F$. Then, since the tangent cone $K_F$ at it is the "difference" of the tangent cones $K_G$ and $K_H$ at the corresponding points of $G$ and $H$, it follows that there are open only the two possibilities for $K_G$ and $K_H$:

(1) at least one of the cones $K_G$ and $K_H$ fails to reduce to a plane or a dihedral angle;

(2) $K_G$ and $K_H$ are dihedral angles with nonparallel edges.

In other cases $K_F$ is inevitably a plane or a dihedral angle.

On a convex surface the set of conical points is at most countable, and so the set of points $A$ for which the first possibility is realized is at most countable.

Suppose that the second possibility is realized. Consider the surface $S$ with equation $z = g(x, y) + h(x, y)$. This $S$ is convex. Its tangent cone $K_S$ at the point corresponding to $A$ is the "sum" of the dihedral angles $K_S$ and $K_H$. Since their edges are not parallel and both angles open upwards, their "sum" is a tetrahedral angle. Consequently, in the second case the point on $S$ corresponding to $A$ is conical. Since $S$ is convex, it follows that the set of these points is at most countable.

Thus, both possibilities can be realized at at most countably many points. Therefore, the set of conical points of a DC surface is at most countable.

The second part of claim (A) follows obviously from the similar properties of convex surfaces. Besides, we should observe that by Theorem 3 claim (A) turns out a corollary to a general theorem on tangent sets (contingencies) by Šmidov and Verčenko [2].

Claims (B) and (C) are also obvious corollaries to the corresponding properties of convex surfaces. For the latter they are established quite simply.

(The first part of claim (B) for convex surfaces is easily deduced from the fact that the limit of supporting planes is a supporting plane. The second part also

follows since if we take the supporting planes at $X$ and $Y$ and verify that when the angle between it and $P$ tends to zero, so does the angle between the chord $XY$ and $P$.)

**Theorem 5.** *The difference of convex functions has second differential almost everywhere: the right (or left) partial derivatives $f_x$ and $f_y$, existing everywhere by Theorem 3, are almost everywhere differentiable; furthermore, wherever this holds, $\frac{\partial f_x}{\partial y} = \frac{\partial f_y}{\partial x}$ and, in addition, the convergence of the ratios of the differences $\frac{\Delta f_x}{\Delta s}$ and $\frac{\Delta f_y}{\Delta s}$ to the derivatives $\frac{df_x}{ds}$ and $\frac{df_y}{ds}$ in every direction is uniform in all directions $\Big($i.e., $\left|\frac{\Delta f_x}{\Delta s} - \frac{df_x}{ds}\right| < \varepsilon$, where $\varepsilon$ depends only on the displacement $\Delta s = \sqrt{\Delta x^2 + \Delta y^2}\Big)$.*

This follows from the similar properties of convex functions $\big($see (3) and (4)$\big)$.

For DC surfaces Theorem 5 means that at almost all points they enjoy all local properties of twice differentiable surfaces (the Dupin indicatrix, Meusnier's, Euler's, Rodriguez's theorems, and so on.)

**Theorem 6.** *If $a(x, y)$ is the difference of convex functions, then $\frac{df(a+\alpha s, b+\beta s)}{ds}$ is always a function of $s$ of bounded variation.*[2] *This means that the section of the surface $z = f(x, y)$ by the plane parallel to the axis $z$ represents a curve with bounded swerve. By the latter we understand the least upper bound of the sums of angles between the (right and left) tangents at the successive points of curve.*

(For a regular curve this is the integral of the curvature, and for a broken line, the sum of complements of its angles to $\pi$. All quantities are taken without sign, only in absolute value).

In a plane section of the surface $z = f(x, y)$ we obtain a curve that is the difference of convex curves. Every convex curve or, more exactly, every convex function of one variable, has monotone derivative. Consequently, for the difference of convex functions the derivative has bounded variation.

The first claim of the theorem is verified; the second just translates it into different terms.

The convex function of one variable is characterized by the fact that the derivative is monotone. The difference of monotone functions is nothing but a function of bounded variation.

Hence, we immediately deduce

**Theorem 7.** *In order for a function of one variable to be the difference of convex functions, it is necessary and sufficient that the function be the integral of a function of bounded variation.*

We are unaware of any similar characterization of the differences of convex functions of two or more variables.

Also, we are unaware of any geometric definition of DC surfaces. In order for a plane curve to be the difference of convex components, it is necessary and sufficient that its swerve (or, if so wished, the variation of the swerve) be finite. We think that something similar should hold for surfaces. Is the property indicated in Theorem 7 characteristic of DC surfaces and, accordingly, differences of convex functions?

[2] In any case, this holds on a closed interval inside the domain of the function, since as the point approaches the boundary, the derivative can become infinite.

Furthermore, if a function $y = f(x)$ is the difference of convex components, then finding its convex components and, moreover, the minimal ones, reduces to finding the derivative and its variation. Indeed, $\frac{1}{2}\big[\mathrm{var} f'(x) + f'(x)\big]$ and $\frac{1}{2}\big[\mathrm{var} f'(x) - f'(x)\big]$ are monotone components of the derivative $f'(x)$, and the integrals of them yield those minimal convex functions whose difference is $f(x)$.

By the minimal convex components of a function $f$ (of arbitrarily many variables) we understand convex functions $g$ and $h$ such that

(1) $f = g - h$,

(2) $g$, $h \leq 0$,

(3) for every convex $g$ and $h$ satisfying the first two conditions, $g \geq g_1$ and $h \geq h_1$.

Here it is convenient to pass to the other understanding of a convex surface (function): as one lying below its every chord. The functions $g' = -g$ and $h' = -h$ are convex in this sense and the definition reduces to the conditions:

(1) $f = h' - g'$,

(2) $g'$, $h' \geq 0$,

(3) if $g'_1$ and $h'_1$ are convex and satisfy (1) and (2) then $g' \leq g'_1$ and $h' \leq h'_1$.

In this case the intuitive meaning of the definitions becomes clearer: the surface $z = g'(x, y)$ $\big($and respectively $z = h'(x, y)\big)$ lies below the plane $(x, y)$ and, together with that, below every surface $z = g'_1(x, y)$.

The existence of minimal components for every function that is the difference of convex functions follows from the next fact.

If two functions $h'_\xi$ are convex in the second sense and nonnegative then the function $h'$ equal for all $x$ and $y$ to the infimum of the values of $h'_\xi$ is also convex. This is just an analytic expression of the fact that the intersection of convex sets is convex.

Thus, the minimal convex components always exist provided that the function is the difference of convex functions. For a function $f$ of one variable these components are the integrals of the monotone components of its derivative $f'$ $\big($the integrals of $\frac{1}{2}[\mathrm{var} f' + f']$ and $\frac{1}{2}[\mathrm{var} f' - f']\big)$.

We can give a different method for constructing these "convex components" of a function $f(x)$. We construct the maximal convex function $g_1(x)$ such that $f(x) \geq g_1(x)$. This function is drawn as the curve subtending the curve $y = f(x)$ as the boundary of the convex hull. Furthermore, we construct the function $g_2(x)$ from $g_1(x) - f(x)$ in the same way as $g_1(x)$ is constructed from $f(x)$. Then we construct $g_3(x)$ in the same way from $g_2(x) - g_1(x) + f(x)$, and so on. The sums of $g_i(x)$ with

odd and even indices yield convex components of the function $f(x)$:[3]

$$f(x) = \sum_{i=1}^{\infty} g_{2i-1}(x) - \sum_{i=1}^{\infty} g_{2i}(x).$$

The convergence of the series $\sum g_i(x)$ at least at one point is a necessary and sufficient condition for $f(x)$ to be the difference of convex functions.

We have no method for finding "convex components" of a function of two variables, not only minimal, but arbitrary components (if it is known that the function is the difference of some convex functions).

Thus, two problems remain open:

1. Give a characterization of the differences of convex functions and DC surfaces.

2. Give a method for finding convex components of a function which would always converge when the function is the difference of convex functions and diverge otherwise.

## 4. Convergence and Approximation of DC Surfaces

For differences of convex functions it is natural to introduce the concept of "strong convergence", understanding by this that $f_n$ strongly converges to $f$ if there exist convex functions $g_n$, $h_n$, $g$, and $h$ such that $f_n = g_n - h_n$, $f = g - h$, and $g_n \to g$, $h_n \to h$. This convergence automatically turns out uniform, since convex functions always converge uniformly.[4]

We consider no convergence other than the strong convergence as defined. Accordingly, we can speak of the strong convergence of DC surfaces.

Since convex surfaces admit approximation by analytic and polyhedral convex surfaces, it follows that we have

**Theorem 8.** *Given a DC surface, there exists a sequence of analytic (polyhedral) DC surfaces converging strongly to it.*

This opens up the possibility of studying a DC surface by passing to the limit from analytic or polyhedral surfaces.

**Theorem 9.** *Consider some surfaces $F_n$ strongly converging to a surface $F$, and take some points $X_n$ on $F_n$ converging to a point $X$ on $F$. If $F$ has tangent plane $P$ at $X$ then the tangent cones of $F_n$ at $X_n$ converge to $P$. But if the tangent cone at $X$ is a dihedral angle $V$ then the limit of every converging sequence of tangent cones at $X_n$ is a dihedral angle with the same edge as $V$ (but it need not coincide with $V$ and can be different for different sequences; it is not excluded, and we should*

[3]This is based on the following remark. If a function $h(x)$ is convex and so is $h(x) - f(x)$ then $h(x) - g_1(x)$ is convex as well. Indeed, wherever $g_1(x) \neq f(x)$, i.e., the curve $g_1$ goes below $f$, the curve $g_1$ reduces to a segment, i.e., there $g_1(x)$ is a linear function, and therefore, $h(x) - g_1(x)$ is certainly convex. Also, wherever $g_1(x) = f(x)$, we have $h(x) - g_1(x) = h(x) - f(x)$, and therefore the latter is convex as well. It is easy to verify that at the points of transition from $g_1 = f$ to $g_1 < f$ the convexity is preserved.

Suppose now that $f(x) = h(x) - k(x)$, where $h$ and $k$ are convex functions and, moreover, the minimal ones. Then $h(x) - g_1(x)$ is convex by the remark just established.

Furthermore, we verify that $h(x) - g_1(x) + f(x)$ is convex and so $h - g_1 + g_2$ is convex as well, and so on.

[4]Convergence of convex functions is equivalent to convergence of the convex surfaces they represent, or, which is the same, of the corresponding convex bodies. Convergence of bounded closed sets consists in their deviation tending to zero, and this precisely means uniform convergence.

*take this more as a rule, that it is a plane). In addition, in every closed domain inside the projection of the surfaces $F$ and $F_n$ onto the plane $(x, y)$ the slopes of the tangent lines are uniformly bounded for all surfaces $F_n$.*

This theorem follows from the fact that converging convex surfaces enjoy these properties; for convex surfaces we can deduce these properties easily since the limit of supporting planes is a supporting plane. In particular, at the points where the tangent cone is a dihedral angle, all supporting planes pass through its edge, and consequently, all limits of supporting planes, and together with them the limits of tangent cones, must pass through the same edge.

In the subsequent theorems we consider not the whole surface $F$, but only a domain on it whose boundary avoids the boundary of $F$ itself. This domain can, of course, cover almost the whole surface with the exception of an arbitrarily narrow strip along the boundary (considering this domain is equivalent to considering a surface which admits an extension in all directions as a DC surface).

Restricting expositions to the above-indicated type of domains, by Theorem 4 (C) and the last part of Theorem 9 we guarantee bounded derivatives and avoid the complications arising as we approach the boundary of the surface since on the boundary it can completely lose its regularity.

**Theorem 10.** *Consider some surfaces $F_n$ strongly converging to a surface $F$. Consider a rectifiable curve $L$ on $F$ (not approaching the boundary of $F$), and some curves $L_n$ on $F_n$ sharing with $L$ the projections onto the plane $(x, y)$. Then $L_n$ are rectifiable curves and their lengths converge to the length of $L$.*

Proof. Take the equations

$$z = f(x, y) = g(x, y) - h(x, y), \quad z = f_n(x, y) = g_n(x, y) - h_n(x, y).$$

for $F$ and $F_n$, with convex functions $g, \dots, h_n$ such that $g_n \to g$ and $h_n \to h$. Denote by $\lambda$ the common projection of $L$ and $L_n$. By hypotheses, $\lambda$ stays away from the boundary of the common projection of the surfaces $F$ and $F_n$. Therefore, at the points lying under $\lambda$ the slopes of the supporting planes of the convex surfaces $z = g_n(x, y)$ and $z = h_n(x, y)$ are bounded (otherwise, in the limit we would obtain a vertical supporting plane inside the limit surfaces $G$ and $H$, which is impossible). Together with the slopes of the supporting planes, the slopes of chords are bounded, i.e., the ratios of the form $\frac{\Delta g_n}{\sqrt{\Delta x^2+y^2}}$, and thus so are the ratios $\frac{\Delta f_n}{\sqrt{\Delta x^2+y^2}}$.

The curve $\lambda$, as the projection of a rectifiable curve $L$, is rectifiable. Denote by $\zeta$ the arc length of $\lambda$. If the equation of $\lambda$ is $x = x(\zeta)$, $y = y(\zeta)$, then take $z = f_n\big(x(\zeta), y(\zeta)\big)$ as an equation for $L_n$. By what we have already proved, the ratios $\frac{\Delta f_n}{\Delta \zeta}$ are bounded on $\lambda$. And then, as it is known, (5) the curve $L_n$ is rectifiable and its length is expressed as

$$s(L_n) = \int \sqrt{1 + \left(\frac{df_n}{d\zeta}\right)^2}\, d\zeta. \tag{9}$$

By analogy for $L$ we have

$$s(L) = \int \sqrt{1 + \left(\frac{df}{d\zeta}\right)^2}\, d\zeta. \tag{10}$$

The derivatives $\frac{df_n}{d\zeta}$ and $\frac{df}{d\zeta}$ exist almost everywhere (in the sense of the measure by the arc length $\zeta$ on $\lambda$), and are clearly bounded since so are the ratios $\frac{\Delta f_n}{\Delta\zeta}$. In addition, $\lambda$, as every rectifiable curve, almost everywhere has tangent line. Consequently, $L_n$ and $L$ have tangent lines almost everywhere, and their slopes are bounded. Since the union of countably many sets of measure zero has measure zero, all curves $L_n$ and $L$ simultaneously have tangents almost everywhere.

According to Theorem 4, the set of conical points of $F_n$ and $F$ is at most countable. Therefore, we can exclude the consideration of all conical points lying on $L_n$ and $L$ either. Moreover, under this condition at every point where $L(L_n)$ has tangent line $F(F_n)$ either has tangent plane or its tangent cone reduces to a dihedral angle and, furthermore, the tangent curve goes along the edge of this angle.

Take the points $X$ and $X_n$ on $L$ and $L_n$ lying over the same point $\xi$ of $\lambda$. The points $X_n$ converge to $X$. Denote by $P$ the vertical plane (parallel to the $z$-axis) passing through the tangent line to $\lambda$ at $\xi$. The tangent lines to $L$ and $L_n$ are the intersections of $P$ with the tangent cones of $F$ and $F_n$. As we pointed out, each of these tangent cones can be either a plane or a dihedral angle. In the latter case its edge lies in $P$.

By Theorem 9, if the tangent plane exists at $X$ then the tangent cones at $X_n$ converge to this plane. Therefore, the tangent lines to $L_n$ at $X_n$ converge to the tangent line to $L$ at $X$.

But if the tangent cone at $X$ is a dihedral angle then its edge $q$ lies in every limit of tangent cones at $X_n$. Together with that, it lies in $P$. Thus, the intersection of the tangent cones at $X_n$ with $P$ converges to $q$. These intersections are precisely the tangent lines to $L_n$.

Consequently, in this case too the tangent lines to $L_n$ at $X_n$ converge to the tangent line to $L$ at $X$.

Thus, we have proved that the tangent lines to $L_n$ converge to the tangent lines to $L$ everywhere but a set of measure zero in the sense of the arc length $\zeta$ on $\lambda$. This means that the derivatives $\frac{df_n}{d\zeta}$ converge to $\frac{df}{d\zeta}$ almost everywhere. In addition, we saw that these derivatives are bounded. Thus, resting on a theorem on the convergence of integrals, we can conclude that the integrals in (9) converge to the integral in (10), and so we have the convergence of lengths: $s(L_n)$ to $s(L)$.

Therefore, the proof of the theorem is complete.

From this theorem we deduce a theorem on the convergence of intrinsic metrics of strongly converging DC surfaces. The intrinsic metric of a surface $F$ is determined as follows. On assuming that every pair of points on $F$ can be connected by a rectifiable curve on $F$, take as the distance $\rho_F(XY)$ between two points $X$ and $Y$ on $F$ the greatest lower bound for the lengths of curves lying on $F$ and connecting $X$ and $Y$. By the intrinsic metric we understand the function $\rho_F$ of pairs of points defined in this way whose values yield intrinsic distances between points on the surface.

**Theorem 11.** *Every DC surface possesses intrinsic metric.*

Take two points $X$ and $Y$ on $F$. Connect their projections onto the plane $(x, y)$ by a broken line. To each segment of the broken line there corresponds an arc of a plane section of $F$. It is rectifiable since by Theorem 7 the swerve of its tangent line is bounded. In result, the points $X$ and $Y$ are connected on $F$ by a rectifiable

curve,[5] and then $\rho_F(XY)$ exists as the greatest lower bound for the lengths of these curves.

**Theorem 12.** *The intrinsic metrics of strongly converging surfaces uniformly converge to the metric of the limit surface. In more detail: Consider some surfaces $F_n$ strongly converging to a $F$; moreover, according to the condition indicated above, the surfaces $F_n$ and $F$ are domains inside some DC surfaces. Take some points $X$ and $Y$ on $F$ and some points $X_n$ and $Y_n$ on $F_n$ with the same projections $\xi$ and $\eta$ onto the plane $(x, y)$. Then we may regard the distances $\rho_{F_n}(X_nY_n)$ and $\rho_F(XY)$ as functions of the pair of points $\xi$ and $\eta$. These functions converge uniformly.*

Proof. Given two points $X$ and $Y$ on $F$, take $X_n$ and $Y_n$ on $F_n$ with the same projections $\xi$ and $\eta$ onto the plane $(x, y)$. By the definition of $\rho_F(XY)$, for every $\varepsilon > 0$ there exists a curve $L$ on $F$ connecting $X$ and $Y$ of length $s(L)$ satisfying

$$s(L) < \rho_F(XY) + \varepsilon. \tag{11}$$

The curves $L_n$ on surfaces $F_n$ with the same projections onto the plane $(x, y)$ are rectifiable by Theorem 10, and

$$s(L) = \lim_{n\to\infty} s(L_n). \tag{12}$$

Together with that, these curves join $X_n$ and $Y_n$, and the definition of the distances $\rho_{F_n}(X_nY_n)$ ensures that

$$s(L_n) \geq \rho_{F_n}(X_nY_n). \tag{13}$$

Comparing (11)–(13), we obtain

$$\rho_{F_n}(X_nY_n) < \rho_F(XY) + \varepsilon,$$

and since $\varepsilon$ is arbitrary, it follows that

$$\limsup_{n\to\infty} \rho_{F_n}(X_nY_n) \leq \rho_F(XY). \tag{14}$$

On the other hand, by the definition of the distances $\rho_{F_n}(X_nY_n)$ on each surface $F_n$ we can indicate a curve $L'_n$ connecting $X_n$ and $Y_n$ and satisfying

$$s(L'_n) < \rho_{F_n}(X_nY_n) + \varepsilon. \tag{15}$$

According to (14), all $\rho_{F'_n}(X_nY_n)$, and so $s(L'_n)$ as well, are uniformly bounded. Therefore, out of the curves $L'_n$ we can choose a converging sequence, and the length of its limit curve $L'$ will satisfy

$$s(L') \leq \liminf_{n\to\infty} s(L'_n). \tag{16}$$

The curve $L'$ joins $X$ and $Y$, and so[6]

$$s(L') \geq \rho_F(XY). \tag{17}$$

Comparing (15)–(17), and keeping in mind that we can take an arbitrarily small $\varepsilon$, we obtain

$$\rho_F(XY) \leq \liminf_{n\to\infty} \rho_{F_n}(X_nY_n). \tag{18}$$

[5]In Theorem 10 we essentially proved the following much stronger claim: a curve on a DC surface possessing a rectifiable projection is rectifiable itself.

[6]Here we use the assumption that $F$ itself is a closed domain inside some DC surface. Otherwise, the curve $L'$, which is the limit of $L'_n$, while getting to the boundary of the surface might be leaving $F$.

Together with (14), this implies that $\lim \rho_{F_n}$ exists, and

$$\rho_F(XY) = \lim_{n\to\infty} \rho_{F_n}(X_nY_n). \tag{19}$$

Thus, the convergence of metrics is established, and it remains to show that this convergence is uniform. But we verify the equicontinuity of metrics, and the equicontinuity of converging functions implies uniform convergence. We have the following statement.

*If for a surface $F$ the slopes of the tangent semiaxes (derivatives in every direction) are at most some $M$ then the distance $\rho_F$ between the points $X(x, y)$ and $X'(x + \Delta x,\, y + \Delta y)$ is at most*

$$\sqrt{1 + M^2} \cdot \sqrt{\Delta x^2 + y^2}.$$

Indeed, suppose that $L$ is a plane section of $F$ connecting the points $X$ and $X'$. Then $\rho_F(XX') \le s(L)$, while (10) yields

$$s(L) = \int \sqrt{1 + \left(\frac{df}{d\zeta}\right)^2}\, d\zeta \le \sqrt{1 + M^2} \cdot \sqrt{\Delta x^2 + \Delta y^2},$$

so that

$$\rho_F(XX') \le \sqrt{1 + M^2} \cdot \sqrt{\Delta x^2 + \Delta y^2}. \tag{20}$$

Together with that, for all points $X$, $Y$, $X'$, and $Y'$ on $F$ we have

$$\left|\rho_F(XY) - \rho_F(X'Y')\right| \le \rho_F(XX') + \rho_F(YY'). \tag{21}$$

Together with (20), this implies that the metric $\rho_F$ satisfies the Cauchy–Lipschitz condition with a constant depending only on the bound $M$ for the slope of tangent lines.

According to Theorem 9, for all surfaces $F_n$ strongly converging to $F$ the slopes of tangent lines are uniformly bounded. Thus, the metrics of these surfaces are equicontinuous, as required.

Remark. From the convergence and equicontinuity of metrics it is easy to deduce the following supplement to Theorem 12. For every $\varepsilon > 0$ there exist $N$ and $\delta > 0$ such that, as soon as $n > N$ and the distances from the points $X_n$ and $Y_n$ on $F_n$ to the points $X$ and $Y$ on $F$ are less than $\delta$, it follows that $\left|\rho_{F_n}(X_nY_n) - \rho_F(XY)\right| < \varepsilon$.

**Theorem 13.** *If some surfaces $F_n$ converge strongly then the absolute integral curvatures (i.e., the integrals of the absolute value of the Gaussian curvature) are jointly bounded for all domains with the same projection $G$ not approaching the boundary of the projection of $F_n$.*

If $z - g_n(x, y) - h_n(x, y)$ are equations for $F_n$ with convex functions $g_n$ and $h_n$ then, as we have already seen in the proof of Theorem 9, the derivatives $\frac{\partial g_n}{\partial x}, \ldots, \frac{\partial h_n}{\partial y}$ are bounded in every domain $G$ not approaching the boundary of the projection of $F_n$.

By this remark, Theorem 13 turns out to be a corollary to

**Theorem 13a.** *Suppose that $z = f(x, y) = g(x, y) - h(x, y)$ is an equation of a DC surface, where the functions $g$ and $h$ are convex and twice differentiable. Then the absolute curvature $\Omega$ of the surface, i.e., the integral of the absolute value of its*

*Gaussian curvature, is bounded and the bound depends only on the bound $M$ for the derivatives of $g$ and $h$.* $\big[$*Namely,* $\Omega < 2\pi(1+8M^2)^{3/2}\big]$.

PROOF. The Gaussian curvature is expressed as

$$K = \frac{f_{xx}f_{yy} - f_{xy}^2}{(1+f_x^2+f_y^2)^{3/2}}. \tag{22}$$

Since $f = g = h$, it follows that

$$f_{xx}f_{yy} - f_{xy}^2 = (g_{xx}g_{yy} - g_{xy}^2) + (h_{xx}h_{yy} - h_{xy}^2) - (g_{xx}h_{yy} + g_{yy}h_{xx} - 2g_{xy}h_{xy}),$$

or

$$|f_{xx}f_{yy} - f_{xy}^2| \le |g_{xx}g_{yy} - g_{xy}^2| + |h_{xx}h_{yy} - h_{xy}^2| + |g_{xx}h_{yy} + g_{yy}h_{xx} - 2g_{xy}h_{xy}|. \tag{23}$$

But since $g$ and $h$ are convex functions, their second differentials are nonnegative quadratic forms, and this is known to imply that all three quantities whose absolute values appear here on the right are themselves nonnegative.[7] Therefore, by simple transformations we can replace (23) with

$$|f_{xx}f_{yy} - f_{xy}^2| \le (g_{xx}+h_{xx})(g_{yy}+h_{yy}) - (g_{xy}+h_{xy})^2,$$

or putting $g + h = k$, with

$$|f_{xx}f_{yy} - f_{xy}^2| \le k_{xx}k_{yy} - k_{xy}^2. \tag{24}$$

The integral of the absolute value of the Gaussian curvature is

$$\Omega = \iint |K|\, ds = \iint \frac{|f_{xx}f_{yy} - f_{xy}^2|}{(1+f_x^2+f_y^2)^{3/2}}\, dx\, dy \le \iint (k_{xx}k_{yy} - k_{xy}^2)\, dx\, dy. \tag{25}$$

Since $k = g + h$ and $|g_x|, \dots, |h_y| < M$, it follows that

$$|k_x|, |k_y| < 2M, \quad (1 + k_x^2 + k_y^2) < 1 + 8M^2.$$

Therefore,

$$\iint (k_{xx}k_{yy} - k_{xy}^2)\, dx\, dy < (1+8M^2)^{3/2} \iint \frac{k_{xx}k_{yy} - k_{xy}^2}{(1+k_x^2+k_y^2)}\, dx\, dy. \tag{26}$$

The integral on the right-hand side is nothing but the integral curvature of the surface with the equation $z = k(x, y)$, which is convex since $k$ is the sum of the convex functions $g$ and $h$.

The spherical image of this surface is included into on a hemisphere since it is uniquely projected onto the plane $(x, y)$. Consequently, its integral curvature is at most $2\pi$. Thus, estimating the integral on the right-hand side of (26), and combining (26) with (25), we obtain

$$\Omega \le 2\pi(1+8M^2)^{3/2}.$$

This completes the proof of Theorem 13a and, therefore, Theorem 13 as well.

Theorems 12 and 13 provide a foundation for studying some deeper properties of DC surfaces.

Namely, resting on Theorems 12 and 13, we can apply to DC surfaces the general theory of intrinsic geometry of surfaces or "manifolds of bounded curvature" whose foundations appeared in my article "Foundations of intrinsic geometry of

[7]For the first terms this is obvious. But if by a linear transformation of the variables $x$ and $y$ we obtain $h_{xy} = 0$ at this point, then the third quantity reduces to $g_{xx}h_{yy} + g_{yy}h_{xx}$, which is nonnegative since $g_{xx}, \dots, h_{yy} \ge 0$.

surfaces" [6] and even earlier were sketched in the last section of my book [1]. Thus, Theorems 12 and 13 open up a path to thoroughly studying the intrinsic geometry of DC surfaces. Here belong, apart from the main results of [6], some results of the notes [7, 8], as well as many others so far unpublished. In addition, we obtain the possibility of studying the relation of the intrinsic geometry of a DC surface to its "extrinsic" geometry, i.e., the properties of itself and figures on it as figures in space.

Theorem 14, proved in the next section, is a simple example from this area.

## 5. On the Tangent Cone

We defined the tangent cone of a surface $F$ at $A$ as the cone formed by the limits of secant semiaxes emanating from $A$. It is known that this definition is equivalent to the following: the tangent cone of a surface $F$ at $A$ is the limit of surfaces obtained from $F$ as dilations centered at $A$ as the coefficient of dilation increases indefinitely.[8]

This view of the tangent cone turns out more advantageous in a series of questions. Using it and resting on Theorem 12 on the convergence of metrics, we verify that a DC surface is "infinitesimally isometric to its tangent cone".

By this we mean, more exactly, the following result:

**Theorem 14.** *Consider a DC surface $F$ and its tangent cone $K$ at some point $A$. Project $F$ onto $K$ in the direction of the $z$-axis. Given $\varepsilon > 0$ there is $\delta > 0$ such that, as soon as the distance from two points $X$ and $Y$ on $F$ from $A$ is less than $\delta$, it follows that*

$$\big|\rho_F(XY) - \rho_K(X'Y')\big| < \varepsilon \max\big[\rho_F(AX), \rho_F(AY)\big], \tag{27}$$

*where $\rho_f$ and $\rho_K$ are the distances measured on $F$ and $K$, while $X'$ and $Y'$ are the projections of $X$ and $Y$ onto the cone $K$.*

Speaking intuitively, (27) means that the projection under consideration in an infinitely small neighborhood of $A$ is an isometric mapping; according to (27), the difference of the distances on $F$ and $K$ is infinitely small relative to the distances to $A$.

Proof[9]. Subject the surface $F$ to a dilation centered at $A$. Denote the coefficient of dilation by $\lambda$. Denote the resulting surfaces by $\lambda F$, and the points obtained from some point $X$, by $\lambda X$.

Take the circle of radius 1 centered at the projection of $A$ onto the plane $(x, y)$ and consider only the parts of surfaces lying under this circle. Project the surfaces $\lambda F$ onto $K$ along the $z$-axis.

As $\lambda \to \infty$, the surfaces $\lambda F$ converge to the tangent cone $K$ at $A$. Therefore, by Theorem 12 for every $\varepsilon > 0$ there is $\lambda_0$ such that, as soon as $\lambda > \lambda_0$, for all points $B$ and $C$ on $\lambda F$ and the corresponding projections of the points $B'$ and $C'$ on $K$ we have

$$\big|\rho_{\lambda F}(BC) - \rho_K(B'C')\big| < \varepsilon. \tag{28}$$

Now take the points $X$ and $Y$ on $F$ going into $B$ and $C$:

$$B = \lambda X, \quad C = \lambda Y. \tag{29}$$

[8]The equivalence of both definitions can be established in an obvious fashion.

[9]This proof verbatim repeats the proof of the same theorem for convex surfaces in [1].

Under the dilation centered at $A$ the cone $K$ goes into itself, while the projections of $X$ and $Y$, into the projections of $A$ and $B$:

$$B' = \lambda X', \quad C' = \lambda Y'. \tag{30}$$

The dilation under consideration increases all distances by a factor of $\lambda$, and so the distances on $\lambda F$ and $K$ satisfy

$$\rho_{\lambda F}(BC) = \lambda_{\rho_F}(XY), \quad \rho_K(B'C') = \lambda_{\rho_K}(X'Y'). \tag{31}$$

Inserting this into (28), we obtain

$$\big|\rho_F(XY) - \rho_K(X'Y')\big| < \frac{\varepsilon}{\lambda}. \tag{32}$$

Take $X$ and $Y$ so close to $A$ that

$$\max\big[\rho_F(AX), \rho_F(AY)\big] < \frac{1}{\lambda_0} \tag{33}$$

and put

$$\max\big[\rho_F(AX), \rho_F(AY)\big] = \frac{1}{\lambda}, \tag{34}$$

so that $\lambda > \lambda_0$.

Since dilation with coefficient $\lambda$ increases distances by a factor of $\lambda$, for $B$ and $C$ this means that

$$\max\big[\rho_{\lambda F}(AB), \rho_{\lambda F}(AC)\big] = 1.$$

Therefore, $B$ and $C$ remain in the domain under consideration (lie inside the circle of unit radius), and so our conclusion applies to them: for $X$ and $Y$ we have (32) with $\lambda$ defined in (34). Thus,

$$\big|\rho_F(XY) - \rho_K(X'Y')\big| < \varepsilon \max\big(\rho_F(AX), \rho_F(AY)\big). \tag{35}$$

This holds for all points satisfying (33). Therefore, once we only put $\frac{1}{\lambda_0} = \delta$, (33) becomes equivalent to

$$\max\big[\rho_F(AX), \rho_F(AY)\big] < \delta. \tag{36}$$

Consequently, if (36) holds then so does (35), and this is precisely the claim of the theorem, which is therefore established.

In closing, note without proof some important results established by Theorem 14:

(I) Every geodesic (the shortest curve on every sufficiently small segment) on a DC surface at every point has right and left tangent lines; at every point where the surface has tangent plane, the geodesics have ordinary tangent lines (the right and left tangent lines coincide).

(II) The angle between the geodesics emanating from a point $A$, defined intrinsically,[10] equals the angle between their tangent lines measured on the tangent cone at $A$.

(III) In order for a curve on a DC surface to have direction at the initial point $A$, it is necessary and sufficient that it have tangent line at $A$. The angle between two curves emanating from $A$, defined intrinsically, equals the angle between their tangent lines measured on the tangent cone.

The proofs of these statements rest on quite deep conclusions about the intrinsic geometry of DC surfaces.

[10]For convex surfaces, I gave [1] definitions of an angle between curves and the direction of a curve avoiding the differentiability assumption. These definitions carry over verbatim to DC surfaces, as indicated in [7].

In the case of convex surfaces, their proofs appeared in [1]. Liberman originally proved [9] the first of them in full generality by a beautiful geometric argument which, however, rests crucially on the convexity of surfaces under consideration, and so fails to carry over to other types of surfaces. Our argument is quite different, and so it yields a new proof of Liberman's theorem for geodesics on convex surfaces.

In fact, we directly prove the general claim (III), of which (I) and (II) are only particular cases.

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# ALEXANDROV OF ANCIENT HELLAS

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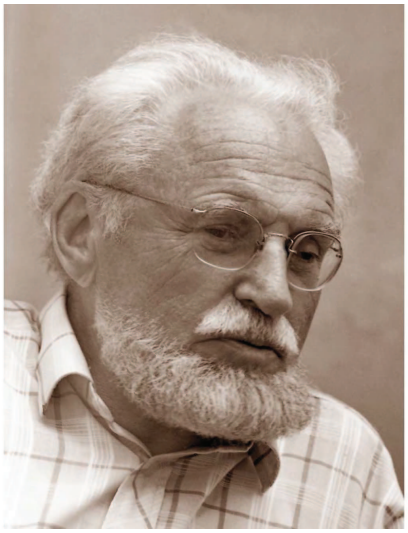
Abstract. This is a short overview of the life and contribution of Aleksandr Danilovich Alexandrov (1912–1999). Most attention is paid to his general outlook and ethical principles.



The photo by V. T. Novikov.

Kutateladze, S.S., Alexandrov of Ancient Hellas.



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**Life's Signposts.** Aleksandr Danilovich Alexandrov was born in the Volyn village of the Ryazan province on August 4, 1912. His parents were high school teachers. He entered the Physics Faculty of Leningrad State University in 1929 and graduated in 1933. His supervisors were Boris Delauney (1890–1980), a prominent geometer and algebraist, and Vladimir Fok (1898–1974), one of the outstanding theoretical physicists of the last century. The first articles by Alexandrov dealt with some problems of theoretical physics and mathematics. But geometry soon became his main speciality.

Alexandrov defended his PhD thesis in 1935 and his second doctorate thesis in 1937. He was elected to a vacancy of corresponding member of the Academy of Sciences of the USSR in 1946 and was promoted to full membership in 1964.

From 1952 to 1964 Alexandrov was Rector of Leningrad State University. These years he actively and effectively supported the struggle of biologists with lysenkoism. Genetics had been in the syllabus of LSU in the 1950s whereas this happened in the other domestic universities only in 1965. The name of Rector Alexandrov is connected with the uprise of the new areas of science such as sociology and mathematical economics which he backed up in the grim years. Alexandrov was greatly respected by established scholars as well as academic youth. "He led the University by moral authority rather than the force of direct order," so wrote Vladimir Smirnov (1887–1974) in the letter of commendation on the occasion of Alexandrov's retirement from the position of Rector.

In 1964 Mikhail Lavrentyev (1900–1980) invited Alexandrov to join the Siberian Division of the Academy of Sciences of the USSR. Alexandrov moved with his family to Novosibirsk where he found many faithful friends and students. By 1986 he headed a department of the Institute of Mathematics (now, the Sobolev Institute), lectured in Novosibirsk State University, and wrote new versions of geometry textbooks at the secondary school level. Alexandrov opened his soul and heart to Siberia, but was infected with tick-borne encephalitis which undermined his health seriously. From April of 1986 up to his death on July 27,1999, Alexandrov was on the staff of St. Petersburg Department of the Steklov Mathematical Institute.

**Contribution to Science.** Alexandrov's life business was geometry. The works of Alexandrov made tremendous progress in the theory of mixed volumes of convex figures. He proved some fundamental theorems on convex polyhedra that are celebrated alongside the theorems of Euler and Minkowski. While discovering a solution of the Weyl problem, Alexandrov suggested a new synthetic method for proving the theorems of existence. The results of this research ranked the name of Alexandrov alongside the names of Euclid and Cauchy.

Another outstanding contribution of Alexandrov to science is the creation of the intrinsic geometry of irregular surfaces. He suggested his amazingly visual and powerful method of cutting and gluing. This method enabled him to solve many extremal problems of the theory of manifolds of bounded curvature.

Alexandrov developed the theory of metric spaces with one-sided constraints on curvature. This gave rise to the class of metric spaces generalizing the Riemann spaces in the sense that these spaces are furnished with some curvature, the basic concept of Riemannian geometry. The research of Alexandrov into the theory of manifolds with bounded curvature prolongates and continues the traditions of Gauss, Lobachevsky, Poincaré, and Cartan.

The Mathematics Subject Classification, produced jointly by the editorial staffs of Mathematical Reviews and Zentralblatt für Mathematik in 2010, has Section 53C45 "Global surface theory (convex surfaces à la A. D. Aleksandrov)". None of the other Russian geometers, Lobachevsky inclusively, has this type of acknowledgement. Alexandrov became the fist and foremost Russian geometer of the twentieth century.

**Sources of Geometry.** It is impossible to grasp Alexandrov's outlook without turning to the roots of his cherished science. He wrote in 1981 that "the pathos of contemporary mathematics is the return to Ancient Hellas." His favorite slogan was "Retreat to Euclid!"

Geometry is part of the culture of the ancient world. The traces of any epoch transpire in its most abstract conceptions. It is impossible to grasp the elementary basics of nanotechnology of quantum logic out of the the modern cultural tradition. The hints of time are reflected in evolution of an arbitrary scientific system. Geometry was invented to meet various human needs. Its mystic, explorative, and economic sources coexisted in the common cultural environment of the man of the pre-Bible times. The strongest quest of geometry stemmed from the cadastral surveying aimed at regular taxation. The famous harpedonaptae of Egypt were tax agents who used ropes for measuring the tracts of land. The tricks and techniques of harpedonaptae were used in construction. Pyramids were erected long before the abstract definition of the geometrical form of a pyramid.

Bewildering is the history of the abstract geometric concepts of point, monad, figure, and solid which came from the remote ages. We are rarely aware of the fact the secondary school arithmetic and geometry are the finest gems of the intellectual legacy of our forefathers.

There is no literate who fails to recognize a triangle. However, just a few know an appropriate formal definition. This is not by chance at all, since the definition of triangle is absent in the Elements. Euclid spoke about three-lateral figures, emphasizing that "a figure is that which is contained by any boundary or boundaries." Clearly, his definitions remind us of the technology of cadastral surveying of his times. It is worth observing that the institution of property is much older than the art and science of geometry. To measure a tract of land from outside is legitimate whereas trespassing the borders is forbidden. The ancient rope stretchers had similar restrictions for measuring the constructions like pyramids. Clearly, the surveyors of the Kheops pyramid would mum every single word about the interiors of this building.

In the modern parlance, we say that Euclid considered convex figures and solid bodies. The concept of convexity seems quite elementary today. Some part of a plane or space is called convex provided that no straight line segment between any two points of this part lies within the object under consideration. If we drive three stakes in a tract of land and stretch a lasso whose loop surrounds the stakes, we will single out a triangle. The harpedonaptae did exactly the same, but the interior of the tract to be measured might be inaccessible to the surveyors without permission of the owner. Nowadays we also measure property and levy taxies but any unauthorized attempt to stretch a rope within somebody's property is still a felony of trespassing on land. The definitions of Euclid are listed among the immortal witnesses of the ancient economic relations.

**Geometry as a Basis of Science.** Geometry deals with the quantitative and qualitative properties of spatial forms and relations. The criteria for equality of triangles provide instances of qualitative geometric knowledge. Finding lengths, areas, and volumes exemplifies quantitative research.

The abstraction of a straight line in geometry can be attributed to intuitive perceptions. Any straight line is a "length without breadth" perceived as a whole. There are points on every straight line, and the straight line is complete, which is not postulated as obvious without much fuss or circumlocution. The reals of the ancients appeared as processes rather than completed figments of intuition. Each real is either a completed process of combining units/monads or an incomplete process of measuring noncommensurate quantities.

Science has confronted the problem of counting the continuum since remote ages. The incommensurability of the side and diagonal of a square became an outstanding discovery of Euclidean geometry.

When our ancestors had demonstrated the absence of any common measure of the side and diagonal of a square, they understood that rational numbers are scarce for practical purposes. It is worth recalling that the set of rational numbers is equipollent with the collection of natural numbers. This means that all rational numbers comprise a countable set, thus serving as an instance of the cardinal number that we use to express the size of the imaginary collection of all entries of the natural series. The most ancient idea of the potential infinity in the form of consecutive counting turned out insufficient for quantitative analysis in geometry.

The straight line segment has decomposed in points within the convergence theory of Fourier series. To measure parts of the segment with transfinite numbers is the problem of the continuum in the same sense in which the ancient tried to commensurate the diagonal and side of a square. The discovery that the side and diagonal of a square are incommensurable is the height of mathematics as awesome and ethereal as the independence of the fifth postulate, the axiom of choice, and the continuum hypothesis.

The incompleteness of the rationals led to no inconvenience prior to geometry. Humans had no inborn conceptions of the reals. Insufficiency of the rationals was revealed only in the practice of measurement. Geometry in the times of its onset was directly tied with the need of tax levying and cadastral surveying. The mathematics of harpedonaptae must possess the power of law. The requirement of standardized reports and universal measurement, rather than whatever a priori ideas, led to the construction of a complete collection of reals. The mathematical intuition of the ancients was based on the conception of a straight line segment as a judicially correct definition of a rope stretching taut between two stakes to be used as an etalon for measuring. Measure theory stems from geometry, the latter originated with the judicial procedures that required the extreme definiteness and unicity of application. The logic of Aristotle followed geometry and reflected the methodology of geometry.

**Retreat to Euclid.** Alexandrov accomplished the turnround to the ancient synthetic geometry in a much deeper and subtler sense than it is generally acknowledged today. The matter is not simply in transition from smooth local geometry to geometry in the large without diffentiability restrictions. In fact Alexandrov enriched the methods of differential geometry by the tools of functional analysis and measure theory, driving mathematics to its universal status of the

epoch of Euclid. The mathematics of the ancients was geometry (there were no other instances of mathematics at all). Synthesizing geometry with the remaining areas of the today's mathematics, Alexandrov climbed to the antique ideal of the universal science incarnated in mathematics.

Alexandrov overcome many local obstacles and shortcomings of the differential geometry based on the infinitesimal methods and ideas by Newton, Leibniz, and Gauss. Moreover, he enriched geometry with the technique of functional analysis, measure theory, and partial differential equations. Return to the synthetic methods of *mathesis universalis* was inevitable and unavoidable as illustrated in geometry with the beautiful results of the students and descendants of Alexandrov like Misha Gromov, Grisha Perelman, Alexei Pogorelov (1919–2002), and Yuri Reshetnyak.

**Geometry and Alexandrov's Outlook.** Geometry appeared as a result of human activities. Geometry was invented to organize human's life and change it for the better. Human is the starting point, the creator, and the aim of life. The general outlook of Alexandrov was determined by his scientific views that were formed in studying geometry. It is not by chance that the ideas of Karl Marx's Theses on Feuerbach enchanted Alexandrov.

Alexandrov was not a man of the past, but he was not ashamed of the past. He was able to discern his own misconceptions and eliminate them. He never concealed his own mistakes but tried his best to repair them if possible. He was interested in what they had done rather then what they had been doing. He never made a vain boast and always hated meritocratism. His attitude to truth was dynamic and based on principle.

Everyone trusts themselves, whatever cirecumlo9cution notwithstanding. Alexandrov was capable of extending the practice of trust to the others, using the presumption of decency which acts up to the first infringement. Alexandrov himself was a man of honor whose statements deserved acceptance without proof in much the same way as one's own words. Alexandrov put trust higher that proof.

**Alexandrov's Ethics.** Synthesizing geometry with the other areas of mathematics, Alexandrov elevated to the antique ideals of the unique science and placed the scientific stance in the center of his ethical views.

Alexandrov's contemplations about morality are connected with opposing religious belief and scientific search. The genuine human with the earthly needs rather than an ideal abstraction occupies the center of Alexandrov's outlook. It is the human seeking for truth and creating the circumstances of life. The human who is the source and the aim of life, Alexandrov emphasized the openness of science as well as its principle refutation of all forms of dogmatism and subjectivism innate to belief.

Alexandrov hated all crooks, "marxism-borne" popes and inquisitors who used science for mean and greedy ends. There is a precipice of repulsion between science and power. Power confronts freedom which is the essence of mathematics. Alexandrov viewed science as the tool that liberates humans from material burdens and untether them intellectually. Geometry taught Alexandrov universal humanism. He liked the words of Paul the Apostle and repeated that "there is neither Greek, nor Jew" in geometry. Humanism, responsibility, and scientific stance are the ingredients of the perfect morality by Alexandrov. Human is the source and aim of everything. That is the essence of universal humanism. Human is responsible for everything. That is the meaning of responsibility. The scientific stance as human's statement

free of subjectivism is that which makes the foundation of morality. Alexandrov's staunch principles made predictable and tragic his fate. The defense of truth is a heavy cross and a lonely service. Alexandrov often felt himself "red carpet clown." Misunderstanding and mockery are the rewards of an alive hero. Time shows all in due proportion. Alexandrov will remain in history as a noble knight of science.

**Alexandrov and the Present Day.** Alexandrov emphasized the criticism of science and its never-failing loyalty to truth. Science explains "how the thingummy's actually going on" with greatness and modesty, using experience, facts, and logic. The love and hatred to Alexandrov stem from the same sources. His reviews and opinions were welcome and appreciated, but his approaches and areas of research were silenced if not scorned. He was accused of zionism, but many bet and counted upon his antisemitism. His communistic beliefs were blasphemed obscenely, but he was humbly requested to write a letter or two to the Central Committee of the Communist Party of the USSR or the Party journal *The Communist*. His philosophical essays were spit upon furiously, but the same despisers required that their students used Alexandrov's popular writings at the final examinations in philosophy which were obligatory for admittance to the public maintenance of theses. The professorship of St. Petersburg is full of raptures about the palace, fountain, and park ensemble of Peterhof, but most of Alexandrov's colleagues will never forgive the sage decision of Rector Alexandrov who suggested to build a new university campus in Peterhof. During the years of Gorbi's *perestroika* Alexandrov was accused in confessing lysenkoism but decorated with the Order of the Red Banner of Labor for his efforts in safeguarding and propelling genetics and selection in the USSR. So were the scales of Alexandrov's personality.

Alexandrov's life spanned the rise and fall of the Soviet Union. Complicated if not paradoxical ideology of communism views the individual freedom as necessity understood within a collective. Collectivism tends to transform into the hegemony of standardization and totalitarianism in much the same way as individualism brings about the tyranny of absolutism and globalization. Dictatorship, as the simplest form of universal subordination, becomes the inevitable instrument of individualism and collectivism. Collectivism reveals itself as altruism in morality, generating mysticism in the realm of reasoning. The creed of individualism is egoism and rationality. Alexandrov's ideas oppose rational egoism, abstract objectivism, and mystical dogmatism. Humanization of science as the vector of its progress is the most attractive ingredient of Alexandrov's views of the future of science and society.

The universal humanism of the geometer Alexandrov, stemming from the heroes of antiquity, will always remain in the treasure-trove of the best memes of the humankind.

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# MEMORIES OF ALEXANDR DANILOVICH ALEXANDROV

YU. E. BOROVSKY

Abstract. This is a tribute to Alexandr D. Alexandrov on the occasion of the centenary of his birth.

Presented here are excerpts of my memories about Alexandrov. My experience shows that if I decide to do anything completely, I will never do it at all.

Everyone who knew Alexandrov was well familiar with his hospitality. His Leningrad home at the Field of Mars was a permanent meeting place for our entire seminar. The tradition continued in Akademgorodok, although our get-togethers in Alexandrov's house included fewer people.

Alexandrov was for me what sociologists call a "reference group": thinking about my possible behavior, I always considered how Alexandr Danilovich would look at it. At his seminars, participants discussed not only a variety of mathematical problems, but also physics, philosophy and moral issues. I do not know whether my ideas about physics have evolved as a result of those seminars, or if they initially had beep similar to the ones of Alexandrov. Nevertheless, the fact that I am as much an expert in the *general theory of relativity* as professionals working in that field, is the result of Alexandrov's seminars. As for his lectures in *differential geometry*, his concept of *infinitesimally close points* was extremely important, because these points are used in various fields of mathematics since the time of Isaac Newton. Scientists employ them in mathematical analysis, differential geometry, algebra, and algebraic geometry in both classical and modern versions (Grothendieck's schemes). The same is true about nonstandard analysis, with its idea of approximation of

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Borovsky, Yu. E, Memories of Alexandr Danilovich Alexandrov.



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arbitrary objects with those finite (which is the underlying idea of Alexandrov's work about approximation of surfaces).

Alexandrov was the only person in my life with whom I could discuss everything. He knew a lot and had his own view of any subject. This is true for diverse subjects as history of mathematics, physics and biology. It was from Alexandrov that I learned about works on time circuits.

Unlike many scientists, Alexandrov never "stole" other people's work; rather the contrary. If a joint article included his name, it meant that the main idea and results had been his. But even in that case his name might not be mentioned among the authors.

Now, I'd like to describe an episode. A year after the death of Nikolai Vladimirovich Efimov (1910–1982), the Moscow Mathematical Society held a seminar in his honor. Most of the leading Soviet mathematicians and many of their foreign colleagues were present. One of the presentations was the joint work by A. D. Alexandrov, V. N. Berestovsky and I. G. Nikolaev "Generalized Riemannian spaces."[1] Alexandrov walked up to the podium, and began his speech with the words: "Alexandrov is a history here; the authors of the main results are Nikolaev and Berestovsky. That's why the presentation will be done by Nikolaev." The core of the joint work was *manifolds of bounded curvature*, the branch of geometry which was originated by Alexandrov in the 1930s. He received Stalin's State Award for that discovery.[2] The joint work under discussion at the seminar was based on the results of that theory. The findings of Igor Georgievich Nikolaev (who, by the way, had worked under Alexandrov's advice) were significant, but not nearly as important as those by Alexandrov. Later I asked Alexandrov why he had suggested that Nikolaev make the presentation. His answer was as follows: "Nikolaev is not appreciated enough in the Institute of Mathematics; I wanted to support him."

On my third or fourth year at the university, I participated in the seminar of Dmitriĭ Konstantinovich Faddeev (1907–1989) which was devoted to representation of Fëdorov groups. In connection with that seminar, I read the book by A. D. Alexandrov, B. N. Delone and N. N. Padurov "Mathematical Foundations of the Structural Analysis of Crystals and Determination of the Basic Repetition Parallelepiped by means of Röntgen Rays." There was one section in the book that I could not understand. So I approached Alexandrov after his lecture on differential geometry and asked about it. Alexandrov looked like a person who had never heard of that problem. "Where did you get that?" he asked me. "From your own book." "From which book?" I named the title. "Ah, I have forgotten about that already," he replied. The book was published in 1934.

Although I graduated from Leningrad University as a straight A student, one of the best in mathematics and the best in social and political disciplines, I was not admitted to the postgraduate program. I knew that it was at least partially due to a slander told about me by a Komsomol activist, Pavilainen by name, the man whom I had never met in person. A year later, when I was vacationing in a sanatorium in Komarovo (a suburb of Leningrad), Alexandrov walked into the cafeteria during lunch. Busy with his duties of Rector of Leningrad State University, he had made a special trip to Komarovo to invite me to join the doctoral program under his supervision.

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[1]Later this talk was published in *Russian Math. Surveys*, Vol. 41 (1986) No. 3, pp. 3–44.

[2]Editor's note: In 1941.

There was a predicament concerning my first (i. e. Kandidat) thesis. It contained a small lemma which was so easy for me that later I completely forgot about it, and never referred to it. What I did not understand then, was that without that lemma, my entire thesis looked as if it had a false ground. One of my opponents was Olga Alexandrovna Ladyzhenskaya (1922–2004), whom everyone justly considered a genius. She could determine the value of the results as well how justified they were, by just glancing at a thesis, and so she did not need to read the thesis thoroughly. Naturally, Ladyzhenskaya decided that my thesis had a false ground, and she did not conceal her opinion. Ladyzhenskaya was famous not only for her outstanding research in mathematics, but also for grasping the issue immediately and never making mistakes, and Alexandrov respected her greatly for that. On the other hand, he appreciated me as a mathematician. He convened a seminar, and invited Ladyzhenskaya. I made a long and detailed presentation of my thesis. Ladyzhenskaaya listened silently, until I reached that unfortunate place. "And why is that?" she asked. That is the exact moment when I finally remembered that I had a lemma on that subject, so I opened my thesis and read it. That was enough for Ladyzhenskaya to change her opinion of me to the opposite, and she always expressed that opinion to other people. Thanks to her references, Sergeĭ L′vovich Sobolev (1908–1989) offered me a job in his Institute of Mathematics in Novosibirsk.

I have always respected Ladyzhenskaya, so I'd like to tell another story about her. Alexeĭ Mikhaĭlovich Vinogradov was proving his second (Doctor) thesis. He was a topologist and a student of Sergeĭ Petrovich Novikov, and his thesis was based on the most sophisticated tools and methods of algebraic topology—which in my opinion is the most difficult field of mathematics after algebraic geometry. Ladyzhenskaya was the only opponent who was not a topologist, and she was the only one who did not show up in person, but had mailed her review of the thesis: she always refused to travel by air. After all opponents made their presentations and Ladyzhenskaya's one was read aloud, Vinogradov approached me and said: "Ladyzhenskaya understood my thesis better than anyone else."

Alexandrov often visited our home in Akademgorodok. Once he dropped in when my wife was out of town, and I offered him a piece of salted ham that I served wrapped in paper, just the way it had been kept in the refrigerator. Alexandrov reacted to such a catering with the following phrase: "The fact that you have dared to serve me, an academician, pig's fat on a dirty paper, shows that you do not consider me a rascal to whom one needs to cringe."

Alexandrov wrote articles against the death penalty. During one of his birthday celebrations, many people from different Soviet republics (I had never met about half of them before) gathered in his one-family house in Akademgorodok. One of the topics at the party was the death penalty. I expressed an opinion that the death penalty should not be abolished. Alexandrov got angry: "You are a rascal, aren't you? They execute people for currency violations!" He then named other articles of the Criminal Code which are punishable by death. I replied that, in my opinion, people should not be executed for currency violations, but only for common crimes. "What do you mean by *common crimes*?" asked Alexandrov calmly. "Capital crimes against a person," I responded. Alexandrov said nothing to that.

When the VAK (the Supreme Attestation Committee of the USSR) had procrastinated approval of my Doctor thesis for three years, Alexandrov and

Sobolev wrote to the VAK, requesting it to expedite the decision. After this letter, I was summoned to the VAK, and six weeks later my thesis approved finally.

Alexandrov had the prestigious sports title "Master of Sports of the USSR" in mountaineering which was not just a sign of honor but awarded by the government only for some recoded high achievements in sports. If he hadn't gotten sick with encephalitis and lost some of his mobility due to a food injury, he would have lived much longer. Here is one of the episodes he told me. Once while climbing a mountain, he decided to rope down simply holding a rope without other means of protection, since the descent looked easy. Before rappelling, he touched the hook that was holding the rope—and the hook fell out of the rock! That's how Alexandrov escaped certain death.

Alexandrov defended Igor′ Andreevich Poletaev (1915–1983) against persecution, and eventually took him with his entire laboratory into his department. It was partly due to his efforts that Ladyzhenskaya elected as a full member of the Academy of Sciences of the USSR. He was the man who persuaded Jean Leray (1906–1998) to write a reference about Ladyzhenskaya's works that played a critical role in her election. He told me how he had been helping geneticists at the time when genetics was not recognized in the Soviet Union and the scientists working in the field were persecuted. When Vadim Delone (1947–1983), a grandson of B. N. Delone (1890–1980), fell out of favor with the authorities, he lived at Alexandrovs'. It was the time when nobody else wanted to have anything to do with Vadim, since people feared repressions for mere acquaintance with him. Alexandrov helped Revol′t Ivanovich Pimenov (1931–1990) as well as other scientists who were persecuted by the authorities.

He tried very hard to help my son-in-law when he got in trouble. After Alexandrov's death, my daughter told her daughter: "Alexandrov was the only person who tried to help your father."

During one of my final trips to Leningrad, I visited Alexandrov in a hospital. I told him about a problem I have been working on since then (both the claim and my idea of how to prove it). He was the only specialist with whom I ever discussed that unfinished work. I wanted very much for Alexandrov to see my work published—but it was too late.

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# ЛЕММА ЗЮССА И ОБРАТНЫЕ ЗАДАЧИ

В.П. Голубятников

Abstract. On the ocassion of the centenary of Alexandr D. Alexandrov's birth, we describe some aspects of geometric tomography that we discussed with him.

**Keywords:** convex bodies, visible bodies, transformation groups.

**1**. В известной работе Александра Даниловича Александрова [1] «К теории смешанных объемов выпуклых тел» по ходу изложения основных результатов приводится принадлежащее И.М. Либерману изящное доказательство одного утверждения Вильгельма Зюсса [8] (см. также [2]):

**Лемма**. *Если проекции двух выпуклых тел при любом направлении проектирования равны и параллельно расположены, то и сами тела равны и параллельно расположены.* (Речь идет о телах в не менее чем трехмерном пространстве, так как лемма заведомо неверна для выпуклых областей на плоскости.)

Подобный круг вопросов мне довелось изучать, занимаясь на первый взгляд совсем другой проблематикой. В августе 1977 года я поступил на работу в лабораторию обратных задач математической физики ВЦ СО АН СССР. Начальное условие сотрудничества, которое было мне сформулировано заведующим лабораторией Ю.Е. Аниконовым, звучало примерно так: половину рабочего времени Вы тратите на задачи лаборатории, а оставшуюся половину — на Ваши топологические хобби. Мне очень повезло, оказалось, что эти «хобби» никак не противоречили интересам лаборатории, наоборот! После обязательной по тем временам двухнедельной отработки в деревне я приступил к исполнению служебных обязанностей, и для начала Юрий Евгеньевич предложил мне геометрическую задачу, имеющую отношение к теории распространения сейсмических волн:

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Golubyatnikov, V.P., Süss's Lemma and Inverse Problems.



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*Пусть $G$ — гомеоморфная шару область в римановом пространстве. Если любые две граничные точки этой области соединяются в $G$ единственной геодезической линией, то и две любые точки из $G$ соединяются в этой области единственной геодезической.*

В двумерном случае это утверждение было доказано В.А. Топоноговым [9]. Сохранив общий план доказательства и используя гомотопическую технику (нестягиваемость сферы), я получил доказательство этого утверждения и для многомерных пространств.

Поскольку в те времена начинали приобретать большую популярность томографические проблемы, следующая серия задач, которую тогда же, в 1977 году, мне предложил Ю.Е. Аниконов, была связана с обобщениями леммы Зюсса на другие группы преобразований проекций выпуклых тел. В конечном итоге ([4, 5]) была сформулирована такая проблема:

*Если у двух (выпуклых) тел $V, W$ в $n$-мерном евклидовом пространстве (вещественном, либо комплексном) проекции на любую $k$-мерную плоскость, $(1 < k < n)$ совмещаются преобразованиями из некоторой группы $G$ линейных преобразований $k$-мерного пространства ($\mathbb{R}^k$ или $\mathbb{C}^k$), то насколько сильно могут при этом отличаться тела $W$ и $V$? Какими преобразованиями объемлющего пространства они могут переводиться друг в друга?*

В лемме Зюсса фигурировала группа параллельных переносов.

Довольно скоро мне удалось разобрать два простых случая, в которых у двух выпуклых многогранников проекции на любую гиперплоскость совмещаются

1. сохраняющими ориентацию изометриями;
2. сохраняющими ориентацию подобиями.

Были рассмотрены также вопросы минимизации необходимых проекционных данных.

В те далекие времена одной из самых больших знаменитостей Академгородка был «молоточник» — маньяк, подстерегавший на лесных тропинках одиноко идущих женщин. Судя по слухам, тактика его нападений была однообразной: он шел вдогонку очередной своей жертвы, бил ее молотком по голове, и далее действовал по обстоятельствам. Его фоторобот был развешан во всех людных местах Академгородка и многие мои знакомые находили в нем некоторое портретное сходство со мной, о чем всякий раз радостно мне сообщали.

В октябре 1977 года я рассказывал на семинаре у Александра Даниловича о своих первых достижениях в области обобщений леммы Зюсса, а также и о единственности геодезических. Во время доклада в аудиторию постоянно заглядывали какие-то люди, никогда ранее в геометрических кругах не появлявшиеся. Оказалось, они выслеживали молоточника и его сообщников!

Диспетчер главного корпуса НГУ Дина Харламовна позднее рассказывала мне, что к ней забежала одна дама и потребовала срочно вызвать милицию: «Там на третьем этаже молоточник беседует с каким-то седым стариком!»

**2**. В середине 80-х годов мне удалось получить более сложную версию леммы Зюсса:

*Если у двух выпуклых компактных тел проекции на любую двумерную плоскость ориентируемо изометричны* (*или ориентируемо подобны*) *и не имеют симметрий относительно поворотов, то эти тела совмещаются в объемлющем пространстве либо параллельным переносом, либо центральной симметрией* (*соответственно, либо параллельным переносом, либо гомотетией, возможно с отрицательным коэффициентом*).

В последнем случае коэффициент подобия проекций *à priori* не предполагался постоянным, не зависящим от плоскости, на которую осуществлялось проектирование.

Рассказал я Александру Даниловичу об этих результатах и попросил его представить в Доклады Академии Наук СССР соответствующий текст. Однако, он довольно тонко польстил мне — отказался представлять подготовленную статью в Доклады АН с такой примерно мотивировкой: «В этом направлении не было продвижений лет 50, значит, все это либо никому не интересно, либо очень сложно. В обоих случаях следует публиковать подробное изложение, а не докладную заметку».

В итоге вариации на темы леммы Зюсса были напечатаны в Сибирском математическом журнале, в Математическом сборнике [3], см. также [5], а Александр Данилович представил в Доклады другую мою заметку, посвященную когомотопическим свойствам пространств Тома.

Примерно тогда же мне удалось перенести эти результаты на довольно широкий класс невыпуклых компактных множеств. Название им было придумано во время многочисленных обсуждений с Д.А. Троценко:

*Назовем компактное подмножество $W$ евклидова пространства* **обозримым**, *если каждая точка, не лежащая в $W$, содержится в некоторой прямой, которая не пересекается с $W$.*

Иными словами, если точка не лежит во множестве, то это можно увидеть на некоторой проекции этого множества.

Услыхав на очередном моем докладе такой выразительный термин, Александр Данилович задумчиво повторил его, а потом добродушно проворчал: «Вы бы еще ощупываемые тела придумали». Однако же, во время всех наших последующих встреч он постоянно интересовался дальнейшими продвижениями в этой области.

Были эти результаты обобщены и на другие типы проекционных данных [5, 6], в частности, на *видимые контуры гладких поверхностей*. Эти геометрические объекты можно наблюдать на экране overhead'a, если согнуть лист «прозрачки» — на экране будет видна проекция «линии сгиба». Такие видимые контуры естественным образом появляются при изучении каустик волновых фронтов в сейсмической томографии. Позднее я узнал, что в теории функций многих комплексных переменных есть аналоги и обозримых множеств и объектов, которые вполне естественно было бы называть ощупываемыми.

Отмечу, что упомянутое условие отсутствия симметрий у изометричных проекций для меня остается загадкой. Насколько оно существенно, я не знаю до сих пор. А вот в случае ориентируемо подобных проекций условие отсутствия симметрий оказалось необходимым. Соответствующий пример был опубликован в статье К. Петти и Дж. МакКинни [7]. Любопытно, что рукопись их статьи в Португальском математическом журнале ожидала в редакции своего выхода в свет почти 14 лет.

Вопросами единственности восстановления выпуклых тел по формам их проекций занимался также и ученик Александра Даниловича А.В. Кузьминых, получивший ряд соответствующих результатов в предположении о том, что функция ширины реконструируемых тел имеет конечное число максимумов.

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# ИСТОРИЯ

Б. В. ДЕКСТЕР

Abstract. This is a tribute to Alexandr D. Alexandrov on the ocassion of the centenary of his birth.

Во время ректорства Александра Даниловича Александрова (А.Д.) в ЛГУ я был студентом, и содержательных контактов с А.Д. у меня, естественно, не было. Лишь позже, на конференции в Петрозаводске, оказавшись рядом в очереди к окошку администраторши гостиницы, мы обменялись несколькими малозначительными фразами. (Инициатива исходила от А.Д.) Тем не менее, с большой благодарностью к А.Д., я вспоминаю историю, которая в значительной мере определила мою судьбу. А.Д. играет в ней главную роль, хотя на сцене этой истории вообще не появляется и никак не действует. Действует его «дух», его «эффект присутствия». Этот-то эффект и приводит к занимательной коллизии с участием кафедры истории партии. (Прямая речь и прочтенные мысли ниже, разумеется, не претендуют на точность.)

Было это в 1958-м году. Двадцатый съезд — позади, но люди, добившиеся положения в обществе своей верностью делу Ленина — Сталина, продолжали сидеть в своих креслах — хотя и без прежней уверенности в завтрашнем дне. В университете они нервничали даже несколько больше, чем в иных местах: присутствие в стенах ЛГУ энергичного и эксцентричного ректора вызывало беспокойство у каждой университетской мыши. (Все это я осознал, конечно, не тогда, в свои 20 лет, а много позже.)

Я переходил тогда после второго курса Кораблестроительного института на второй же курс мат-меха, — такая любовь к математике у меня вышла. Сначала я получил на мат-мехе твердый отказ и уже настроился закончить свои дни в трюмах кораблестроения. Но в конце лета я решил предпринять еще одну попытку — хотя никаких новых аргументов у меня не было. На этот раз я попал не к замдекана (его не было), а к декану, Н. Н. Поляхову. Поляхов пожал

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плечами и сказал: «Пожалуйста! Только вам придется многое досдать». Я был согласен на все. И меня взяли на мат-мех — с академической задолженностью. И сразу — в колхоз.

Мне нужно было досдать все, чего в Кораблестроительном просто не было: алгебру, геометрию, анализ... А также пересдать — или перезачесть, как повезет, – другие предметы: физкультуру, немецкий, и пр., и пр., и, наконец, — историю партии. Этот последний предмет приводил меня в трепет. Дело тут не только в моей плохой памяти и в отсутствии должного к нему интереса. Но еще и в том, что я не видел в нем логики — которая так помогает запоминать! В Кораблестроительном я потратил на историю больше времени и энергии, чем на математику, начертательную геометрию и черчение вместе взятые. Чтобы зазубрить повестки дня съездов, я придумывал мнемонические правила, считалки, неприличные стихи и т. п. И только таким путем получил четверку с натяжкой. Под конец экзаменатор спросил меня что-то о войне, хотя войны мы, естественно, не проходили. Я, похоже, ответил невпопад. Он долго и молча на меня смотрел, потом спросил: «А ваш отец — воевал?» «Да, — сказал я, — воевал. Погиб на Ленинградском фронте». Тут он, видимо, ощутил неловкость. (Он-то — не погиб.) Он скорбно поморщился, почесал голову тыльным концом авторучки и вывел мне «хор». Это стало мне как бы пенсией за отца.

Не было, не было у меня способностей к этому предмету... А еще я ведь вот как опозорился. По окончании мат-меха я вербовался преподавать в Африку — в порядке помощи странам, сбросившим иго и вставшим на путь соц. развития. И должен был на первом этапе пройти некое историко-политическое собеседование Мой собеседователь был вполне доброжелателен и, склоняясь закончить интервью на положительной ноте, спросил: «Ну, какой был номер последнего съезда?» Если бы он спросил про повестку дня, я бы в грязь лицом не ударил. Но к этому вопросу я совершенно не был готов. И я ошибся. На 2 номера. Собеседователь мой посерел: «Ну, знаете, если вы такое там ляпнете, вас же просто засмеют. Это же позор будет на всю Африку.» И меня отсеяли от Африки.

Да, эта четверка в Кораблестроительном была мой потолок, апогей, мой высший пилотаж! Мой звездный час в истории! (Партии.) Повторить такой подвиг я уже не мог, даже на тройку. Нужно ли говорить, что сразу после экзамена я все немедленно и целенаправленно забыл? Только перезачет мог меня спасти.

Недостающую математику я изучил в колхозе и по возвращении в Ленинград благополучно досдал. И начались мои перезачеты. Начал я, естественно, не с истории партии. На кафедре физкультуры никаких проблем не возникло. Немецкий мне тоже перезачли, хотя и менее охотно. И вот, наконец, она осталась одна — история партии.

С колотящимся сердцем, сжимая в потной руке зачетку с уже удавшимися перезачетами (важный прецедент!), я вступил в бесконечный коридор истфака и остановился у двери с красной стеклянной табличкой «Кафедра истории КПСС». Постучав и не получив ответа, я заглянул внутрь. Это была приемная. Вдоль левой стены стоял длинный старомодный диван (кожа и темное дерево тонкой резьбы), вероятно, унаследованный от Петербургского университета. В глубине, у окна, что-то печатала секретарша, а между нею и диваном в стене была высокая дверь, обитая дерматином поверх толстой ваты. Я робко приблизился к секретарше и сказал, что хотел бы видеть завкафедрой.

Выяснив причину этого моего желания, она сказала: «Иван Иваныч (И.И., условное имя) сейчас занят. Посидите, пожалуйста (кивок на диван). Я вас позову». Сказано это было вполне доброжелательно, и я расценил это как доброе предзнаменование. Я сел поближе к дерматиновой двери и начал ждать.

Может быть, это у них — обычное дело, и они перезачитывают всем не моргнув? Возможно, так со мной бы и вышло, если бы не дьявольское стечение обстоятельств. Тут я должен вернуться к кожаному дивану. Он не был пуст, когда я вошел. В другом его конце сидел утомлнного вида мужчина средних лет в темном костюме и при галстуке. Я никаких почтительных улыбок ему не послал и, вообще, видел его лишь боковым зрением, оставляя его как бы по ту сторону своих земных хлопот. Вероятно, это был преподаватель истории партии, зашедший в перерыв со вкусом посидеть. («Историк», как представился Воланд.) И вот какая вышла история.

Некоторое время он неприязненно меня разглядывал, а потом, из своего потустороннего далека, отнесся ко мне так:

— А И. И. вам не перезачтет.

Тут бы мне вздохнуть попечальнее, развести руками и кивнуть в знак покорности судьбе. Но этому я так и не научился. К тому же мне казалось, что, при описанной диспозиции сил, «историк» уже никак не может встрять между мною и И.И. Я сказал:

— А почему же он мне не перезачтет?

— А потому, молодой человек, — был ответ, — что история партии преподается в университете более углубленно, с бóльшим вниманием к первоисточникам, и, к тому же, у нас на историю отводится больше часов, чем в техническом ВУЗ'е.

— Хм... — Про часы я не знал. Да и так ли это? — Но я ведь на мат-мех перехожу, а там это дисциплина... — я сделал паузу чтобы пошевелить в воздухе пальцами, — не является профилирующей.

— Так значит, история партии — второстепенный предмет?

— На мат-мехе — да, — сказал я с гордостью за мат-мех.

— Ах вы так думаете? — Он почти вскричал.

— Да, я так думаю.

Тут «историк» встал, проследовал мимо меня к дерматиновой двери и за нею скрылся. Я похолодел. Он вышел минут через 5, но на диван больше не сел, а взглянув на меня с затаенным торжеством, вышел в коридор. Кошмар! Что я наделал! Исход был предрешен.

Спустя 10 минут И.И., глядя себе на нос, промямлил то же самое: более углубленно,... первоисточники,... больше часов. Помявшись, я сказал:

— У меня по истории КПСС — четверка... Не могли бы вы перезачесть мне... с тройкой?

Это торгашеское предложение вызвало искреннее возмущение И.И. Ощутив свое моральное превосходство, он отлепил взгляд от своего носа и посмотрел мне прямо в глаза:

— Стыдитесь, молодой человек! Вы на кафедре истории Партии! Торг здесь не уместен! И, я думаю, вам будет особенно полезно освежить в памяти вехи нашей истории!

Все было кончено. Я вышел.

И только уже на улице, под сводами истфаковской колонады, я сжал голову руками и стоял так, закрыв глаза и не в силах сдвинуться... Что делать?...

Что делать?... Почитать, что ли, Ленина? Чернышевского? Мои «кораблестроительные корабли» были уже сожжены... Так что, — в Красную армию?

Дня 3 я пребывал в полной прострации, курил по две пачки в день и пил: с друзьями, и в одиночку, и на Невском 27 — с кем попало. А на четвертый — я принял решение: бороться! Пойду к Александрову! Запишусь на прием, выстою в очереди, если надо, и бухнусь в ноги: защиты и справедливости!

Ведь он же — математик, талант! (Это я уже знал.) Он должен меня понять! Не может же он в глубине души не презирать эту касту, которая «приводит прошлое в соответствие с настоящим» и отнимает у студентов столько времени этим «приведенным» прошлым! Разумеется, он мне этого не скажет. И, может быть, даже отчитает за лень. Но, возможно, он все же щелкнет за сценой каким-нибудь тумблером и повернет какое-нибудь колесико в мою сторону.

Конечно, в любом случае мне уже не добиться от кафедры КПСС того, чего добивался Коля Остен-Бакен от Инги Зайонц.(Любви, см. «Золотой теленок».) Но, если А.Д. проявит какой-то интерес к моей судьбе, то, может быть, они, хотя и не перезачтут, но не убьют меня совсем? И, хоть на второй моей попытке, выведут мне жалкую тройку?

Я зашел в ректорат и выяснил, как попадают на прием к ректору. Оказалось, нужно просто явиться в приемные часы. Мне пришло в голову, что хорошо бы принести А.Д. какую-нибудь бумажку: ему-то она ни к чему; но, если он решит мне помочь, то – с бумажкой — ему будет легче. И я написал заявление на имя И.И., в котором повторил свою просьбу о перезачете, и опять предстал перед секретаршей.

— Простите, я бы хотел подать вот это заявление.

Секретарша пробежала заявление и отодвинула голову назад:

— Так И. И. вам же уже отказал...

— Да, отказал. Но я бы хотел получить его отказ в письменной форме.

Ее взгляд нужно было видеть. Тут было много чего: возмущение, гнев, презрение,... беспокойство... Она взяла мою бумагу и сказала зайти через 3 дня.

Эти 3 дня я провел в такой же прострации, как и предыдущие. И вот, в назначенный срок, я снова предстал перед секретаршей. Головы она не повернула. А только выдвинула правый ящик стола и там, не глядя, взяла верхнюю бумагу и протянула ее мне. Это было мое заявление.

Оно уже было изготовлено для меня, как орудие — к бою. Заряд — полный, доворот от основного направления — хх, прицел — уу! (Спецкафедра была тоже на истфаке, совсем рядом. Там, на паркетном полу, изредка обтираемая уборщицей, стояла тяжелая гаубица – для изучения матчасти.) Осталось только дернуть за шнур. За историю нашей Партии – огонь!... Вот тебе, «Пифагоришка»! Двоечник! Лоботряс! Ишь, жаловаться решил! И на кого? — На Партию! Да ты уж не диссидент ли? Ну, погоди, диссидентская морда! Баламут! Узнаешь, почем фунт исторической правды! Кофе тебе будет! Какава!

Я с поклоном забрал свою ксиву и направился вон из приемной. Но до двери я не дошел. Ноги мои вдруг приклеились к щербатому паркету. На несколько секунд я просто потерял способность соображать. Что за черт?... Да, заявление — мое. Но... как это может быть? Я читал резолюцию И. И. снова и снова и не мог взять ее в толк. Эта резолюция — все, что осталось у меня в памяти от истории партии. Я выучил ее наизусть. Вот она (я ничего не добавил и не убавил): «Перезачесть» !!!

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# THE TEACHER

G.M. IDLIS

Abstract. This is a tribute to Alexandr D. Alexandrov on the occassion of the centenary of his birth.

Alexandr Danilovich Alexandrov, Full Member of the Academy of Sciences of the USSR, an outstanding mathematician, head of the Russian school of geometry which came to be known and appreciated worldwide, the disciple of V.A. Fok (Member of the Academy of Sciences of the USSR) and B. N. Delone (Corresponding Member), himself a Teacher for a whole group of scientists, died on July 27, 1999 in Saint-Petersburg, in the age of 86.

I was fortunate enough, being a student, to meet Alexandr Danilovich more than half a century ago, when he came to Almaty to give special courses in geometry in Kirov's State University of Kazakhstan. He gave general lectures on the principle questions of relativity and quantum theories, talking with us, students, during our walks in the foothills of Zailiysk Ala-Tau. My wife and fellow-student Anna Abramovna Zilberberg (unfortunately, she also died in 1999) presented her thesis under Alexandrov's supervision, and later on, when he already became the rector of Leningrad State University (LSU), defended her candidate dissertation there.

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Grigory Moiseevich Idlis (1928–2010) was the head of the department of the history of physics and mathematics in S. I. Vavilov's Institute of the History of Science and Technology of the Russian Academy of Sciences; professor of the history of science department of the Russian State University of Humanities, doctor of physics and mathematics, professor of astrophysics. This is an authorized English version of an article in Russian which appeared in *Академик Александр Данилович Александров. Воспоминания. Публикации. Материалы.* Ред.: Г. М. Идлис, О. А. Ладыженская. М.: Наука, 2002. The English version of the article and the poems it contains were produced by the author's granddaughter, Julia B. Idlis, a graduate of Lomonosov Moscow State University.

Idlis, G.M., The Teacher.



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She was never a very fit person and tired very quickly during mountain trips; then Alexandrov simply carried her in his arms.

Wherever he happened to be, Alexandr Danilovich was always the center of everyone's attention and interest, he invited people to communication. His scientific works also were marked with openness, immediate appeal to the interested readers and invitation to further addition to and generalization of his results; I mean such works as, for instance, his book *Convex Polyhedra* (1950). This book was absolutely unique; alongside with stating the results achieved the author pointed out the possible generalizations and formulated new tasks and questions (by the way, one of these questions was taken as the dissertation theme by my wife). Later on many of the questions outlined in this book were tackled by Alexandrov himself and by his disciples and followers.

Such openness was also characteristic of Alexandrov's seminar in geometry which was held in LSU for many years. It presupposed systematic reports and discussions not only of the already-completed research, but also of the research in progress. All the participants were active and friendly while cooperating in the field. It is interesting to note that V. A. Zalgaller, who for many years had been a senior student of the seminar, a direct disciple and one of the closest collaborators of A. D. Alexandrov, prepared for publication a manuscript of a new revised version of Alexandrov's book *Convex Polyhedra*, Berlin etc.: Springer-Verlag, 2005, with Zalgaller's own commentary and an appendix including two articles by Yu. A. Volkov and an article by L. A. Shor. The publication of this book (both in Russian and in English) is very desirable indeed, since it is going to be a well deserved scientific monument to Alexandrov, this mathematician of genius.

After Alexandrov had visited Almaty for quite a few times there appeared a special seminar on convex polyhedra in the State University of Kazakhstan. Among the members of the seminar were E. P. Sen′kin, A. A. Zilberberg, M. E. Kvachko, V. V. Ovchinnikova, myself and others. Later on Sen′kin, Zilberberg, Kvachko and Ovchinnikova became Alexandrov's post-graduates.

In 1953 Alexandrov published an article in collaboration with Ovchinnikova, which had principal significance: "Observations on the Foundations of Relativity," *Herald of LSU: Mathematics, Physics, and Chemistry* (1953) **11**:4, 95–110.

In 1953 in collaboration with Academicians A. N. Kolmogorov and M. A. Lavrentiev, Alexandrov prepared for publishing a monograph in three volumes under the title *Mathematics, its contents, methods and meaning*, aimed at the mass reader. This was a unique voluminous definitive edition; among its authors were Academicians I. M. Gelfand, M. V. Keldysh, A. I. Malcev, K. K. Mardzhanishvili, S. M. Nikol′skii, I. G. Petrovskii, S. L. Sobolev, D. K. Faddeev, and Alexandrov's Teacher Corresponding Member of the Russian Academy of Sciences B. N. Delone. The monograph first came out in very small numbers (only 350 copies), but, as Semen S. Kutateladze noted, it averted the threat of ideological massacre of Soviet mathematics, which was very real indeed at the time. Only in 1956 the book came out in 7000 copies, and it immediately became a sensation in the literature on mathematics worldwide (suffice it to say that the book saw three new editions only in English).

One of the immediate disciples of Alexandrov, Academician Yu. G. Reshetnyak, wrote an article dedicated to the 75th anniversary of his teacher (*Science in Siberia*,

30.07.1987), where he listed significant quotations from a book by an American geometrician G. Busemann *Convex Surfaces* (1964):

> The aim of this book is to bring the theory of convex surfaces to a wide circle of mathematicians; the theory of convex surfaces has been developing during the recent 25 years, mainly in the USSR, but it remained practically unknown in other countries, at any rate in the USA.
>
> Generally speaking, the main results of the present book, including chapters I and II, belong to A. D. Alexandrov, and the other part of the material appeared under the direct influence of his works.
>
> In the field of the geometry of convex surfaces there can be no doubts as for the priority.

During the last years of his work Alexandrov made important contributions into creation of new textbooks on mathematics for high schools and institutes, which competed with those by Academician A. V. Pogorelov, his former student.

Of special significance were and still are Alexandrov's works on methodological questions of theoretical physics. These concern chronogeometry, axiomatics of relativity, its interpretation as the theory of absolute space-time, and defining the absolute speed of bodies relative to the general field of radiation.

Alexandrov's interests lay outside pure mathematics; he was deeply interested in natural sciences, up to the leading questions of humanities. This can be illustrated by several of his works, for instance, *Scientific Research and Religious Faith* (1974), *Reflection on Economics and Ethics* (1986), *Truth as Moral Value* (1987), *Talks on the History of Science* (three essays published in 1988), and many others. Alexandrov considered mathematics to be a humanitarian science, since it studies "elementary human actions."

He was highly educated erudite, intellectual and outstanding scientist, an expert in world culture with phenomenal memory, a subtle connoisseur of painting, music, poetry, himself a poet, author of beautiful verse, a true Teacher of life and a man of immense courage and surprising willpower, who stood upon his course always and everywhere, with unfailing spirit and passion.

A master of sports in mountain climbing, he celebrated his 70th birthday and the 50th anniversary of his mountaineering practice in 1982 by climbing the Panfilov peak (4300 meters) in the Mountains of Tian Shan.

"Sports interest me, because it make one overcome oneself," he said.

This can be seen in a passage from his long poem *Ascending* which he wrote 10 years before climbing the Panfilov peak, published in his book *Problems of Science and a Scientist's Standpoint* (Nauka Publishers (1988), 507–508):

THE PEAK

The tired hand grabs the last ledge; the effort
Is made. An easy way stretches ahead.
We're on the peak, and this is our glory.
Here we can look around and take a breath.
There is a valley with green meads way down,
The slopes are covered with a thick wood's shawl,
The warmth and peace of this place spreads and tempts us
To choose sweet sleep instead of work and strain.
But close to us, and all around, and further,
Where skies have got over the earth's sharp edge,
There heaves a mass of mountains clad in whiteness,

Shackled in rocks and cliffs and armor of ice.
They bear sharp forms, and sinister cold threatening,
And bold ascent of the forbidding heights,—
The challenge and the call and the hunger of passion,—
Seductive everlasting beauty's call.
The mountains shine with all their diamond facets
And knit the brows of angry beetling rocks;
They call and mesmerize with rigid order
To come to them, to struggle, and to search
The ways for new ascents and climbing upwards,
To fight the obstacles, with difficulty though,
To strive forward again in aspiration,
To calculate, to go, and to risk.
Though there were part most hard to be climbed over,
Where every nerve was tense and strained as string,
We did ascend, o'ercome it all, and here
We are on top. The victory's great flame
Burns in our hearts, the mountains enchant us
And cast their spell of miracle unseen,
Of a fantastic image of creation,
Of inspiration, ecstasy, and charm.
There beams above with eternal joy and power
The space of blue for which there are no words;
The sun caresses us with rays of happy brightness
And burns our skin and dazzles our eyes.
But happiness is transient everywhere;
An hour has passed; it's time for us to leave.
We start again on our long way down.
Farewell, the peak! Ye, mountains, wait for us!

It is cosmic power and ineradicability of the Great (and human) Spirit that he expressed in a poem published about the same time (the same source, p. 508-509)

THE SPIRIT OF MAN

When the last cataclysm causes explosion,
The Earth will perish like a nova, aflame;
Mankind will vanish in the space forever
Without a memory, monument, or funeral feast.
Then scorching gas will whirl at great speed
Into the boundless interstellar space;
The Spirit will leave the Earth anew to wander;
And this will not be the last time it does.
Some day somewhere it will appear again
In flesh and blood for suffering and struggling,
For searching and materialization,
For running upward and returning back.
Thou, Spirit of eternal striving forward,
Which borne the Evil and the Good alike,
Which nothing in the world left outside
The sphere of knowledge and will to transformation,—
Thou, Spirit, do create and convert Nature
Into a battlefield of mighty forces
Which thou begot thyself, so that thou couldst
In struggle gain thee everlasting freedom
And cause a cataclysm, or death, perhaps,

To the inert, stagnant, and worthless matter,
But make the impossibility come true
And once again rise unto life anew.

In all those years Alexandrov often came to Almaty to visit our Mountain observatory of the Astrophysical Institute of the Kazakhstan Academy of Sciences, where I worked after graduating from the university. After 1973 we moved to Moscow and I started working in the Institute of Engineering and Electronic Technology of the Russian Academy of Sciences, and Alexandrov always stayed at our place during his systematic visits to the capital. His visits, contacts with him, edifying conversations about various scientific and general themes and things always were a holiday for us, our children and grandchildren. We witnessed him overcoming all his serious illnesses: the consequences of vernal encephalitis, leg erysipelas, and chronic pneumonia.

Alexandrov has always been and is my unfortunately unattainable ideal of a true scientist, citizen and person. We could only envy his enthusiasm in science, the power of his mind and exceptional efficiency.

In 1945, in the age of 33, before he was elected Corresponding Member of the Academy of Sciences of the USSR, he published 14 scientific works during the span of one year, and in 1956 (11 years, after a period typical of solar and creative activity) he published even more than 17 articles. At the time he already was the rector of Leningrad State University. Every scientist understands how much energy is required for such productive work. Suffice it to say that in 1961, when I was 33 (the age typical of acme), and was working as a vice-director of the Institute of Astrophysics in Almaty, and often was to substitute for the director (V. G. Fesenkov) even before I was appointed director, I managed to publish only 12 scientific works in the space of a year. It was my personal record then, and I was able to break it only in 1997.

Many people were interested in Alexandrov's opinion concerning very different matters. It is said that once his students asked him which scientist could be considered the greatest of all. He answered that it depended on the accepted standards, on the measure and the unit of greatness. If we take for it the time that took people to understand the work of a great man, than Archimedes is the greatest, because he could already integrate. Then, on second thoughts, Alexandrov added: however, if Jesus Christ was a real person, it is he that must be considered the greatest, since he formulated the necessary moral commandments which mankind is yet not ready to grasp. This was not an incidental phrase with Alexandrov. He repeatedly expressed the idea about the true greatness of Christ (and Buddha as well) in his reports and articles.

By the way, in the personal archive of Alexandrov's daughter, D. A. Medvedeva, there is a commandment, formulated by him in the same style on January 23, 1982:

> I do not tell thee to believe blindfold, but to turn to the grasping of what is and how it is; and do not put forth thy prejudice, because in what is and how it is there shines God's wisdom, and in thy prejudice there is only thy wit. And thou cannot put thyself above your God.

Alexandrov himself could not confine himself to the Procrustean bed of narrow specialization in any field of knowledge, and even in the limits of one of the two opposing disciplines— the natural sciences or humanities. He was a "physicist" and a "lyricist" in the best and highest sense of these words, which denote the combination

of the corresponding qualities so rarely seen. He was interested in exceptional ideas in very different fields of knowledge, even paradoxical ones. In those he strived for singling out a rational core (which not every scientist can do because of their narrow specialization). This made him, for example, to help organize L. L. Vasiliev's parapsychological experiments and publish N. A. Kozyrev's controversial works on the so called causal mechanics. In Novosibirsk he took with great interest Yu. I. Kulakov's theory of physical structures and tried hard to publish the corresponding work on mathematics of one of Kulakov's disciples (G. G. Mikhailichenko) in the *Doklady* of the USSR Academy of Sciences, since it was of fundamental significance for the theory.

Alexandrov was keen on discussing the so-called anthropic principle, which was introduced in the modern cosmology in 1957 by me.

Alexandrov was also interested in my deductive conclusion about the necessary mathematical induction of the interconnected periodic systems of ideal fundamental structural elements of matter on the main levels of its natural self-organization: physical, chemical, biological and psychological (mental) levels. He sent an article written by me to Vice-President of the USSR Academy of Sciences Yu. A. Ovchinnikov in order to publish it in the *Doklady* of the Academy in the section of biochemistry.

In 1984 they exchanged letters on this matter, unfortunately, without much effect, because Yu. A. Ovchinnikov could not grasp my rather elementary mathematical arguments:

To Vice-President of the Academy of Sciences of the USSR, Academician Yu. A. Ovchinnikov. Moscow, March 16, 1984

> Dear Yuri Anatolievich,
>
> I enclose a short article written by an old friend of mine—Doctor of Physics and Mathematics, Professor Grigory Moiseevich Idlis "The Common Periodic System of Genetically Encoded Standard Aminoacid Remainders of Biopolipeptides and the Standard Nucleotides of DNA and RNA which Genetically Encode the Former." I told you about this article yesterday when we met at the General Meeting of the Academy, and I would like to ask you to submit it to the *Doklady* of the Academy of Sciences of the USSR in the section of biochemistry.
>
> G. M. Idlis has managed to discover three natural general characteristics with consecutive natural meanings for genetically encoded standard aminoacid remainders of biopolipeptides and the standard nucleotides of DNA and RNA which genetically encode the former. The common system worked out for them turned out to be symmetrical to the limit: it is quadratic as for the number of non-hydrogenous atoms and as for the range of atoms of all kinds and ions in their radicals. Interestingly, according to the latter characteristic, the system turned out to be cyclically closed, that is, periodical.
>
> As far as I can tell, this is an important work, which could become the beginning of a whole new era in understanding the deterministic nature of bioorganic structures. I hope you would consider the article with interest and attention and would think it worth publishing in the *Doklady* of the Academy of Sciences of the USSR as a significant achievement in your field of soviet science.
>
> With best regards, Academician Alexandrov.

Vice-President of the Academy of Sciences of the USSR, Academician Yu. A. Ovchinnikov. April 17, 1984

> Dear Alexandr Danilovich,
>
> I read with great attention the article by Professor G. M. Idlis "The Common Periodic System of Genetically Encoded Standard Aminoacid Remainders of

> Biopolipeptides and the Standard Nucleotides of DNA and RNA which Genetically Encode the Former" enclosed in your letter. Unfortunately, the article has been written by a professional mathematician in a very specialized style. That is why it seems difficult for biochemists, whom the article has been meant for, to understand the arguments. I am not an expert in the given field, and it is hard for me to appreciate the results of the author's research and to recommend the article for the *Doklady* of the Academy. It may be advisable to rewrite the article in order to make it more comprehensible for the majority of chemists and biochemists without sacrificing its mathematical thoroughness before negotiating the publishing.
> Yours, Yu. A. Ovchinnikov

To Vice-President of the Academy of Sciences of the USSR, Academician Yu. A. Ovchinnikov. June 11, 1984

> Dear Yuri Anatolievich,
> Thank you for your kind letter concerning the article by Professor G. M. Idlis "The Common Periodic System of Genetically Encoded Standard Aminoacid Remainders of Biopolipeptides and the Standard Nucleotides of DNA and RNA which Genetically Encode the Former," as well as for advising to rewrite it so that it would be more intelligible for most chemists and biochemists without sacrificing its mathematical thoroughness. I informed the author about your recommendation, and hope that he would rewrite the article according to it, in order to submit it for your consideration once again.
> With greatest respect, Academician Alexandrov

I remembered the way this coordinator of the Soviet biology reacted to extraordinary ideas of other authors (particularly, the works of L. B. Mekler and R. G. Idlis), and I decided not to waste time and effort on trying to convince Yu. A. Ovchinnikov in anything; all the more so that he, as he himself had put it, "was not an expert in the given field" (although it was he who introduced the term "physicochemical biology" in the 1970s, thus stating the necessity of the physicochemical approach to biology in this day and age). I just included the article into more general publications.

In the years of T. D. Lysenko's rule in the Soviet biology, as well as in the subsequent lengthy period of biology in this country under abnormal direction and regulation from above which was exercised by Yu. A. Ovchinnikov, biology had "shrunk" to an extent that one could not help feeling uncertain about its revival being advisable and indeed possible at all. In this respect Leningrad State University was a real oasis of genetics when Alexandrov was its rector.

To celebrate Alexandrov's 87th birthday (he lived up to it only but for a week) the newspaper *Science in Siberia* published an article by seven authors in his memory. Among the authors were three doctors of sciences (Yu. F. Borisov, V. A. Zalgaller, and S. S. Kutateladze) and four Academicians of the RAS (O. A. Ladyzhenskaya, S. P. Novikov, A. V. Pogorelov, and Yu. G. Reshetnyak). In their article, while writing about Alexandrov's scientific contributions, they noted the following:

> With the rise of *perestroika* there appeared people who tried to blame Alexandrov for "Lysenkoism" (being a supporter of Lysenko's policy), which stirred repulse on the part of scientific community. Alexandr Danilovich was deeply moved by the statement of Leningrad Mathematical Society as of March 28, 1989, which read: "Leningrad's scientists remember A.D. Alexandrov's numerous good deeds: his attempts helped to save science and scientists in the years of hardship, which required great personal courage on his part." In October 1990 A. D. Alexandrov, the only mathematician in a group of biologists, was awarded the Order of the Red

> Banner of Labour for his special contribution to the preservation and development of genetics and selection. This unusual reward was caused by the sweeping majority of the country's scientific community estimating highly A.D. Alexandrov's noble work.

Alexandrov showed personal courage while working in Siberia, supporting the grandson of his teacher B. N. Delone—Vadim Delone (1947–1983) who was persecuted by the government for remedial actions, and receiving then disgraced poet Andrey Voznesensky (1933–2010) at his place in the Golden Valley (the academic campus in Novosibirsk) together with Alexandrov's first wife, Marianna Leonidovna Alexandrova.

There are two volumes of Voznesensky's poems in the family archive of A. D. Alexandrov (*The Gaze. Rhymes and Poems.* Moscow: SP, 1972; *Axiomatics.* Moscow: SP, 1990). The first volume was presented by the poet to the Alexandrovs then and the second—later, in 1990, in Tokio, on returning from a trip to Japan with Alexandrov. The books were supplied with the words of the poet which were kindly given to me for publication by Alexandr Danilovich's widow, Svetlana Mikhailovna Alexandrova.

Alexandrov painfully suffered the unjustified attacks of L. Infeld (1898–1968) on V. A. Fok's views[1] upon the bases of general relativity, which some physicists in this country propagated, and initiated our joint work concerning V. A. Fok's contribution to the relativity theory of space, time and gravitation.

Being a true scientist Alexandrov took nothing on trust, questioned and tested everything, but he was a man of wide views and deep judgments. I always liked him. At the very outset, despite the age difference (Alexandrov was 16 years elder than me) we became real friends and could quite seriously discuss any questions be it scientific, philosophical, social-political, or daily. We had much in common. Like Alexandrov, I began my university education at the faculty of physics in LSU. After finishing my first year there I decided to enter the faculty of chemistry as well in order to broaden my scientific horizons. However, the dean of the physical faculty, professor S. È. Frish (1899–1977), who had been Alexandrov's dean, advised me to study at the faculty of mechanics and mathematics instead, deepening my understanding of mathematics. When my parents moved to Alma-Ata I became a student of the Department of Physics and Mathematics in Kazakhstan University; having taken the advice, I studied at two departments there at once: the department of physics and that of mathematics. Both of my dissertations (in physics and in mathematics) were written at the department of theoretical physics under the supervision of assistant professor N. M. Petrova, who was an immediate disciple of Academician V. A. Fok and his former student. One of the dissertations (the mathematical one) *On the Inertial Nature of Harmonic Coordinates in General Relativity* dealt with the question originally raised and investigated by V. A. Fok himself, one of Alexandrov's teachers, who, by the way, was only 14 years elder than his student. Once, when Alexandrov came in Kazakhstan University, another one of his teachers, an older one, came to Almaty. It was Corresponding Member of the USSR Academy of Sciences B. N. Delone (1890–1980), and the moment I saw him I knew who gave Alexandrov his special intonation and expressiveness of speech. Later I worked together with B. N. Delone's daughter, A. B. Delone, in the Institute of Astrophysics of the Kazakhstan Academy of Sciences in Alma-Ata.

[1] V. A. Fok (1898–1974).

As Alexadrov himself admitted, his ideal of industriousness was his father, Danila Alexandrovich Alexandrov, director of a gymnasium. He worked till late hours, was never proud of his noble origin and was eager to lecture in auditoriums and working class clubs.

Like Alexandrov, I had two teachers: Alexandr Danilovich and Academician Vasilii Grigorievich Fesenkov (1889–1972), who in the hard years accepted me as his postgraduate student in the Institute of Astophysics of the Kazakhstan Academy of Sciences and then made me his successor in the institute. Both of us—Alexandrov and myself—made our choice and specialization at last under the continuing influence of our second Teacher, but could not help feeling gratitude to the first. What one gets in one's green years remains with one for the rest of one's life, although one cannot always realize it.

As science progresses its scale is systematically growing in geometric progression, and it is becoming more and more difficult to grasp the full of it. One cannot help turning to the sources of science, to its beginning, and analyze them closer. It is not only the famous biblical saying: "In the beginning was the Word" (the Gospel of John).

Alongside with the basic meaning of the popular but extremely poly-semantic and rather conventional verbal language it should be taken into consideration that the real ultimately refined and universal, and at the same time quite definite (or monosemantic and unconditional) language is that of mathematics. It must be noted in this connection that geometry is first and foremost thought of as this kind of language, although geometry is characterized by the principle of ambivalence of corresponding ultimate concepts—points and platitudes, as well as by Heisenberg's uncertainty principle or Bohr's principle of complementarity in physics and the whole of natural sciences.

It is not without reason that the motto of the Plato's famous Academy was "Let none but geometers enter here." Hence the necessity to turn again and again to Euclid's *Elements*, to Newton's *Mathematical Principles of Natural Philosophy*," to Einstein's special relativity. By the way, in 1959 Alexandrov wrote an article *Relativity as the Theory of Absolute Space-Time*, which dealt with axiomatic cause-effect foundations of the latter.

As Immanuel Kant wrote in his *Metaphysical Foundations of Natural Science* (1786), "natural science in the proper sense of the word suggests first and foremost the metaphysics of nature." At the same time, while dividing natural science (the science about nature) into a rational science in the proper sense of the word similar to mathematics (that is, the a priori pure fundamental science) and science in the transferred sense similar to systematic art (that is, empirical, applied science), he maintained that "any particular doctrine about nature contains as much science in the proper sense of the word as it contains mathematics."

Similarly, according to Paul Lafargue (1842–1911), Karl Marx (1818–1883) thought that "science only achieves perfection when it uses mathematics."

Natural science appears particularly effective when approached systematically and mathematically. Kant in his early but outstanding work on natural science *Universal Natural History and Theory of the Heavens or An Essay on the Constitution and the Mechanical Origin of the Entire Structure of the Universe Based on Newtonian Principles* (1755) made a rightful observation: "He who investigates various spheres of nature purposefully and regularly discovers such

properties which remain concealed and unnoticed when observations are made without order and system."

Of course there are systems and there are other systems; however, according to René Descartes (1598–1650), in the long run everything is interconnected:

> One should realize that all sciences are bound together so tightly that it is easier to study them in the aggregate than to treat any of the sciences separately. Consequently he who strives for perceiving truth must not choose a single discipline—for they are interconnected and interrelated—but care only for the increase of the natural light of reason, and the light of reason must be used not simply to solve various scholastic difficulties but to give man's will the power to guide him through everyday fortuity.

D. I. Mendeleev (1834–1907), who concentrated all his efforts on chemistry which, according to Kant, was a systematized art rather than a science in the proper sense of the word, and who was the first to suggest the periodic table of elements, had a right to say: "It's easy to say anything, but one should also be able to prove one's point!"

Atoms of chemical elements made scientists remember the ancient hypothesis that runs through the whole history of natural science already since Leucippus, Democritus, and Plato.

Natural science is actually impossible without the notion of regular—correct—disposition of elements. It was therefore only natural for Plato to equate the traditional four Pythagorean elements (air, earth, fire and water) and one extra substance (distillation "ether") with five known rectilinear convex polyhedrons (tetrahedron, octahedron, icosahedron, cube, and dodecahedron). Plato even maintained that elements can transform into each other while retaining their identical rectilinear convex "triangular" surfaces (something like the theory of conservation of energy!). However, the cube with its square faces and the dodecahedron with its pentagonal faces obviously dropped out of Plato's system of transformation of solids into each other.

In fact not only Plato's solids can be regular—symmetric in all dimensions. The corresponding nonconvex polyhedra, also five in number, are regular too (taking into account the Pacioli—Kepler stellated octahedron which consists of two crossed tetrahedrons and usually is not treated as separate rectilinear polyhedron). Among those ten regular convex and nonconvex polyhedra five (tetrahedron, convex and nonconvex octahedrons and icosahedrons) have identical triangular surfaces. Consequently they can transform into each other.

By the way, as it was shown by the author of this article, the ten solids mentioned above heuristically correspond to the most stable atomic nuclei, electronic atom shells and all typical features of periodic classification of the elements right up to the so-called magic numbers that connote stable atomic and nuclear structures.

Furthermore, on every of the four possible levels of natural selforganization of substance (i.e. physical, chemical, biological, and mental levels) cyclically enclosed periodical systems, analogous in terms of their symmetry and deductively determined due to indispensable mathematical induction of fundamental structural elements of substance begin with the most symmetrical initial system of elementary particles (leptons) and subatomic particles (quarks and antiquarks) whose various charges (colour, electric, and weak) in a certain charge space are set at the corresponding apexes of those four regular convex octahedrons.

The author of this article unintentionally became interested in regular convex polyhedrons apparently at Alma-Ata geometrical seminar based on Alexandrov's monograph *Convex Polyhedra*. At that seminar not only convex polyhedrons were considered but also some regular non-convex polyhedrons were used as visual aids.

Regular convex and nonconvex polyhedra with identical triangular faces are associated with the initial concept of atoms (the indivisible elements of nature that can transform into each other); moreover, they serve as invariants of certain discontinuous transformations (quantum transitions).

It is not without reason that Richard Feynman (1918–1988) in his famous course of lectures paid special attention to the atomic hypothesis which runs through the whole history of natural science:

> If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words? I believe it is the atomic hypothesis (or the atomic fact, or whatever you wish to call it) that all things are made of atoms—tiny particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling if squeezed into one another. In that one sentence, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.

I still remember many episodes of Academician Alexandrov's life. He is as clear in my mind as he was when I sent him the following lines written to his seventy-fifth anniversary:

THE TEACHER
To Academician
ALEXANDR DANILOVICH ALEXANDROV,
my wife's scientific supervisor in her diploma
paper and candidate dissertation,
to our Teacher and old friend of ours,
on his 75th birthday.

What is this vision?
The marvel of being,
Constantly moving,
Climbing, ascending;
Always in search for
The truth and the meaning:
General history,
Ancient aporiae,
Axiomatics of
All mathematics,
Physical principles,
Lyrical sources,
Morals and ethics,
Modern genetics—
All polyhedral
Vividly given
Interconnected
Areas of knowledge.
Incessant, insistent
In thinking and feeling.
Your cross to bear is
The gift of creation,
The torture of daring,
The thirst for knowledge,
The power of longing.
Each peak that you conquer
Makes you see everything
Which is around you
Clearer, distinctly.
But in a moment,
Like a sign or an omen,
There distantly rises
A new peak.
Farther and further,
Over rocks and steep slopes,
Under landslides and stonefalls,
You're headed towards it,
If only not down!

Alexandr Danilovich used to say that "mountain climbing (like life itself) is not a worship for peaks but a conquest of them." He did not worship anything but

truth. At the same time alongside with vital and concrete scientific questions he was always concerned with eternal problems of existence and relationship between science and morality: could morality be scientific or science moral? In 1974 he published a special brochure on this problem: *Scientific Research and Religious Faith*. During my studies of interrelated periodical (cyclically enclosed) systems of fundamental structural elements of substance, presented at four levels of its natural self-organization (physical, chemical, biological, and mental levels), I came to think that there must exist initial—zero—elements with zero values of main characteristics. Such elements must exist at each of the four levels beginning with neutrino (though ignored by physicists for a long time), neutron and antineutron elements of substance still neglected by chemists (though there proved to exist neutron stars) and finally Supreme Reason or Absolute Spirit whose existence was violently denied by orthodox materialists. The first one to share my ideas was Alexandr Danilovich. Just like his father he was a confirmed atheist and got nothing on faith (although as V. I. Vernadsky (1863–1945) had rightly noticed atheism is also a fact of faith). Being no "militant atheist," unlike many of our pseudoscientific journalists, philosophers, and some physicists who deny everything that does not fit the Procrustean bed of orthodox scientific concepts, Alexandr Danilovich did not regard my views as some kind of gibberish. On the other hand, he confided to me that once he had a heavenly vision (a cross); he treated it as an omen of all his misfortunes being deserved by himself (by the way the same thing he told Marianna Leonidovna later).

Corresponding and symmetrically similar definite periodic systems can be deductively defined due to indispensable mathematical induction; moreover, they turn to be cyclically enclosed and not only each taken separately but also as a whole. All these ideas are included in my original syllabus *The Concepts of Contemporary Natural Sciences* (1997) and in some other books on the same subject: *Natural Sciences* (1996) and *The Biography and Genealogy of the Electron* (1997). Actually, the so-called "main issue of philosophy" of whether matter is primary to spirit or vice versa becomes irrelevant. They coexist and cyclically transform into each other. The answer to the question about their relations can therefore resemble the dialectical principle statement of Alexandrov, which Yu. F. Borisov adduces in his memoires: "One of the last statements Alexandr Danilovich made on existential problems was that matter is fundamental and spirit is primary." By the way, the recently deceased B. V. Raushenbach (1915–2001) shared the same idea when talking about the dualism of matter and spirit.

Since the course *The Concepts of Contemporary Natural Sciences* became compulsory for all humanitarian departments of colleges more than a hundred of manuals by different authors have been published in this country. Still one of the best introductions to that course are Alexandrov's "Discussions on the History of Science" written by him as far back as 1971 while preparing the course of lectures on the history of science at Novosibirsk University (published in 1988) and the three studies: *Science from Its Birth to the Present Time*, *Rise of the Modern Science*, and *Science of Today*. These works are published in the supplement to the Russian edition of the collection of articles in memory of A. D. Alexandrov.

As he got older his charm got no weaker. No matter whether he looks back at human history staying by the Pyramid of Khufu after having returned from India via Egypt, or represented as a patriarch in the vast expanses of his land, or

as a mountain-climber, swinger, scientist, tribune, or thinker—in all his photos he invariably arouses admiration.

Having become the rector of Leningrad State University A. D. Alexandrov (according to his personal records) visited foreign countries 17 times altogether.

1. Italy. September, 1953. A conference on differential geometry.
2. Denmark. December, 1954. Lectures at the University of Copenhagen.
3. GDR. January, 1955. A conference on relativity.
4. Poland. May, 1955. Anniversary of Warsaw University.
5. Switzerland. 1956, June, 1955. A conference on relativity.
6. India. February, 1956. A conference on mathematical education.
7. Italy. March, 1957. Cultural delegation, Conference Italy–USSR.
8. Canada. July–August, 1957. Lectures at summer school.
9. England. September, 1958. Mathematical congress.
10. USA. April, 1959. Delegation to Harvard University.
11. India. January, 1960. A conference on mathematical education.
12. Italy. 1961. Boltzan prize committee.
13. Italy. March, 1961. Political delegation.
14. Switzerland; Italy. March, 1962. Bolzano prize committee and lectures in Rome.
15. Switzerland. June, 1963. Conference on differential geometry.
16. Greece. November, 1963. Delegation.
17. India. December, 1964. UNESCO lectures.

After he was forced to resign his post of the rector had to move to Siberia to the Siberian Division of the Academy of Sciences of the USSR for almost a quarter of a century. In August 1966 he managed to visit the Czechoslovak Republic (conference on differential equations); after that he in fact was prohibited to leave this country. In 1975 he was elected a member of the eldest Italian National Academy of Sciences of the Forty (XL) which was found as far back as 1782. Only in 1990 Academician Alexandrov got an opportunity to come to Rome to a session of the Academy of Sciences of the Forty.

Григорий Моисеевич Идлис (1928–2010)

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# EACH OF US IS RESPONSIBLE FOR EVERYTHING

O. M. KOSHELEVA

Abstract. This is a tribute to Alexandr A. Alexandrov on the occassion of the centenary of his birth.

The first time I saw A.D. was during the award ceremony for the winners of the Novosibirsk Mathematics Olympiad. A.D. was giving a keynote talk at this ceremony, and this talk was wonderful, original, and absolutely non-standard. He said:

"When a mathematician tries to prove a theorem, he feels the same feelings as a young man in love. Just like a young man always thinks about the girl he loves, similarly, a mathematician always thinks about his theorem, he dreams about this theorem as a young man dreams about his beloved."

Not only his talk was unusual, his appearance was very unusual too. He had a sizable beard, and while he was talking, he was stroking his own beard.

He told us how he understood what is a world-wide fame. In the 1930s, when he was a young professor in Leningrad, he could not get into an overcrowded tram, so he was hanging outside, trying not to fall off. A policeman noticed him, ordered him off the tram, and asked for his ID. When A.D. showed his professor ID, the policeman, instead of fining him (the usual punishment at that time), saluted him and said: "Please continue your experiment, comrade professor." Many years later, when A.D. was visiting Calcutta, India, one of the local professors asked him: "Is it true that the following story happened to you in 1930s?" and repeating the tram story. "This was," said A.D., "when I realized what people mean by a world-wide fame."

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After that, I attended many of his talks: his class lectures, his presentations at the seminar on space-time geometry, his lectures on history of science. I even interviewed him for the University newspaper. He invited me into his house. The door was opened by Tanya who A.D. half-jokingly called his "housekeeper" — after a character from Astrid Lindgren's story "Karlsson-on-the-roof" that was very popular in Russia. What impressed me the most was the luxury of a separate house, the luxury that seemed unbelievable after the overcrowdedness of the communal apartments and dorms where I and most of my friends lived all our lives. There was a fireplace in the room, a bear skin on the floor. During the entire interview, Tanya sat in the corner playing solitaire. I prepared my questions beforehand, but instead, A.D. talked himself. He told me many interesting things—about science, about women in mathematics. In his attitude towards women, A.D. was very old-fashioned: "Sometimes, it is possible for a woman to succeed in mathematics, but God created women for a different purpose. Of course, there are exceptions; one of them is Olga Ladyzhenskaya, she is a beauty, she is full of charm but at the same time she has a very strong will." A.D. described how he became interested in science. For him, this interest started when he was six years old. He was excited about Native Americans, wanted to become a geographer, but then somewhy started doing physics and mathematics instead—and then realized it was too late to change to a different science. It may sound strange but already at the age of six, he decided that he would be a scientist, and he told everyone "When I will be a professor ..."

During the interview, he was in a very good mood, he was all charm, he even served us a very good dinner. But when later I started going to his seminar, I noticed that he could often be very abrupt. After one such seminar, I wrote down in my diary what he said: "Everybody is a scoundrel, everybody is rotten. There may have have been only one good person—Jesus Christ. Even Einstein was a scoundrel, he should have left the United States after they A-bombed Japan against his will."

Overall, his attitude towards religion was unusual. I remember that in December 1974, he gave a talk at the joint meeting of Departments of Mathematics and Philosophy where his book *Religious Faith and Scientific Search for Truth* was being discussed.

A.D. said that during this discussion, he felt "like a clown in a circus who has to entertain the audience every time when others are not ready."

"A book is published under Alexandrov's name but it is only half mine—the censors deleted a half of what I wrote and replaced this half with a text that I did not write."

"Why is this topic important? Because, in the eyes of most people, science has become a Big Bad Beast, a Tyrannosaurus Rex who can do a lot of bad things to them. It is a wild, mysterious, unpredictable dinosaur that overcomes people with information."

"In his 1910s article in the classical Russian Brockhaus and Efron Encyclopedia, Vladimir Soloviev, a prominent Russian philosopher, defines faith as something that cannot be acquired or changed by neither observing facts nor listening arguments."

"The often repeated statement that faith is necessary for people to behave morally is a shame. Morality should itself be the basis of a human behavior; if a good behavior is not based on the inherent moral feelings, only on faith, then under different circumstances, the same faith can—and often does—justify amoral behavior."

"In ancient India, people noticed that everything falls, but the Earth does not fall. Their sages explained that the Earth rests on the backs of four huge elephants. The elephants do not fall because they are standing on the shell of an even larger turtle, the turtle is floating in an ocean, etc. We can continue forever and never come to a final answer. And later on, scientists discovered that the Earth does not rest on anything, it is there all by itself. The same is with morality."

"In human society, we often have to work together. Two religious believers who hold different sets of beliefs can never convince each other, because their faiths do not come from facts or logic and cannot be changed by facts or logic. So, the only way these two believers can work together is when one forces the other by physical force. Thus, faith leads to violence."

"At first, faith was not inconsistent with the search for scientific truth. A pure faith, without prejudice, with readiness to believe in anything—such a faith is, in some sense, equivalent to the scientific search for truth."

"Science is different from religious faith. With scientists, you can talk objectively and calmly about everything. Why? Because when a scientist encounters facts that contradict his previous beliefs, he does not stubbornly stick to his beliefs, he can change them."

He ended his lecture with a citation from his book:

"According to modern science, all the events in the world are—directly or indirectly—connected to each other. As a result, each individual is—to a smaller or larger extent—connected to everything that is happening in the world. So, the actions of every individual can influence (and does influence)—to a smaller or larger extent—all the events in the world. And if each of us can influence each of these events, this means that each of us is responsible for each of these events."

To me, this became the definition of a place of a human being in this world: each of us is individually responsible for everything that is happening in the world. Each of us is responsible for everything.

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# LARGER THAN LIFE

V. YA. KREINOVICH

Abstract. This is a tribute to Alexandr A. Alexandrov on the occassion of the centenary of his birth.

To adequately describe Alexander Danilovich is very difficult, but it is OK: everyone will describe him from his or her own perspective, and from that, a reader will be able to reconstruct him as he was—just like one can reconstruct a polytope from its 2D projections.

As in a thick 19th century Russian novel, fate prepared me for my meeting with A.D. slowly but surely. For the first time, I heard about A.D. in Summer 1969. It was right before I started taking my classes at the Mathematics Department of St. Petersburg University. With my friend Nikolai "Kolya" Vavilov, we were having a good time at the Used Academic Books bookstore on the Liteiny Prospect. We enjoyed the books, and I enjoyed Kolya's stories about the mathematicians who wrote them. When we came across a book on space-time geometry by Revolt Pimenov (1931–1990), Kolya said: "Of all his students, A.D. Alexandrov probably valued Pimenov the most. And probably cursed him the most. Did you know that Pimenov spent several years in the Gulag?" I was not surprised: "Many people were sent there by Stalin, and many good mathematicians among them." "This is true, but Pimenov was jailed after Stalin's death, when Khrushchev was in full power." That was unusual indeed.

When I became a student, I started attending Pimenov's seminar on space-time geometry. A.D.'s name was mentioned very often there. And then Ludwig Faddeev taught us physics. This course really impressed us. This was probably the first course where we felt that it was no longer just a preparation for interesting things that

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would come later—this was it, the real mathematics, and its real applications. And when he talked about special relativity, Faddeev mentioned, among other things, that there was a theorem proved by A.D. Alexandrov about causality-preserving transformations. That was a sign of quality, almost a sign of eternity.

And then I was having my (officially required) practice with Pimenov, in a Northern town of Syktyvkar to which he was by then exiled—this time by Brezhnev, a practice which the Math Department not only officially approved but even financially sponsored. Throughout the entire semester, whether I was listening to Pimenov's lectures or simply talking with him, I could feel the almost visible presence of "Danilych," like a kind and powerful grandfather, like a guardian angel; and the best praise for my work was "This is worth sending to Danilych."

Suddenly Alexander Danilovich appeared in St. Petersburg. At first there was a flier—of an unusually large size, then came his lecture on geometry of space-time. A large audience was packed, and unexpectedly, at first—instead of formulas and theorems—there pour common sense explanations why all this is important; motivations that we liked so much in American mathematical papers—for the first time we heard them during a talk. I simply fell in love with his style—and with his personality.

And the next morning, another shock: in the Math Department, where everyone is against the communist despots but where everyone is quietly whispering his or her against—because in every student group there in a known KGB informer who always gets failing grades but somehow is never expelled—suddenly loudly, very loudly: "Your Buravtsev is a scoundrel! Dared to inform on me that I was discussing philosophy instead of mathematics! For God's sake! Who wrote the article about geometry of space-time in the *Philosophical Encyclopedia*? Alexandrov wrote. What other philosophy do they want? They should have thanked me for the talk instead of writing their dirty denunciations!"

And then there was the first National Conference on Geometry of Space-Time in Novosibirsk Akademgorodok in October 1974. Pimenov's exile was over, so he was able to come too. Kolya Vavilov was right: Pimenov was the most valued student of A.D. At the banquet, the first toast A.D. proposed was about Pimenov: "In geometry, we study conformal transformations a lot, but everything non-conformal is also important. So let us drink the health of Revolt Pimenov!"

I liked A.D., I liked Akademgorodok, and it looks like A.D. liked me too, because he sent me an official offer to work with him. As required by the police regulations about changing the place of residence, I officially renounced my right to live in St. Petersburg, but in Novosibirsk, in spite of the official invitation, the authorities did not hire me and did not grant me the right to live there. I had no place to live, no money to live on. And I could not return to St. Petersburg because I no longer had the right to live there.

And A.D. started to fight for me. "I could let you live in my house but two bears cannot live friendly in the same den"—and, by the way, he was right. He helped me get a bed in the dorm—and then the authorities threw me out; he helped me become a lecturer at Novosibirsk University, but when he went on a business trip, I was immediately fired—KGB called the University Rector and ordered him to.

Sometimes I felt like there was no way out. I felt like a jailer in a story we learned at school about Lenin's first prison term: "Your struggle is useless, young man, the

regime is as solid as these stone walls." It felt like that, that there was a stone wall all around. It felt that it was impossible to break through it.

But A.D. never gave up. His advises were sometimes unusual, but often helpful. Once Brezhnev said, in his speech, that young scientists need nurturing—the next day A.D., with a copy of this speech printed in the *Pravda* newspaper, goes to the regional Communist Party Committee to argue for me—and "Vladik, go to the Youth Communist League committee, ask them to help."

Sometimes it helped. More often it did not, and then A.D. cheered me by his usual "Never lose your optimism because the worst is still ahead."

I was not the only one whom A.D. tried to help. There were many like me. Sometimes, he succeeded, then he was happy as a clam. How many times I heard his boastful stories and, at the end, right before the final good news: "And all these communist scoundrels... ."

Not only in the more tolerant Brezhnev times he succeeded in fighting the scoundrels, in Stalin's times he was also sometimes successful. When, during Stalin's rule, in the language of those days, "an ideological bomb exploded on the mathematical front" and a "discussion" threatened mathematics—a "discussion" similar to what had just happened in biology where the losing researchers were summarily shot—A.D. managed to convince the bosses that instead of a discussion, the book should be published under the title *Mathematics, Its Content, Method, and Meaning.* What happened then followed a story about a 13th century philosopher and teacher of life Hodja Nasreddin who, in an effort to save his life, volunteered to teach the King's donkey to read in 20 years. When his friends got worried, Hodja said: "Do not worry, in 20 years one of us will be dead: either I myself, or the King, or the donkey." While the book was being prepared, did die the bloody tyrant, and the book—the book is still very much alive. Every year, Dover Publishers send me a catalog, where among the Einsteins and the Hilberts there is this old (but clearly not outdated) book by A.D.

A.D. also liked to tell a story how in a mountaineer camp, at the elevation of 10,000 feet, he authorized a student to be enrolled at Leningrad University. We all had a special attitude towards alpinism. Somehow we believed that an alpinist, a person who risks his life to save others, cannot be a scoundrel. A professor could be a scoundrel, we all knew that, we witnessed it every day, but an alpinist was something sacred, pure. Many encyclopedias had articles about A.D., and each article listed his numerous prizes and awards, but there was one award that A.D. was most proud of—that the National Alpinist Federation certified him as a Master of Alpinism. To us, this award was as convincing a proof of his goodness as a halo above a saint's head.

A.D. did a lot of good, but a saint he wasn't. He liked to chase after girls, he had a short temper and often got very angry—although Pimenov told me "I do not know anything more harmless than A.D.'s anger." For us, his anger was indeed harmless, but for the scoundrels—far from that.

And the scoundrels replied in kind. They chased him ruthlessly and tirelessly like hunters chase a bear. They voted against his students at Ph.D. defenses. They spread false rumors, very clever false rumors so that we young people would not support him. That he was supposedly an Anti-Semite, and that he was brown-nosing Communist dictators. We, who were close to him, knew that these rumors were false but there were many who believed in such a nonsense.

For us he was a father-like figure, who could help you when you needed help, who could give you *Doctor Zhivago* to read—a Nobel-prize winning book for reading which you could still get a jail term under the Communist rule. For those who believed in God (a no-no under the Communists), A.D. was a person to whom you could come for a traditional Easter celebration and not be afraid of getting fired. For us he was a person who hosted Vadim Delone (1947–1983) after Vadim got out of jail where he was placed for defending the freedom of the Czechs in 1968. For us, he was proudly walking with his head straight, but many viewed this same posture as self-promotion.

This attitude hurt him, and he felt bad, but what really made him feel bad was when the scoundrels hurt us, his students, whom he tried to protect as much as his own children.

And when he felt very bad, two things helped him: love and science. I remember, once he had a collision with another skier, his Achilles' tendon was badly injured, and we came to visit him. We could see that he was in pain, so much in pain that he wanted to shriek. I am not sure whether we understood that he was not in a mood for nice words, or whether he himself asked us to leave, but what I do remember well is that before leaving, overcoming his pain, A.D. asked Alexander Kuzminykh to stay and discuss some math—and while they were discussing math, the pain went away! And then his beloved Svetlana came to nurse him and adore him, and he was happy again.

And I? Did I adore him? Not always. In politics, it was not A.D. who was my hero, but Sakharov—a knight in shining armor, a person who never tells lies, a saint.

But in mathematics, I did adore him. His ideas, his intuition: I was simply awed! For months, I would think about an idea, and when I finally made a step forward and started explaining it to him—he would immediately come up with the complete solution! I was awed by him, I still am. In English, there is a phrase for which there is no direct equivalent in Russian—"Larger than life." This is what he was: larger than life.

What made the man he was? Maybe the fact that he always looked at all the problems from the viewpoint of eternity. "From our entire civilization," he once said, "only Christ and Buddha will remain forever ..."

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# TRAITS

S. S. KUTATELADZE

Abstract. This is a tribute to Alexandr D. Alexandrov on the occassion of the centenary of his birth.

Reminiscences and memoirs comprise a special kind of fiction with lies and boasts unavoidable. The latter were disgusting for A.D.[1] to an extent that leaves no room for envying the authors who provide their written recollections about A.D.

It happened so that I had a privilege and honor of constant communication with A.D. from the end of the 1970s up to his death. Writing reminiscences is by far much easier after many years' elapsing. However, my elder friends had managed to convince me to reflect some details of the Siberian period of A.D.'s life.

I have had many opportunities of writing about A.D. in the traditional (and not fully traditional) forms of scientific publicism. I am happy that he never reproached me for this, and so I guess that I may skip the task of surveying his scientific contribution.

Many events in which I observed A.D. and sometimes participated in secondary roles were not so long ago as to become an impartial history. Not all of them deserve inspecting over for revival and plunging into once again.

Perusing my personal archives, I decided to select just a few items reflecting those traits of A.D.'s personality that were revealed in our contacts.

I will be glad if the lessons of A.D.'s life help someone to hold on or to settle some pending crisis as they have readily done for me...

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This is an authorized English version of an article in Russian which appeared in *Академик Александр Данилович Александров. Воспоминания. Публикации. Материалы.* Ред.: Г. М. Идлис, О. А. Ладыженская. М.: Наука, 2002.

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[1]Sounds in Russian like "under" with the letters "n" and "r" omitted and both syllables stressed equally.

## Anger and Self-Criticism

A particular trait of A.D. I wish to emphasize is the physiological reaction of anger to danger, assault, or offence. These circumstances are well known to bring about the emotion of fear (pale face, cold wet, etc.). The military commanders of the ancient times often enrolled in their forces the warriors whose reaction to danger was anger.

A.D. exhibited the classical examples of the emotion of anger: his face reddened, the chest threw out, and he showed the bared teeth. A.D. understood quite perfectly how he intimidated those who provoked his anger. At that I never saw his unjustified fits of temper. Many years of acquaintance with A.D. cultivated the strong stereotype: Everyone hating A.D. is a potential if not complete scoundrel. As regards his students, friends, and relatives, A.D. was exceptionally kind, even tender, very attentive, and scrupulous.

A man of passion, A.D. always remained self-critical. I had an opportunity to write that self-criticism is a necessary test for intelligence. Every now and then A.D. reconsidered his attitude to people and events in accord with the ideals of morality he proclaimed: universal humanism, responsibility, and scientific outlook.

Providing a small illustration, I can recall that A.D. voted against the admittance of my Kandidat thesis to the formal procedure of public maintenance in 1969. Moreover, he supplied no motivation whatsoever. An open negative vote of an academician happens rarely on such an insignificant occasion as the admittance of somebody's Kandidat thesis. My thesis was submitted in analysis under the title *Related Problems of Geometry and Mathematical Programming*. Its topic was close to the research of A. D. Alexandrov in the theory of mixed volumes and the research of L. V. Kantorovich in optimization and ordered vector spaces. Clearly, I was not the only person impressed by the unmotivated demarche of A.D.

I thought that my article could be of interest to A.D. (the formal review of a "leading mathematical organization" was written by V. A. Zalgaller; and my main technical result was an extension of one unpublished idea by Yu. G. Reshetnyak in measure theory). I was rather nervous making my talk at the public maintenance. Using an overhead projector in a semi-darkened hall, I cast a casual glance towards A.D. When I had told that my thesis uses the theory of mathematical programming by L. V. Kantorovich and the theory of surface area measures by A. D. Alexandrov, there was some noise from the side benches: A.D. rose and strode out. It is easy to imagine how confused I was after that. However, the vote was unanimous.

After many years, when we had been close with A.D. for a long time, I reminded him of this story. He rebuffed immediately: "This never happened at all." (You should know A.D. to understand his answer properly: when he forgot or doubted something, he always said: "Don't remember." Replying in other words, A.D. had declared the whole episode nil and void.)

It is a pleasure to recall that I had received satisfaction from A.D. in due course. In result of some bizarre machinations of the Higher Attestation Committee of the USSR in the 1970s, my Doctor thesis was sent to extra referral despite its formal approval at the corresponding section of the Committee on the recommendation by E. M. Nikishin. Happily, it was A.D. who was appointed a "black" opponent (which is a Russian equivalent of a "blind" reviewer), and I received his appraisal for the isoperimetric problems with arbitrary constraints on mixed volumes.

## M. A. Lavrent′ev and a Book on the Methodology of Mathematics

Narrating about his participation in the ideological battles of the 1940s and 1950s, A.D. always spoke about the tactics of preemptive blows. One of them deserves recalling.

The Academy of Sciences of the USSR had printed in 1953 a huge volume of about thousand pages under the title: *Mathematics: Its Content, Methods and Meaning.* The Editorial Board of the volume comprised A. D. Alexandrov, A. N. Kolmogorov, and M. A. Lavrent′ev.

The eighteen chapters of the book were intended to the general public and written by thirteen authors. The list of the latter contained I. M. Gelfand, M. V. Keldysh, M. A. Lavrent′ev, A. I. Malcev, S. M. Nikol′skiĭ, I. G. Petrovskiĭ, and S. L. Sobolev. The run of 350 copies was exceptionally small those days. Besides, each copy was enumerated and the front page contained the index of the copy and the extraordinary signature stamp "Published for Discussion."

Sufficiently many copies of this book were printed free of classification only in 1956, and the book became an issue in the world mathematical literature. Suffice it to say that the translation of this book was reprinted thrice in the USA (the last time in 1999).

Clearly, such an extraordinary collection had rather nontrivial reasons for its compilation. The aim of this project consisted in defending mathematics from the antiscientific attacks that were typical of those days in the Soviet Union.

To strike a severe preemptive blow on the pseudo-scientists of Marxism which try to harass the development of science in this country and to get rid of them possibly for ever was an almost successful plot of the book. Avoiding strictly professional nuances, the world-renowned leaders of mathematics gave in this book a detailed and thorough analysis of such fundamental general aspects of their science as the subject of mathematics and the nature of mathematical abstractions, interaction between pure and applied mathematics, relationship between mathematical research and practice, etc. The book remains to stand as one of the heights of the methodology of mathematics.

The soul of this project was A.D. In addition to the two special chapters on curves and surfaces and on abstract spaces, he made a "promising beginning" — wrote the lengthy introductory chapter "A General View of Mathematics" with an impressive analysis of the challenging philosophical problems of mathematics.

The work on this book drove A.D. and M. A. Lavrent′ev closer. By the invitation of M. A. Lavrent′ev, A. D. Alexandrov joined the staff of the Siberian Division of the Academy of Sciences of the USSR in 1964. A.D. was proud of the fact that M. A. Lavrent′ev had solely nominated him as a candidate to a full member of the Academy and freed him from all bureaucratic formalities. When A.D. became aware that L. V. Kantorovich was nominated for the same vacancy, he began to refuse to participate in the elections. However, M. A. managed to convince A.D. to stop refusing. Sage Mikhail Alekseevich turned out to be right: Both were happily elected (the Bylaws of the Academy made room for such an outcome those days).

## Bertrand Russell and a Preventive War Against Russia

At the end of the 1970s the plan was under discussion of publishing a volume of the articles of A.D. on the general issues of science and other articles of publicism. This plan led finally to his book *Problems of Science and a Scientist's Standpoint.*

The release candidate No. 1 was surely the article "A General View of Mathematics." A.D. asked me to look it through for shortening. Reading the article thoroughly, I felt much doubt about the following excerpt omitted from all publications after 1952:

> *In the bourgeois society we can encounter the scientists that turn into obscurantists professing political reaction and antiscientific obfuscation rather than progress and knowledge. An example of this degeneration is one of the founders of the so-called "logical positivism"—Russell, an English philosopher and mathematician. He declared fifty years ago that "Mathematics may be defined as the subject where we never know what we are talking about, nor whether what we are saying is true." In other words, mathematics has no real content according to Russell. The real content of his own views Russell revealed completely when he began to call for an atomic war against the Soviet Union several years ago. A perverter of science and self-conceited epigone of forlorn idealistic systems who instigates mass destruction—that is the true face of this "logical positivist."*

In my opinion of those days Russell was one of the leaders of the Pugwash movement, a dedicated peace warrior, and a Nobel prize winner. In no way he was reminiscent of a perverter of science who instigates mass destruction. Frankly speaking, I thought that A. D. swallowed a tasty bait of the propagandists of the CPSU in the first years of the Cold War.

With a hardly concealed spite and glee I told A.D. that the reader needs a precise reference to the words of Russell and smugly requested his explanations. In fact, I attacked him impudently in the trite style of the "presumption of dishonesty." He was definitely offended. He snapped back sharply that the episode did take place but he could not remember any details. I must confess that these explanations convinced me of nothing at all.

In the new millennium I tried to use the omnipotence of the Internet to settle the problem finally by search machines. Without any effort, I found out that one of Russell's statements about the A-bomb appears in textbooks as a standard example of a "false dilemma."

> *Either we must have war against Russia before she has the atom bomb, or we will have to lie down and let them govern us.*

Also, Russell included in his book *The Future of Science, and Self-Portrait of the Author* as of 1959 his interview for BBC Radio:

> *Q. Is it true or untrue that in recent years you advocated that a preventive war might be made against communism, against Soviet Russia?*
> *RUSSELL: It's entirely true, and I don't repent of it now. It was not inconsistent with what I think now.... There was a time, just after the last war, when the Americans had a monopoly of nuclear weapons and offered to internationalize nuclear weapons by the Baruch proposal, and I thought this an extremely generous proposal on their part, one which it would be very desirable that the world should accept; not that I advocated a nuclear war, but I did think that great pressure should be put upon Russia to accept the Baruch proposal, and I did think that if they continued to refuse it it might be necessary actually to go to war. At that time nuclear weapons existed only on one side, and therefore the odds were the Russians would have given way. I thought they would ... .*
> *Q. Suppose they hadn't given way.*

> *RUSSELL: I thought and hoped that the Russians would give way, but of course you can't threaten unless you're prepared to have your bluff called.*

It is a pity that A.D. will never hear the words of my repentance.

## A. P. Aleksandrov and a Polemic about an Article by N. P. Dubinin

A.D. was engaged in defense of science and particular scientists in his Siberian period as well. Many persons he drew out of the screw presses of the scientific and would-be-scientific rascals who made their careers in the 1970s and 1980s. I am reluctant to tell these stories whose analogs are familiar to the majority of scientific groups in this country.

What I want to recall here is the valiant standpoint of A.D. in regard to the article by N. P. Dubinin "Biological and Social Heredity" which was published in *The Communist* (1980:**11**).[2]

A.D. had appraised this composition as an "outstanding piece of antiscientific literature." I am convinced that to read the article by N. P. Dubinin and the relevant controversy is as vital for a young scientist of any specialty as the perusing of the shorthand record of the notorious August Session of the Lenin All-Union Academy of Agricultural Sciences (VASKhNIL in the Russian abbreviation) which took place in 1948.

Avoiding to narrate the whole composition of N. P. Dubinin, I just pinpoint one of the ideological conclusions of his article:

> *Without clear understanding of the genuine scientific basis for the problem of man, it is impossible to properly place the vicious essence of neoeugenical ideas in a disguise of new discoveries in natural sciences and in particular in molecular biology and genetics. Moreover, this problem is such that the coincidence of the truth criterion and the party spirit is most conspicuous here.*

A.D. found primarily repulsive the attempt at making the party spirit the test for truth and refused to consider it as a slip of the tongue. His worst premonitions came true: the editorial comment on the discussion around the article by N. P. Dubinin read later in *The Communist* (1983:**14**):

> *The main criterion for evaluating the philosophical meaning of pieces of theoretical research is their ideological orientation, the purity of the class-characteristic, ideological, and methodological standpoints.*

Practice as the ultimate test for truth was doomed for a pompous funeral and complete oblivion.

A.D. tried to profess his views of the article by N. P. Dubinin actively: he made talks on methodological seminars in various scientific institutions and attempted in vain to publish his arguments. Fortunately (this happened quite rarely to A.D.), he was supported by A. P. Aleksandrov who then held the position of the President of the Academy of Sciences of the USSR and let A.D. get the floor at the General Assembly of the Academy of Sciences of the USSR on November 21, 1980 (a version of the speech of A.D. and the reply by N. P. Dubinin are published in the *Herald of the Academy of Sciences of the USSR* (1981:**6**)).

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[2]The official journal of the Central Committee of the Communist Party of the USSR. The latter will be the "Party" in what follows.

A.D. always emphasized that the cause of science is to find out "how the thingummy's actually going on." He pursued the same approach in this particular case:

> *The real problem consists in studying which sides of psyche depend on heredity or social environment and to which extend. However, N. P. Dubinin closes this problem as regards normal persons, leaving it open only for medical genetics in regard to abnormal persons.*

A.D. told me after the Assembly that Anatoliĭ Petrovich answered to A.D.'s application for having the floor as follows: "Do you want to bite off Dubinin's head right away or after the break?" As far as I could remember, A.D. was eager to accomplish the task immediately... These days A.D. gave me a galley proof of the draft of his speech. Below I present the end of this manuscript which remained unpublished for a couple of decades:[3]

> *I have said now what I wished to say, but I harbor heavy doubts: maybe, it was unnecessary to speak all this out and in so strong words at that. In fact, it is clear that the attempts of Academician Dubinin will not affect serious scientists and practitioners. Therefore, they will hardly influence our biology and medicine.*
>
> *However, this view is not fully accurate. Academician Dubinin used a high rostrum and it is not completely excluded that some assistant professor reading human genetics in some medical college will be called to responsibility for "attempting"—in the words of Dubinin—"to revise and waste the marxist teaching of the unique social nature of man."*
>
> *Furthermore, the question is placed on the agenda of the honor of science and our personal honor: Do we agree to yield to the resurrection of the vicious style and battle against science which reigned here about thirty years ago?*
>
> *Everyone can make a mistake and even speak up in rigmarole. What really matters in the long run are the basic principles of science themselves rather than some particular mistakes. Marx observed that anyone I call a base person who strives to adjust science to external and alien aims—irrespective of whatever delusions science might cling to.*
>
> *The objects of concern are precisely the main principles of scientific research: impartiality and honesty. We cannot let them be dismissed so loudly and impudently.*

## S. L. Sobolev and a Polemic about an Article by L. S. Pontryagin

The year 1980 was rich in events!

The journal *The Communist* (September 1980:**14**) published the article by L. S. Pontryagin "About Mathematics and the Quality of Teaching Mathematics." This composition still arouses the emotions as sharp as those stirred up by the article of N. P. Dubinin. Moreover, both in the same volume of the journal produce an unforgettable adore.

The article by L. S. Pontryagin was supplied with a routine editorial comment that explained the genuine meaning of the article to those who failed or hoped to fail to grasp it:

> *...the author is right in opposing vehemently not only the exceeding devotion to abstract constructions in teaching mathematics and in mathematics itself but also the pseudoscientific speculations related to the false treatment of its subject.*

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[3]See A. D.Alexandrov *Articles of Various Years*. Novosibirsk: Nauka, 2008, 491–495.

> *Noncritical adoption of foreign achievements in relatively new branches of mathematics and hypertrophy of general importance of these achievements to science as a whole have led to overrating the results of many mathematical studies and in some cases to the idealistic treatment of the essence of the subject of this science, to the absolutization of abstract constructions, and to the belittling of the gnosiological role of practice. Exceeding devotion to abstractions of the set-theoretic stance has started disorienting the creative interests of students and academic youth.*

It was impossible to consider such a rhetoric casual and innocent. Indeed, *The Communist* **18** had published a note by Academician I. M. Vinogradov, Director of the Steklov Mathematical Institute of the Academy of Sciences of the USSR. This note said in particular that

> *The Scientific Council of the Steklov Institute is satisfied with the statement of the journal The Communist in the form of the article by Academician L. S. Pontryagin... The Scientific Council of the Steklov Institute supports the statement of the journal The Communist and believes that it will serve the cause of improvement of teaching in secondary school...*

I find it appropriate to cite a few lines from my diary for reconstructing the intensive but stale atmosphere of that span of time.

14.10. A.D. called me in the evening and told about an article in *The Communist* **14**: Pontryagin vs. secondary school, S. L. [Sobolev] + L. V. [Kantorovich] + an editorial comment on idealism in mathematics.
15.10. M. A. Lavrent′ev passed away.
18.10. I read the nasty article by Pontryagin in the morning and dropped in on A.D. in the evening to talk this over.
24.10. The ninth day—the funeral of Mikhail Alekseevich [Lavrent′ev]...
25.10. Zelmanov was flunked by secret ballot in the Institute. We discussed this and *The Communist* with A.D. at length.
26.10. A.D. dropped in on me and gave me the second part of his textbook. Then I dropped in on him. A.D. wants to retire.
30.10. ...G. P. [Akilov] crashed his car but slightly. A.D. told me that S. L. has written a reply to *The Communist*. Yu. F. [Borisov] called me about extreme points.
3.11. At V. L. [Makarov]'s seminar in the morning. Then I visited S. L.—about *The Communist*. He showed me his reply. Next—at S. L.'s seminar with a Vietnamese. Then another conversation with S. L. about the article (in the breaks I talked to A.D.). S. L. spoke eloquently but slightly incoherently about sets and cardinality.
4.11. S. L. called me about arranging a meeting of the Scientific Council [of the Institute of Mathematics] vs. Pontryagin.
5.11. [The Scientific Council] unanimously condemned the Thesis Maintenance Council. The speakers were Serebryakov, Yu. G. [Reshetnyak], A.D., et al....
10.11. The whole day was full of discussions with S. L. and A.D. about Pontryagin and also about Reshetnyak and Zelenyak (in view of a scandalous meeting of the Academic Council to take place tomorrow [in NSU]).
11.11. An anniversary of Bourbaki. Yu. G.['s department in NSU] was reduced by 40% ...

12.11. A seminar with A.D. about Lenin's speech at the III Convent + vs. Pontryagin... Yu. G. discussed $L_p$ with me.
24.11. I looked for the list of the members of the Scientific Council with L. M. [Krapchan]. A.D. has arrived—he spoke against Dubinin at the General Assembly. Dubinin replied... A seminar about attractors with Ustinov (from Obninsk).
28.11. [The Scientific Council] unanimously supported the appeal by Zelmanov. Celebration of the 20 years of the M[athematical-]E[conomical]D[epartment].
3.12. I dropped in on S. L. about the resolution. He told that he will move it himself.
8.12. I dropped in on S. L. with A.D., V. A. [Toponogov], and V. V. [Ivanov]. Discussion vs. Pontryagin.
12.12. [We were] pretty close to adoption of the anti-Pontryagin resolution [at the philosophical-methodological seminar]. I dropped in on A.D. in the evening to talk this over.
15.12. I discussed the resolution with S. L. Then at his seminar... Bokut′ called me in a break about his troubles. Shirshov recommended Ershov for the party membership...
23.12. Talking everything over the whole day out with Tikhomirov who just arrived. Mainly in the anti-Pontryagin mood.
24.12. Theses' maintenance: [V. N.] Dyatlov (yea 18 nay 0 abstained 0) and [G. G.] Magaril[-Il′yaev] (yea 17 nay 0 abstained 0)... Everything was pretty nice... Booze & noise at Dyatlovs'...
25.12. Short discussion with A.D. in the morning... [The Scientific Council] adopted the anti-Pontryagin text + there will be a letter to *The Communist* to be prepared by A.D. + Yu. L. [Ershov] + [S. I.] Fadeev!

Such were the circumstances we lived in those days.

I remember the extraordinary stamina of A.D. (which was predictable) and Sergeĭ L′vovich (which was unexpected for me). The latter startled me on November 3, giving his reply to *The Communist*: "I am interested in your opinion but you should bear in mind that I have already mailed my reply." On the same occasion he showed me a copy of an analogous letter to somebody in the leadership of the Central Committee of the CPSU (seemingly, this was M. V. Zimyanin).

Many participants of these events are still alive. Some of them have changed for the better (and the rest of them still have a good chance to do the same). That is why I am reluctant to describe all details of the vehement struggle for a noble answer to the article by L. S. Pontryagin. I mention only that the crucial ingredient was the titanic joint efforts of Aleksandr Danilovich and Sergeĭ L′vovich.

In result, the Scientific Council of the Institute of Mathematics unanimously (sic!) adopted at its meeting on December 25, 1980 the resolution that read in particular as follows:

> *The Scientific Council announces its disagreement with those who informed the Editorial Board of The Communist about the situation in the science of mathematics which gave grounds for the editorial comment on the article by L. S. Pontryagin to make accusations of the noncritical adoption of foreign achievements, formalistic craze, disorientation of academic youth, and the false treatment of the subject of mathematics. Mathematics is an indivisible whole*

*and deterioration of its more abstract fundamental part resembles proscription of chromosome heredity theory, treatment of cybernetics as a "science of obscurantists," and prohibition of using mathematical methods in economics on the basis of false pseudoscientific arguments. Mathematics is a very serious matter of paramount importance for the development of our society. Therefore, treating it and judging it requires great responsibility.*

There was some cool in the relations of A.D. and S. L. that year (but I am disinclined to reveal the reasons behind this yet). Therefore, it happened so that the drafts of the resolutions were prepared with me acting as an intermediary. I keep these drafts with the scars of those "shuttle operations" in remembrance of the unforgettable material lesson of struggling for scientific truth.

It is worth observing that E. I. Zelmanov whose Kandidat thesis was rejected by secret vote as mentioned above acquired the Fields Medal a few years later.

The standpoint of Sergeĭ L′vovich was reflected by *The Communist* in the phrase: "Comments are still coming. Among them some are written in a polemic style: the letters by Academician S. L. Sobolev, Assistant Professor P. V. Stratilatov, and Professor Yu. A. Petrov." The chant "Academician Sobolev, Assistant Professor Stratilatov, and Professor Petrov" was our catch-phrase for a few years.

We attempted to print a booklet with the resolution of the Scientific Council and a detailed version of the report by A. D. Alexandrov "About the Article by L. S. Pontryagin in *The Communist* (1980:**14**)." Our attempts were unsuccessful—we were opposed by V. A. Koptyug. [4]

A.D. showed me a personal memo by V. A. Koptyug in which the latter—a censor (sic!)—reproached A.D. for a "persecutor's tone" and refused to publish the report.

Despite this the scientific community became aware of the standpoint of Siberian mathematicians: at Sobolev's request the copies of the resolution and A.D.'s report were sent to the principal mathematical institutions of this country.

Something similar happened later to A.D.'s book *Problems of Science and a Scientist's Standpoint* whose publication was procrastinated by the chiefs of the Siberian Division and became possible only after interference of P. N. Fedoseev who knew A.D. rather well and strictly obeyed academic etiquette in this matter.

## Sic Transit...<br>or Heroes, Villains, and Rights of Memory

April 25, 2003 is the date of the centenary of the birth of Andreĭ Nikolaevich Kolmogorov. The personality and creative contribution of this genius man to the world science and Russian culture are so eminent that the tiniest bits of recollections of anything related to him might be of avail to those pondering over life and its principles.

For many years I have heard requests of my friends and colleagues to present for the public my whatever partial overview of the circumstances and events invoked by Merzlyakov's article "The Right of Memory" and in particular the polemic between A. D. Alexandrov and L. S. Pontryagin this article had stirred up. The story to tell is rather ugly and to plunge into it again, reviving the bygones, brings about much discontent and displeasure.

[4]The Chairman of the Siberian Division of the Academy of Sciences from 1980 to 1997.

Unfortunately, the historical nihilism of these days intertwines rather tightly with nihilism in morality. "The past crimes are buried in the past. The past is absent at present. Therefore, the past crimes are absent now. So, let bygones be bygones." This sophism brings about the opinion that nobody could recall and take into account the crimes of the past in view of the period of limitations. This is correct but partly. The murderer remains a murderer for ever irrespective of whether or not he committed a negligent homicide and was relieved from persecution or has served his punishment and lives with no record of conviction. The thief is still a thief although she returned back the things she had pilfered and was relieved from punishment. No fact of assassination or theft is ever destroyed by whatever decisions about it. No error disappears unless it has been repaired. Always evil is to forget the past and its lessons... These arguments drove me to the decision of narrating about this gloomy episode of the past.

Merzlyakov's article appeared on February 17, 1983 in the newspaper *Science in Siberia* of the Presidium of the Siberian Division of the Academy of Sciences of the USSR. Yu. I. Merzlyakov (1940–1995), an established algebraist, with the titles of Doctor of Sciences and Professor, had a bit of reputation in the theory of rational groups. He was not an ordinary personality devoid of literary and other gifts and so won quite a few admirers. His article served many years as a credo of the Novosibirsk branch of the notorious "Memory" society, an informal nationalistic group sprang to life in the early years of Gorbi's perestroĭka.

To grasp the undercurrents of Merzlyakov's article completely is practically impossible for anyone far from the Russian mathematical life of those days. Moreover, the understanding of and attitude to this text varied drastically from capitals to province. Despite this, all Russian mathematicians clearly saw the implication of the following excerpt of Merzlyakov's article:

> *Academician Lev Semënovich Pontryagin is a brilliant exemplar of a scientist and patriot of these days. The International Astronautical Federation elected him an honorary member side by side with the cosmonauts Gagarin and Tereshkova for his outstanding scientific contributions. Skipping any description of all aspects of the versatile activities of L. S. Pontryagin,*[5] *I will dwell upon a single problem of a national-wide scale, the teaching of mathematics in secondary school. It was exactly L. S. Pontryagin who vehemently pointed out, in particular on the pages of The Communist,* [6] *the evil implications of the sharp turn to the course of the extreme formalization of mathematics which was imposed on our schools in 1967 and oriented consciously or unconsciously to the accelerated intellectual development (with an equally fast achievement of the utmost limits of creativity) nontypical of the majority of the country's population. The flood of responses to the statement by L. S. Pontryagin* [7] *has demonstrated that his criticism was quite timely and fair. In particular, Vice-President of the Academy of Sciences of the USSR Academician A. A. Logunov ascertained on the session of the Supreme Soviet of the USSR in*

[5]The initials seem abundant to the English eye but they reflect the style of the Russian polemics in which the presence of initials brings about some extra respect to the persons in question whereas the absence of initials clearly demonstrates slight indifference, disrespect, or even neglect. Every Russian professor knows that the initials of Gagarin are Yu. A., and the initials of Tereshkova are V. V. To keep the flavor of the polemic I preserve the authors' rules for placing initials in the Russian originals throughout.

[6]*The Communist*, 1980:**14**, p. 99–112.

[7]*The Communist*, 1980:**18**, p. 119–121; 1982:**2**, p. 125–126.

> *October 1980 that there is a grieve situation about the teaching of mathematics in secondary school and to learn mathematics from the present-day textbooks "can destroy any interest in not only mathematics but also exact sciences in general." (I remark parenthetically that the leader of the reform received the prize of 100,000 dollar in 1980 from the state with which the USSR had severed all diplomatic relations exactly in the year of the beginning of the reform.)* [8]

The rest of the article was mainly inspired by the outright scandalous situation in the midst of logicians and algebraists of Novosibirsk and in the whole mathematical community of Siberia either. The point was that the retirement of S. L. Sobolev was pending from the position of the director of the Institute of Mathematics of the Siberian Division of the Academy of Sciences of the USSR. This evoked the battles for power and better places under the sun which were typical of the academic community of those days.

I am disinclined to dwell upon the other details of Merzlyakov's article since I fully agree with the estimate of Sobolev who expressed his attitude to the hysterics by Merzlyakov as follows: "The role of Savonarola befits no twentieth-century scientist."

Sobolev forwarded his sagacious and valiant letter from Moscow to the management of the Institute on March 9. He rejected the slander against Kolmogorov and justly gave a negative estimation of the whole article. I had an opportunity to read this hand-written page of a copy-book which unfortunately was unwelcome by some of the addressees, concealed for a long time, and made public by S. K. Godunov only after fierce battles and conflicts at the meeting of the Scientific Council of the Institute on April 18. The principled and uncompromising position of Sobolev seemed to the many less important than the opinion of local Party leadership. A few iterations under the pressure of petty Party bonzes brought about the official position of the management of the Institute which recalled the merits of Kolmogorov while observing that Merzlyakov appropriately posed the problems of patriotism.

Patriotism and slander... A notorious mixture...

Some unpleasant general thoughts are in order now about professionalism and mathematicians. Professionalism requires absolute devotion to profession and, absorbing personality, tends to impoverish the latter. Professionalism appears amidst mathematicians rather early whereas the upbringing of necessary moral qualities is often far from a fast and easy matter (mathematicians are next of kin to sportsmen in this respect). Of little secret are the elements of gossip, jealousy, and envy encountered the world over even among the first mathematicians. Hatred to the gifts of the others is often mixed or replaced with xenophobia, racism, antisemitism, and similar elements of the same sort. These phenomena are still far from rare nowadays. The oversensitive reaction to the slightest traits of the presence or absence of antisemitism was and still is a litmus test of "friend-enemy" in Russia irrespective of whether this is right or wrong. I believe that to grasp correctly the tension of the events after Merzlyakov's article is impossible without the clear understanding of the above circumstances of the Russian life.

By the way, somebody told me that the then editor-in-chief of the newspaper *Science in Siberia* tried to justify himself on explaining that he had slightly deviated from the standard routine of accepting materials for publication in order to insert

[8] *Notices of the AMS* (1981) **28**:1, p. 84.

Merzlyakov's article in the issue on the Day of the Soviet Army because he viewed it as exceptionally patriotic. In our midst we have called these views "slanderous patriotism" since then. Mixing love for the Fatherland with slander is always characteristic of "the last resort of a scoundrel."

The Moscow mathematical community reacted to Merzlyakov's article immediately and adequately in general. The understanding prevailed that the lampoon could strike the health of Kolmogorov which was already shaken seriously. Surely, nobody showed the newspaper to Andreĭ Nikolaevich but his 80th anniversary approached rapidly and Merzlyakov's article could provoke some undesirable predicaments: for instance, there might have been no ceremonial decoration from the government which could be noticed by Kolmogorov, stirred up his analytical interest and investigation with possibly unfavorable aftereffects to his health.

Another circumstance helped to the spreading of a noble reaction: The article appeared on the eve of the General Assembly of the Academy of Sciences of the USSR in Moscow where several copies of the issue of the newspaper were delivered immediately. The exceptionally sharp reaction against slander and political snitching was revealed by the leading mathematicians: A. D. Alexandrov, S. M. Nikol′skiĭ, S. P. Novikov, Yu. V. Prokhorov, S. L. Sobolev, L. D. Faddeev, and many others.

Already on March 14 there appeared the first written response by Alexandrov with an analysis of Merzlyakov's article. Characterizing the article as objectively anti-Soviet and subjectively base, Alexandrov demonstrated the necessity of terminating all instances of slander and political insinuation. Closing his response, Alexandrov wrote:

> *Yu. Merzlyakov himself has clearly deserved the right of memory. Since at least some of his statements are so evil and monstrous that might go down in history.... We have thus seen that Merzlyakov's article is an objectively anti-Soviet, subjectively base, rude, and antipatriotic composition, its every appeal to patriotism notwithstanding.*
>
> *Let us abstain from judging the author severely but rather pity him since we observe an indubitable pathological case. Only a perverted mind and turbid imagination can bring about such a flood of insolence and mud! Renegades, domestic emigrants, immature moral viewpoints halfway from amoeba to cave-dweller, a shitting bull, a beast, a toady-like mediocrity of a petty shop-keeper and, to crown all these, the monstrous image of villains that crawl to loot the wounded as description of the "horde" of scientific workers and, in particular, his fellow colleagues. Well, that is the limit: an obvious pathology.*

We are to pay tribute to the Mathematics Division of the Academy of Sciences of the USSR and personally to Yu. V. Prokhorov who was an initiator and editor of the following Resolution of the Bureau of the Mathematics Division as of March 25, 1983:.

> *Academician Yu. V. Prokhorov informed the body about a recent article in the weekly newspaper Science in Siberia of the Presidium of the SDAS*[9] *of the USSR (No. 7 of February 17, 1983) by Yu. I. Merzlyakov, Sc.D. on the staff of the Institute of Mathematics of the SDAS of the USSR. This article contains a uniquely*

[9]The abbreviation of " Siberian Division of the Academy of Sciences."

*decipherable insinuation against Academician A. N. Kolmogorov, an outstanding Soviet scientist.*

*The floor for discussion was taken by Academicians S. M. Nikol′skiĭ, V. S. Vladimirov, S. P. Novikov, A. A. Samarskiĭ, S. L. Sobolev, and L. D. Faddeev; Corresponding Members of the Academy of Sciences of the USSR A. V. Bitsadze, I. M. Gelfand, A. A. Gonchar, and S. V. Yablonskiĭ. All of them unanimously condemned the indecent insinuations of Merzlyakov's article and qualified them as slander against one outstanding scientist and patriot. It was also observed that the article contains insinuations against a number of other Soviet mathematicians.*

*The Bureau of the Mathematics Division of the Academy of Sciences of the USSR HAS DECIDED*

1: *to observe that the article of Sc.D. Yu. I. Merzlyakov "The Right of Memory" in the newspaper Science in Siberia of the Presidium of the SDAS of the USSR contains slander against one outstanding scientist/mathematician and Soviet patriot;*
*to observe that the article contains a number of indecent attacking allusions to other Soviet mathematicians.*

2: *to call upon the Presidium of the Siberian Division of the Academy of Sciences of the USSR to take due measures pertinent to Item 1.*
*The Resolution of the Bureau of the Mathematics Division of the Academy of Sciences of the USSR was adopted by a unanimous vote.*

The bushes of provincialism were already full-fruited in Siberia those days, and the solicitude for the honor, dignity, and health of Kolmogorov together with counteraction against the filthy things like antisemitism seemed to the chosen few to be negligible as compared with the prevailing sentiments for their own career, success, fame, and prosperity. The following story of Alexandrov looks like a joke nowadays: one of the top bosses of the Siberian Division responded to the protest and indignation against Merzlyakov's article with the sincere question: "Who is that Kolmogorov guy?" One can easily imagine our reaction...

On March 28 there was a meeting of the Presidium of the SDAS of the USSR. The official letter of the Institute, bearing the signatures of the three deputy directors and the party secretary, was announced together with the second milder letter of Sobolev who was in Moscow. The "Savonarola" letter was never mentioned. Unfortunately, the official copy of the Resolution of the Bureau of the Mathematics Division did not arrived at Novosibirsk (the time of facsimile communication had not come yet). Alexandrov briefed the audience about this Resolution. However, not without reason it is said:"you're nobody till somebody gives you a sheet of paper." V. A. Koptyug, never feeling anything positive towards Alexandrov, moderated the discussion with reference to the unclear standpoint of the Institute of Mathematics and the absence of the Moscow Resolution in writing. Of no avail were the vehement statements of members of the Presidium Academicians G. K. Boreskov, S. S. Kutateladze (1914–1986), and A. N. Skrinskiĭ who condemned the slander against Kolmogorov and insisted on a principled reaction. In result there was adopted a rather insipid resolution which stated that the editorial staff of the newspaper made a serious mistake by publication of Merzlyakov's article "written in the style inadequate to the spirit and aims of the newspaper." That was how slander had become a style in the opinion of a part of the then leadership of the Siberian Division.

The efforts of the supporters of Kolmogorov brought about a tactical success: the Decree of the Presidium of the Supreme Soviet of the USSR was signed on April 22 upon the decoration of Academician A. N. Kolmogorov with the Order of the October Revolution for his great contributions to the development of the science of mathematics and the long-term and fruitful pedagogical activities on the occasion of the 80 years of his birth. It seems to me that Kolmogorov had never become aware of Merzlyakov's article.

Of great importance to Novosibirsk was the publication in the issue of May 12 of the newspaper *Science in Siberia* of an article about Kolmogorov which was written by S. L. Sobolev, A. A. Borovkov, and V. V. Yurinsky. Their article ranked Kolmogorov as one of the most eminent mathematicians on the twentieth century, an outstanding teacher, an ardent patriot, and the founder of his scientific school of a worldwide reputation and few analogs in the history of science. The authors particularly emphasized the undisputable influence of Kolmogorov on the development of mathematics in Siberia.

This did not close the case however. "The Special Opinion of L. S. Pontryagin" was made public already on April 30. In this article Pontryagin expressed his disagreement with the Resolution of the Bureau of the Mathematics Division (he was a member of the Bureau but missed the meeting on March 25 since he was ill). He refuted the accusation against Merzlyakov of slandering Kolmogorov and estimated the article "generally in the positive since it summons up citizenship which is in great demand of our scientists."

In particular, Pontryagin wrote:

> *I ascertain that the statement of Yu. I. Merzlyakov about Kolmogorov, even in deciphered form, cannot be viewed as slander. It does not ascertain any causal relation between the failure of the reform and the awarding of the prize. But the thought about a causal relation can be borne in upon the reader.*
>
> *It was after this meeting already that I received some responses to the Yu. I. Merzlyakov article. One of them showed disapproval (by Academician A. D. Alexandrov) and three of them showed approval (by Academician/writer Leonov; Mathematician, ScD V. D. Mazurov; and the chiefs of the Mechanics and Mathematics Department of NSU:*[10] *Dean M. M. Lavrent′ev and Secretary of the Party Bureau D. E. Zakharov).*

"The Special Opinion" pinpointed a few rare facts of public subscription to soiling Kolmogorov's reputation. Pontryagin's text full of the bits of an open polemic with Alexandrov raised the question: "Whom does A. D. Alexandrov defend so vehemently in his response?". There was little doubt that Alexandrov would leave this question rhetorical.

Alexandrov finished his response to Pontryagin on May 28. Confirming his view of Merzlyakov's article as politically slanderous insinuation, Alexandrov wrote:

> *In my response to Merzlyakov's article I characterized this as baseness and I reiterate: this is baseness, and the meanest baseness at that.*
>
> *Academician Pontryagin is not a young man and he knows the intended consequences of such baseness in the times of the year 1937. He could know in particular that Nikolaĭ Nikolaevich Vavilov, a great Russian scientist/biologist, died in prison since someone casted a political slanderous innuendo about him. Now*

[10]The abbreviation of Novosibirsk State University.

> *Academician Pontryagin supports the revival of political slanders and insinuations and even discerns some "citizenship" in them. However they were condemned by our party and people long ago. It is the Bureau of the Mathematics Division that revealed the genuine citizenship by repulsing Merzlyakov's slander. The "citizenship" in the sense of Pontryagin was revealed already in his article in The Communist where he spread slander against our mathematics. Now it is revealed once again in his "Special Opinion" supporting baseness and slander against not only A. N. Kolmogorov but also the whole school of our scientists which supposedly incorporates a crawling horde of the most monstrous careerists and villains...*

The copies of the March Resolutions of the Bureau of the Mathematics Division and the Presidium of the SDAS of the USSR were displayed on the advertisement board of the Institute of Mathematics of the SDAS of the USSR from July 2 to July 7. So ended the crisis of "patriotically slanderous citizenship" at Novosibirsk in 1983.

The above events in the history of science in Russia may be compared only with the so-called "Case of Academician N. N. Luzin." The pivotal distinction of the year 1986 from the year 1936 lies in the fact that the personality of Kolmogorov had morally united the overwhelming majority of the Russian mathematicians who shielded their professional community from slander and political insinuation.

**Sic transit separation.**

## Science at the Center of Culture

A.D. was a person with a universal outlook. Through much suffering he did achieve a perfect system of views that allowed him to analyze the general philosophical problems and meet the challenges of contemporary life.

I had many opportunities to attend his public lectures which always evoked a vivid response of any audience. I recall his brilliant talk at the conference "The Place of Science in the Modern Culture" which was arranged in Academgorodok near Novosibirsk in the end of April of 1987.

A.D. titled his talk "Science at the Center of Culture" so biting a part of the audience with antipathy to science. In my files there still reside some records of the main points of his talk. I insert a few of them here.

- *We live in the age of science.*
- *False theses: science beyond culture; science as next of kin to utopia and ideology; science as a tool for dehumanization.*
- *This is a spite of philosophers. A philosopher is an unsuccessful scientist full of mania grandiose.*
- *Science occupies the center of culture. Objectively, science is a system of knowledge and conception... Man must stand in the center of science. Man is not only a creator but also an object and ultimate aim of research and thought. Science asks not only "How?" but also "What for?".*
- *Truth is a tool of good. Science leads to truth and its entire credenda appeal to the mind so liberating human mentality.*

A.D. knew much about religion, always contrasting religious belief and scientific search. With love to precise definitions innate in mathematicians, he often cited the following words by Vl. S. Solov′ëv from the article "Faith" in the Encyclopedic Dictionary by F. A. Brokgauz and I. A. Efron:

> **Faith** *(philos.) means the acceptance of something as being true with the resolution surpassing the power of external proofs by fact and formal logic.*

A.D. was fond of reiterating that he believes in nothing. This statement usually called about the retort of the audience: "Neither in Communism?" which always gained the affirmative answer of A.D. It goes without saying that the lectures of A.D. were often accompanied with sneaking letters to various local party committees.

A.D. had explicated his views of interrelation between religion and science in the booklet "Scientific Search and Religious Belief" which was published by Politizdat in 1974 with run of many thousand copies. It seems to me that this article does not lose its actual value nowadays in the time of an unprecedented blossom of mysticism and pseudoscience.

## O. A. Ladyzhenskaya and a Struggle Against the Last Insinuations

At the end of the 1980s Aleksandr Danilovich suddenly became a target of some slanderous attacks that ran as far as accusations of "lysenkoism." Yu. G. Reshetnyak and I were compelled to write much about A.D. Hatred to calumniators boiled in our soles. However, we worked at ease feeling the inspiring warmth of final exposition of a just-proven new theorem. In the most critical moments of controversy we readily found out many objective facts witnessing the intellectual honesty of A.D. and his devotion to serving science and taking care of the fates of his fellow scientists.

Stuffed up with concocted reminiscences, massaged citations, archived data full of sneaking and quasi-sneaking letters to "competent authorities" and having mastered up many tricks typical of a barrister, I grew up to appraise the moral standpoint of O. A. Ladyzhenskaya tied with A.D. by many years of friendly relations.

In the spring of 1989 I happened to be in Leningrad at the peak of controversy about Alexandrov's "lysenkoism." Olga Aleksandrovna asked me to visit her in LOMI (the Russian abbreviation for the Leningrad Department of the Steklov Institute). In contrast to the majority (including some friends of A.D. who always requested the objective proofs of A.D.'s innocence), Olga Aleksandrovna rejected from the very beginning all my attempts to show papers, compare figures, etc.: "Sëma! I need none of this stuff. Tell me only what we must do for A.D."

It seemed to me that the formal position of Leningrad's mathematicians will be important for A.D. Olga Aleksandrovna agreed with this opinion. She was then a deputy chairperson of the Leningrad Mathematical Society (LMS) (and the chairperson was D. K. Faddeev).

Soon after that V. A. Zalgaller sent me to Novosibirsk the following statement of the LMS which was adopted unanimously at the meeting of March 28, 1989:

> *Concerning the publication by the journal Energy (1989:***1***) of a letter of Academician of the Siberian Division of the Academy of Sciences of the USSR V. E. Nakoryakov, the Leningrad Mathematical Society (LMS) announces that the letter by V. E. Nakoryakov contains slander (refutable by proof) and an attempt to defame Academician A. D. Alexandrov, a member of the LMS and an outstanding mathematician. The Leningrad scientists remember many good deeds by A. D. Alexandrov: his efforts helped to save science and his fellow scientists in the grim years which required his great personal fortitude.*

A.D. was touched with this statement. Also, it was a great help for Yu. G. Reshetnyak and me in the public polemics of those years.

The reader seeking for more detail can restore the main events by looking through the corresponding publications in the *Herald of the Academy of Sciences of the USSR* (1989:**7**; 1990:**3**) and the relevant articles in the issues of the newspaper *Science in Siberia* of March 10 and October 13, 1989.

When a decade has elapsed, sharp contrast transpires between the figure of silence (*aposiopesis*) of the top officials of the Academy such as V. A. Kirillin, V. A. Koptyug, G. I. Marchuk, et al. and the behavior of the scientists who consider the defense of the honor of a colleague against slander as their personal duty.

I keep a few letters that were unpublished in view of the standpoint of the then Academy bosses. I cherish the words of my long-term friend V. M. Tikhomirov, a professor at Lomonosov State University in Moscow:

> *I am sure that A. D. Alexandrov belongs to those who have always served the forces of good. I wish to express through your newspaper my feeling of admiration for him, his brilliance, intellectual gift, and human generosity. I've never heart that Aleksandr Danilovich caused harm to the persons he met in life but I heard that he helped them and promoted the development of science.*
> *Of utmost importance for me are the words by V. I. Smirnov, a person of unsurpassable moral standards, who wrote that A. D. Alexandrov controlled the University using the power of moral authority!*

There is no denying that the attitude of contemporaries meant much to A.D.

I have no desire to expatiate upon this story even though it had a "happy end": In October of 1990 A.D., the only mathematician in a group of biologists, was decorated for special contribution to preservation and development of genetics and selection in this country.

The Decoration Decree appeared by the initiative of Professor N. N. Vorontsov who then hold the position of the Chairman of the Governmental Committee for Nature of the USSR. In a lengthy interview to the newspaper *Izvestiya* as of November 3, 1990 Nikolaĭ Nikolaevich testified:

> *Aleksandr Danilovich was the Rector of Leningrad State University and he made much for preservation and development of genetics. He invited to LSU many of those expelled for their scientific views from other cities. Young persons simply fled to Aleksandr Danilovich Alexandrov to gain custody. The courses of lectures in LSU differed drastically from the Lysenko rigmarole that was delivered (and, I am afraid of that, is still delivered) by the teachers of agricultural colleges. This determined the general atmosphere of the academic life of Leningrad.*
> *Alexandrov took care of the level of science as a whole. All scientists know: liquidation of one of the branches will bring about repercussions on the entire frontiers of science. That is why in many running years, many physicists and mathematicians wrote letters to the Central Committee of the Party about the importance of genetics. By the way, when somebody says that A. D. Sakharov was late in taking the road of political struggle, it is not true. His name appeared in the letter of physicists of 1953 together with the names of Kapitsa, Semënov, and Varga. This letter was handled to Khrushchëv by Kurchatov. The letter of physicists was followed by a letter of mathematicians: Kolmogorov, Sobolev, Alexandrov, and Lavrent′ev. I was a first-year postgraduate when I collected their signatures.*

## The English Language

Another not universally known trait deserves mentioning. A.D. was a person of a discriminating artistic taste with a poetic gift. He wrote many poems and plays but most of them are lost since he had an absolute memory and wrote them down only at somebody's request or to make a present of them.

A.D. was in full command of the English language: He delivered lectures, cited classics, and even wrote poems in English. S. I. Zalgaller saved in her memory the following lines:

*My heart is full of burning wishes,*
*My soul is under spell of thine,*
*Kiss me: your kisses are delicious*
*More sweet to me than myrrth and wine.*
*Oh lean against my heart with mildness,*
*And I shall dream in happy silence,*
*Till there will come the joyful day*
*And gloom of night will fly away.*

Not later than in 1944 A.D. had made this interpretation of a celebrated poem that was written by A. S. Pushkin in Russian as far back as in 1825 and soon became an immortal romance by M. I. Glinka.

It is curious but one of our first conversations in the mid-1960s ran in English (I was a freshmen; and A.D., a brand-new academician). I recall the presence of some "English-speaking" diplomat in the hall of a small canteen in the Golden Valley where we dined those years. A.D. remarked that it is indecent to use the language that is not comprehended by everyone present and we proceeded in English.

I also recall an episode of the 1970s when on some occasion I cited a few lines of the 66th sonnet by W. Shakespeare in English, and A.D. continued recital in a flash. This took place long before the famous Georgian "Repentance" by T. Abuladze.

The circumstances of the beginning of the 1990s drove me to writing a short booklet on English grammar to alleviate the burdens of life for my friends who were seeking some sources of nourishment. A.D., always a very attentive reader, pinpointed a slip in a King James's citation of Ecclesiastes.

And in June of 1993 he sent me the following lines in a sloppy handwriting:

*Since legs, nor hands, nor eyes, nor strong creative brain,*
*But weakness and decay oversway their power,*
*I am compelled forever to refrain*
*From everything but waiting for my hour.*

He has never sent me any verses since then...

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# MEMOIRS ON A. D. ALEXANDROV AND HIS LENINGRAD GEOMETRY SEMINAR

V. A. ZALGALLER

Abstract. This is a tribute to Alexandr D. Alexandrov on the occassion of the centenary of his birth.

*In science, new stems grow only from a powerful root*

## 1. FIRST ACQUAINTANCE

In 1937, enrolment in the Mathematics and Mechanics Faculty of Leningrad University (Mat-Mekh) was unusually high: 210 students. The "big Chemistry lecture hall" in the main building on the Neva embarkment hardly contained us. At the beginning, analytic geometry was taught by Professor A. R. Kulisher (1878–1937). But in September, he was already arrested and replaced by a 25 year old Professor Alexandr Danilovich Alexandrov (hereafter referred to as A.D.). An area in front of the blackboard in this hall was separated from the rest of the hall by a wooden barrier with a wicket for the lecturer. Once, deciding to play a joke on their professor, the students locked the wicket prior to his arrival. As he approached the wicket, A.D. looked at the audience, made a step back, jumped over the barrier and imperturbably started his lecture.

The subject was simple. But A.D. talked in passing about affine geometry and mentioned some unsolved problems. Strong students were interested and attracted by that. A.D. was a lecturer from science, from creativity, but not from routine teaching. A.D. could have fun, too. To a party of my fellow-students, A.D. came with his postgraduate students S. P. Olovyanishnikov (1910–1941) and I. M.

This is an authorized English version of an article in Russian which appeared in *Академик Александр Данилович Александров. Воспоминания. Публикации. Материалы.* Ред.: Г. М. Идлис, О. А. Ладыженская. М.: Наука, 2002.

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Liberman (1917–1941). All three were wearing tuxedo and cylinders. Out of the entertainments initiated by them, I remember the selection of the "Queen of the Ball." They lifted the "Queen" in her chair and carried her around the hall.

The two students achieved outstanding results. When coming from the battle front in 1941, they defended their theses but later were killed in the war. After the war, A.D. published their works and a story about their lives.

In my second year, A.D. lectured on Differential Geometry (this time, instead of an arrested Associate Professor V. I. Milinski) and taught a lab in my group of students. In the Department of Geometry, in contrast to Analysis, there was no established standard with respect to labs. A.D. suggested to all so inclined not to come for the labs but instead solve a series of difficult problems. A few of my friends and I accepted this suggestion and had the pleasure of discussing solutions with A.D. At the end of the third year, some of us had own results. The most creative of my friends was P. O. Kostelyanets (1920–1943). He had found a way of solving a problem that was raised by A. N. Kolmogorov (1903–1987) in *Russian Mathematical Surveys*. Kostelyanets perished in battle in 1944. His research was completed and prepared for publication by another student of A.D., Yu. G. Reshetnyak.

In the fall of 1940, by a Komsomol call-up, I (like many of my peers) transferred to a newly created Aviation Institute. We then joined the army, and, for 5 years, I did not see A.D.

In the late fall of 1945, as an ex-serviceman, I came back to Leningrad University and started immediately to attend the already functioning Geometry seminar led by A.D.

## 2. ALEXANDROV IN 1945

A.D. was a widely educated man. He was lucky with his learning environment. His father was the principal of a gymnasium and his mother was a honored Biology teacher. He had mastered German and English in his childhood. (AD. even wrote verses in English.) He had a subtle understanding of and love for classical music...

In the Physics Faculty, which he graduated from in 1932, his supervisor in Physics was an outstanding physicist-theorist V. A. Fok (1898–1974)) and in Mathematics—a geometer, crystallographer, algebraist and outstanding pedagogue B. N. Delone (1890–1980). In 1932, when A.D. started his work in the RIMM (Research Institute of Mathematics and Mechanics) at the Mat-Mekh, he already had some publications in Optics and had taken part in writing the book *Mathematical Foundations of the Structural Analysis of Crystals*.

A.D. was highly influenced by the books: *Space, Time and Gravity*' by V. A. Fok, *Methods of Mathematical Physics* by R. Courant and D. Hilbert, *Theory of Convex Bodies* by T. Bonnesen and W. Fenchel. He was also influenced by his acquaintance in the RIMM with S. Cohn-Vossen (1902–1936), a German refugee and a student of Hilbert who used visual methods in the theory of geodesics on surfaces.

Delone involved A.D. in mountaineering and A.D. acquired the state title "Master of Sports" in mountaineering. He also drew A.D.'s attention to the aforementioned book by Bonnesen and Fenchel. This book was issued in 1934 in German and briefly summarized results of the Geometric school of D. Hilbert (1862–1943) and H. Minkowski (1864–1909). It also put forth new fundamental unsolved problems.

As early as in 1936, A.D. advanced considerably the whole Minkowski's theory of mixed volumes.

The Leningrad Department of the Steklov Mathematical Institute (LDMI) was established in 1940, and A.D., then a doctor of physics and mathematics since 1937, became one of its first research employés. In 1938–1940, A.D. got his first results on convex surfaces. In particular, he proved that such a surface has the second differential almost everywhere and developed a general theory of additive set functions.

In spite of his desire to join the army, A.D., as a Doctor of Science, was not allowed to do so.

The years of evacuation of the LDMI to Kazan (1941–1944) were extremely fruitful for A.D. He suggested a visually-geometric approach to a series of problems and solved a part of the most difficult questions of the theory of convex polyhedra and convex surfaces. For these results, he received the Stalin prize for 1942. A.D. returned to Leningrad full of ideas and creative plans.

## 3. A.D.'s SEMINAR IN 1945–1952

The seminar started in the 1945–1946 academic year. In the beginning, the talks were given by A.D. himself. I remember, he familiarized us with such basic concepts as metric, arc length, shortest paths and geodesics, and also with concepts (introduced by him) such as upper angle, existence of a direction of a curve at its starting point etc. In this process, properties of the concepts and their possible generalizations were discussed in depth and problems were suggested. In the beginning, the audience was changing. These years, students of the second to the fourth year attended many seminars choosing a suitable program or a topic for their Master thesis. Eventually, the set of participants stabilized. There was created an atmosphere of equality of all speakers in the face of scientific truth independently of their ranks. This atmosphere encouraged free discussion. In contrast to many modern seminars (let alone conferences) where only final results are reported while the proofs are presented very briefly, in this seminar one could also discuss projects and various approaches to problems. The presented proofs were checked collectively. During breaks, A.D. organized games such as jumping lengthwise from standstill and lifting a chair in a stretched hand. A.D. always suggested to us the value of an independent learning and unity of the whole science. He stimulated our interest to a wide scope of topics of his area. When talking about methodology, he often pronounced the slogan: "Retreat to Euclid!", fighting for visual methods. He posed problems underlining their importance and estimating their probable complexity. In this process, as a joke, he told how his teacher, Delone, maintained colossal importance of each consequent problem; but when A.D. produced its solution, he heard: "Well, this is just a little thing. What is really important is the following question... ."

In 1946–1948, A.D. was writing the two books: *Intrinsic Geometry of Convex Surfaces* and *Convex Polyhedra*. He liked the style of Cohn-Vossen, who first described in his works the problems and results in a very accessible way and only then presented the proofs in detail.

Naturally, strong students, inclined to geometric thinking. clung to A.D. As years passed, the seminar was getting stronger and stabilized in 1952.

A.D. was famous for his exclusive scientific generosity. He granted topics and prospective ideas to his pupils. Mutual generosity became one of the features of this seminar. Openness of discussions made it easier for graduate and postgraduate students to choose topics for their theses and overcome difficulties arising from their immaturity in adjacent areas.

Friendly peer editing of publications of young participants of the seminar was also a common practice. Let me tell briefly about the participants who became Doctors of Physics and Mathematics (a rank higher than Kandidat).

## 4. ALEKSEY VASIL'EVICH POGORELOV (1919–2002)

In the very beginning of the meetings of the seminar, A.D. told with admiration about a young military man who came to him in 1945 for geometric problems and soon brought all the problems solved. That was Pogorelov, a Kharkiv student of the Physics and Mathematics Faculty of KhSU (Kharkiv State University) in 1936-1941, admitted in 1941 to the Zhukovskii Military Aviation Academy in Moscow who, along with his study in the Academy, attended the geometry seminar in Moscow University. Then he was an external post-graduate student of Moscow University and, after getting his Kandidat, a member of the Mathematical Institute of the Academy of Sciences of the USSR (Steklov Institute) working for his degree of Doctor of Physics and Mathematics. Formally, he was not an A.D's student. But the source of his outstanding geometric research lay in A.D.'s bunch of problems. It is not a coincidence that Pogorelov collected his results in a book entitled *Extrinsic Geometry of Convex Surfaces*, echoing the book *Intrinsic Geometry of Convex Surfaces* by A.D.

At every conference dinner or thesis defense banquet, Pogorelov, smiling merrily and facing A.D., pronounced a standard toast: "To my dear teacher!" (To that, A.D. replied once: "When drinking, you are together. But when getting the Lenin prize, you are alone!") A.D. appreciated Pogorelov very much and dreamed to win him over to Leningrad. For one term, Aleksei Vasil′evich even replaced A.D. for his lectures and, in 1955–1956, taught the whole academic year in Leningrad University. However, he returned then to his native Kharkiv.

In 1960, in Kharkiv, there was created an academic Institute of Low Temperatures. Its director B. I. Verkin (1919–1990) organized both physics and mathematics departments. In particular, the Institute has a "Section of Geometry." Pogorelov, who was a full member of the Russian Academy of Science and the National Academy of Sciences of Ukraine and a laureate of the Stalin and Lenin prizes, was its head just until his removal to Moscow. Members of this Section and Pogorelov's postgraduate students were frequent guests of the Leningrad seminar. Moreover, *Ukrainian Geometric Collection* willingly accepted papers of Leningrad geometers.

Pogorelov had not only outstanding geometric and analytical results but also some serious achievements in Engineering. He created a theory of supercritical deformations of thin shells and confirmed it by a series of laboratory experiments. This theory, reported in our seminar, was based on geometry and energy arguments. In 1970, Pogorelov became one of the fist researchers who suggested a new idea of construction of a turbogenerator with a superconducting energizing winding and took an active part in calculations and technical developments of industrial constructions of cryoturbogenerators. Unlike A.D. who liked to overcome difficulties

by looking at problems from a new point of view varying the very setting of the problems and using successful generalizations, Pogorelov preferred in Geometry a straightforward approach to the problems posed precisely before him.

## 5. YURI GRIGORIEVICH RESHETNYAK

As a juvenile, he found himself on the occupied territory. His family, having returned to Leningrad, lived in a very small room, and Yura studied in libraries. He saved time by finishing grades 9 and 10 in one year. A freshman in 1947–48, he was in the A.D.'s seminar since his second year. He studied all the subjects beyond their syllabi. He saved time again by completing the fourth and fifth year programs in one year. He was A.D.'s postgraduate student from 1951 to 1954.

I remember, at the defense of Reshetnyak's Kandidat thesis (on the integral geometry methods in the theory of curves), somebody said to his opponent S. M. Lozinskii (1914–1985): "I think you said that the thesis deserved a degree of Doctor of Sciences." Lozinskii responded: "I did not pronounce that. But I take this opportunity to say—yes, I think so." After his postgraduate study, starting from 1954, Reshetnyak worked in the LDMI. He traveled to office from Pulkovo where he lived in the room of his wife, an employee of the Pulkovo observatory.

Reshetnyak is exclusively conscientious not only in his own research but also when refereeing somebody else's works. He always notices shortcomings (and, out of his generosity, suggests corrections). If the error is gross, he writes to the author. Once, having noticed an inaccuracy in a paper of Academician S. L. Sobolev, he addressed him. In reply, Sergey Lvovich invited Reshetnyak to work in Novosibirsk, at the Institute of Mathematics of the Siberian Division of the Academy of Sciences which was then under construction. Reshetnyak accepted this invitation in 1957.

These days, Yu. G. Reshetnyak is an academician of the Russian Academy of Sciences and a Consultant of the Sobolev Institute of Mathematics in Novosibirsk.

## 6. YURI ALEXANDROVICH VOLKOV (1930–1981)

He lived in Kazan. In his school years, he suffered bone tuberculosis of an arm which prevented him from attending school. Therefore he educated himself by studying textbooks.

When a student of LSU, he spent all his time in the Public Library and, on Wednesdays, perusing the new acquisitions of the Library of the Academy of Sciences. He was perhaps the most well-read amongst A.D.'s students. Once at a conference, Volkov, then a third year student, instantly pointed to a crucial mistake of a venerable speaker on continuum mechanics.

After about a year, the same mistake was discovered by an academician, and the theory was revised. In the dormitory, he helped Physics students to write theses on quantum mechanics. After his graduate study, he remained at the Geometry Department as an assistant professor. He suggested to teach Mechanics students their courses in tensor presentation and give Mathematics students a course on the three mathematical methods of presentation of Quantum Mechanics. Unfortunately, these suggestions were ignored.

In the A. I. Herzen Pedagogical Institute (now University), he taught a special course on comparison of features of the existing texts (foreign inclusively) on geometry. When heading the Geometry Department in LSU (after Yu. F. Borisov's departure to Novosibirsk), he, together with D. K. Faddeev (1907–1989) and V. A.

Rokhlin (1919–1984), created the most closely coordinated curriculum between all Departments. (However, by the reasons of "simplification," this plan held only for two years.)

Volkov has many deep scientific results following the lead of A.D.'s school. In particular, he solved two important problems. On the path dreamed of by many geometers, he proved the existence of a convex polyhedron with a given development by constructing in an abstract class of objects an extremal problem for which the extremum exists by compactness and the extremal object is the desired polyhedron. He also solved a Cohn-Vossen problem, obtaining an estimate of the spatial deformation of a closed convex surface in terms of deformation of its intrinsic metric. Volkov was a modest and reserved man. As a post-graduate student, he avoided trips to A.D.'s dacha. When heading the Geometry Department, he shied away from coming to the Deans office. He prepared his courses perfectly but assumed that the students read the texts. Therefore some of his remarks were not understood by them. He examined his students attentively but with no rush. Students were afraid of him. But his postgraduate students adored him. He helped them skillfully to develop their knowledge, talking to them for hours in the evenings at his home. As generous as A.D., he gave them excellent themes. Here is a possibly incomplete list of his former post-graduate students (all of them having visited A.D.'s seminar): Mekhtiev, an assistant professor in Makhachkala; Dekster, a professor in Canada; Kagan was an assistant professor in the South-West Polytechnic Institute; Podgornova, an assistant professor in Tashkent; El-Etrebi (Saudi Arabia); Abu-Dhabi (Egypt); Nevmerzhitski works in St. Petersburg; Lifshits died in a car accident in 1976; Alekseeva, an assistant professor in the Wood Industry Academy; Kishukov, an assistant professor in Nalchik, Alexandrova; and Brumberg.

Remembering his orphanhood, Volkov was a very thoughtful father of his two sons. He died very young, leaving good memory in everyone who knew him.

## 7. IL'YA YAKOVLEVICH BAKEL'MAN (1928–1992)

He entered the seminar being a second year student in the1947/48 academic year. During his study, he learned not only from A.D. but also from S. G. Mikhlin (1908–1990), V. I. Smirnov (1887–1974), O. A. Ladyzhenskaya (1922–2004), and V. A. Rokhlin. A gifted student, he learned a lot and worked actively in the area involving geometry, partial differential equations, and functional analysis.

Three years after graduating from LSU, attending no postgraduate program, he became a Kandidat in 1954. He received the degree of Doctor of Physics and Mathematics in 1961.

Since 1955, he worked in the Herzen Institute. Since 1963, he headed the Department of Mathematical Analysis. After two separate departments, the Department of Algebra and the Department of Geometry, were formed in 1965, he headed the Department of Geometry. He showed himself as an energetic organizer, inspiring scientific research in the Department and launching his own seminar. He became a member of a ministerial committee on programs for pedagogical institutes, prepared and published together with other members of his Department a series of lecture notes for graduate courses. He published many articles and a monograph on geometric methods in the theory of Monge–Ampere equations which was a

further development of the works of A.D. and Pogorelov. He had many postgraduate students who occupied numerous positions in pedagogical institutes of various cities.

Bakel'man keenly felt some personal restrictions that grew stronger and stronger that time in Russia. He was not invited to a postgraduate program. He was denied membership in the Communist Party of the USSR (abbreviated further as Party or CPSU). When A.D. and Borisov left for Novosibirsk and the Geometry Department of the University was transferred to Volkov, A.D. invited Bakel'man to a position of professor. Even the vacancy was even officially opened for him. But the local Party bosses declined his candidacy. In 1973, he was forced to leave headship of the Geometry Department that was founded by him. He wanted to move then to the Petrozavodsk Pedagogical Institute but the administration there rejected him either. In 1978 he emigrated to the USA with his wife and son.

In the USA, he held a professorship at Texas A& M University, communicated with leading mathematicians, and participated in many international conferences. In 1992, he died tragically in an automobile accident. In 1994 in Texas there was an international conference in his memory.

The following persons were Leningrad postgraduate students of Bakel'man: Verner, a Doctor of Sciences (in the Herzen Institute), Kantor (in the Herzen Institute), Guberman (in Chernovtsy), Cherdak (in Odessa), Kim (in Khabarovsk), Dmitrieva (in Nizhniy Novgorod), Kalinin, Ehpshtejn and Yunolajnen (in Petrozavodsk), Unegov (in Ekaterinburg), Vereshchagin (in Murmansk), Sovertkov (in Nizhnevartovsk), Magomedov (in Makhachkala), Kolyadov (in Cherepovets), and Ponaryadova (in Syktyvkar). Many of them attended the A.D. seminar from time to time.

## 8. YURI FEDOROVICH BORISOV (1925–2007)

In school, he studied for a year at home due to a problem with his leg. Through vigorous training, he became a long distance runner and continued this training all his life. Also, he was a member of a university rowing team. Borisov took part in A.D.'s seminar since 1945. His Kandidat thesis (about metrized manifolds with an edge) was three times as thick as an ordinary one. A merry and thorough man, Borisov relieved the tension by a suitable joke like this: "The result of the work is attained in the result of the work as its result." Far from being touchy, he was a friendly smiling man whose presence created a light atmosphere of cheerful mood. He was A.D.'s postgraduate student in 1948–1951 and then worked in the LDMI. He became a Doctor of Sciences in 1962. He accompanied A.D. in his mountain trips.

When J. Nash and N. H. Kuiper (1920–1994) discovered an unexpected abundance of isometric deformations of surfaces which preserve their $C^1$-smoothness rather than just $C$-smoothness, Borisov conducted a subtle investigation of intermediate smoothness. He proved that such deformations exist in the classes $C^{1,\alpha}$ for $\alpha \leq \frac{1}{2}$ but do not exist for $\alpha$ close to 1. But he was unable to fill this gap. In this connection, he once pronounced this toast: "To failure!" Indeed, in research, failures are numerous but might lead to an important discovery.

In 1960, A.D., overloaded by his Rector's duties, passed headship of the Geometry Department to Borisov.

In 1963, Borisov together with A.D. left for Novosibirsk where he headed the Geometry Department of Novosibirsk University. He worked in the Sobolev Institute of Mathematics up to his death.

The Leningrad postgraduate students of Borisov are associate professors: Dutkevich in St. Petersburg State University, Frangulov in the Herzen Institute, and Rusieshvilli in Batumi. All of them were participants of the seminar.

## 9. VIKTOR ABRAMOVICH ZALGALLER

It is difficult to write about your own self. I was the secretary of the A.D. seminar. Upon graduation from Leningrad University in 1948, I was recommended for postgraduate study but rejected the offer for a few reasons. A. A. Markov (1903–1979), Director of the LDMI those days, offered me a position of a junior researcher. But in the fall, I. M. Vinogradov (1891–1983, Director of the RIMM) cancelled this arrangement, and I started to work (first, in a temporary and then permanent position) in the Applied Mathematics Department headed by L. V. Kantorovich (1912–1986). There, I developed the linear programming and computational methods of the most economical cutting of industrial materials. In this process, I attended plants and wrote with Kantorovich a book which was published in 1951. Its second edition appeared in 1972. I am proud that Kantorovich, when delivering his Nobel lecture had mentioned my name among his assistants.

A.D. was not inclined to deal with engineering problems and, when somebody asked him for an advice in this area, referred him to his students. By his recommendation, still a fifth-year student, I was a consultant in a military-oriented research institute. The topics were diverse. Here is an example of a nonmilitary problem: calculation of cylindric springs of varying rigidity. The problems of this sort were not discussed at the seminar. But outside the seminar, its participants dealt with such problems. Together with Volkov, we studied how to control a burhole by observations of its curvature and torsion. Together with Reshetnyak, we worked out calculation of the shape of rotors of spiral compressors and tools for their processing. Later, I wrote a booklet on the theory of envelopes.

I remember once, after I and Borisov consulted some engineer, he offered us money which we rejected. He said: "How can I reward you?" Then Borisov smiled and said: "Do you have an entrance-gate office? Erect a monument to us over there." All three of us laughed, and the question was closed.

I defended my Kandidat thesis on Geometry in 1951. But only in 1954, I was transferred to the Geometry Department after completion of editing the book *Mathematics, Its Content, Methods, and Meaning*.

A.D. commissioned me with finishing and presenting the theory of "two-dimensional manifolds of bounded curvature." This is a generalization of his geometry of convex surfaces which happens to be the closure of both the class of two-dimensional Riemannian manifolds and polyhedral surfaces. By compactness, many extremal problems are solved in this generalized class. Our joint. paper with A.D. on this issue appeared in *Transactions of the Steklov Institute* in 1962. Another volume of this research containing papers of some students of A.D. appeared later.

I got my academic degree of Doctor of Sciences not by defending a dissertation but on submission of a collection of my works. Later I proved completeness of the

list of convex polyhedra with regular faces which attracted attention of chemist-crystallographers in Moscow and the USA. This was one of the first works where computer calculations were a basis of a theoretical proof.

Later, I wrote books with Yu. D. Burago and articles on convex bodies, lectured in LSU as professor and worked in the LDMI/PDMI until 2000 (when I retired at the age of 79).

My postgraduate students were: Burago (Head of the Geometry Department of the PDMI), Stratilatova (died early), Fedotov (an associate professor at the Agricultural Academy), Trushina, Makeev (an associate professor of the Geometry Department of SPSU—St. Petersburg State University), and Kozlov (Doctor of Sciences, a tutor of the Geometry Department of SPSU).

## 10. REVOLT IVANOVICH PIMENOV (1931–1990)

As a child, he lived in Magadan. His father was a biologist, a specialist on raising foxes, and an apologist of Prince Kropotkin (1842–1921). He moved to Leningrad with his mother. He dreamed to become a historian.

Distrusting the humanitarian programs of those years, he entered the Mat-Mekh and was in the A.D.'s seminar since his 3rd year (1950/51).

A.D. said once that the nature is "omnipossible," i. e., if a nature-looking structure is logically possible, then the nature realizes it in some form. Pimenov became interested in relativity and its generalizations and started to work out the possible instances of space-time structures.

He was a dissident. In his second year in the university, he withdrew from the Komsomol—the Communist youth organization practically mandatory for everybody at age 14. (On this occasion, he was subjected to a psychiatric test.) We resisted his efforts to discuss politics at the seminar. After the university, when he worked in the Library of the Academy of Sciences and taught in the Library Institute, he wrote letters to the local members of Soviets (parliament) demanding their protests against sending troops to Hungary in 1956. Also, he involved his students in making leaflets with such protests for which he was arrested. His mother brought him scientific literature to his cell. The jail supervisor was very much surprised when Pimenov expressed his desire to stay by in a solitary confinement: "I always dreamed to have a private office."

For his mathematical articles composed in his labor camp (the LDMI refereed them), he was released and sent to work in the LDMI. Pimenov had a phenomenal memory. He had mastered 12 languages including Sanskrit, Latin, Czech and also studied Chinese. He was extremely industrious. For his 6 years in the LDMI, he published 20 papers, wrote a monograph *Spaces of Kinematic Type*, defended both his Kandidat thesis and his Doctor-of-Sciences thesis. At the same time, his works on history and labor camp memoirs were appearing in "Samizdat"—the underground publishers. Then he was accused in spreading "Samizdat" and arrested again. Owing to his knowledge of law, he managed to spoil considerably the career of his investigator. The court, where he got acquainted with A. D. Sakharov (1921–1989), sentenced him to exile in Syktyvkar, the capital of the Komi Autonomous Republic. There, Pimenov headed a computer laboratory in the Komi Affiliation of the Academy of Sciences. He was not allowed to teach undergraduate students, but he still had some graduate ones.

In the very beginning of 1990, Pimenov was elected a member of the Supreme Soviet of the Russian Federation from Komi. Until his death (he died of stomach cancer at the end of 1990), he worked actively in Moscow in a committee on the new state constitution. Pimenov was completely exonerated posthumously in 1991.

## 11. ALEXANDR MIKHAJLOVICH ZAMORZAEV–ORLEANSKIJ (1927–1997)

Zamorzaev was a team-mate of Borisov in a rowing team. Partly, he was an algebraist. He came to the geometric seminar as a second-year student. He was an A.D.'s graduate student from 1954 to 1957. I remember a seminar session where two professors, A.D. and D. K. Faddeev, explained the new graduate student how one can work out a complete classification of Belov's crystallographic groups . (These are groups of symmetry of discrete point sets in which some of these points have positive and some, negative charge.) Zamorzaev had carried out this task brilliantly and was awarded E. S. Fedorov's medal in 1973.

After his graduate study, he left for Kishenev where he later became a corresponding member of the Moldavian Academy of Sciences. A.D. especially appreciated that Zamorzaev created a scientific school for studying symmetries (color, spiral, multidimensional, and noneuclidean). Among many of his students in Kishenev, three became Doctors of Sciences: Makarov, Palistrant, and Lungu (heading for many years the Geometry Department of Kishenev University).

## 12. YURI EVELEVICH BOROVSKIJ

A somewhat eccentric man, he was in the seminar since 1951. Borovskij was independent in his choice of topics. He was a graduate student of A.D. in 1953-1956. Upon defending his Kandidat thesis, he inclined to algebraic geometry. In 1960, following Reshetnyak, he moved to Novosibirsk where he became a Doctor of Sciences. He did not have graduate students in Leningrad.

## 13. YURI DMITRIEVICH BURAGO

He attended the seminar since he was a fourth-year student (1956/57). In 1958-1961, he was a graduate student of Zalgaller. He authored many works in A.D.'s area and on the border of analysis and geometry. He is also a coauthor of a series of big monographs and surveys: with V. G. Maz′ya, on potential theory for domains with nonsmooth boundary; with Zalgaller, on geometric inequalities, on criteria of convexity, and on Riemannian geometry; and with M. L. Gromov and G. Ya. Perel′man, on Alexandrov's metric spaces. He is an inveterate motorist. After his graduate study, Burago works in the LDMI (now, after the renaming of Leningrad into St. Petersburg, PDMI). He has had the degree of Doctor of Sciences since 1968.

After the University had moved to Peterhof, the seminar convened in the LDMI. Upon A.D.'s transition to Novosibirsk, the seminar continued its work, and Burago gradually became its leader.

Burago is a participant of many international conferences, a member of the Organizing Committee of the annual geometry conferences in Oberwolfach (Germany). He heads the Department of Geometry and Topology of the PDMI and is a professor of the Geometry Department of the University.

Burago's graduate students are: Buyalo (Doctor of Sciences, on the staff of the PDMI); Ananov (a member of the St. Petersburg's Legislative Assembly); Dmitriev

(an associate professor at the Institute of Finance and Economy); Mostovskii (in Arkhangelsk); Perel′man (once in the PDMI, now freelance); and Petrunin (in the USA).

## 14. ALEKSEI LEONIDOVICH VERNER

He graduated from the Herzen Institute in 1956, was a graduate student of Bakel'man, and participated in A.D.'s seminar since 1954. He became a Doctor of Sciences in 1969. Verner took an active part in creating new school texts on geometry. Kolmogorov, dissatisfied with the practical aspects of the existent school texts, asked Pogorelov to write a text based on an excessive set of axioms. Independently, A.D. wrote in Novosibirsk some parts of a prospective course in school geometry. There were arguments between Pogorelov and A.D. Pogorelov chose a method under which the equalities were defined in terms of the length introduced axiomatically. That led to a compact presentation and quick passage to computations. A.D. considered unacceptable the hiding of the very process of measurement and regarded as basic not computations but visualization and logic. Upon returning to Leningrad, A.D. and Verner (together with a school teacher V. I. Ryzhik) created a set of texts on geometry for all grades—from 6th to 11th,—for both general schools and mathematically oriented schools.

In 1973, Verner took over from Bakel′man the headship of the Geometry Department in the Herzen Institute. For a few years, he was there Dean of the Mathematics Faculty. Many graduate students of Verner and B. E. Kantor attended our seminar. One of them, V. Oliker, is now a professor in the USA.

## 15. A.D.'S POSTGRADUATE STUDENTS FROM OTHER CITIES BY 1963

One of them, formally from the Pedagogical Institute, was A. Ya. Yusupov (1913–1990). He was Head of the Geometry Department in Bukhara. In 1949–1951, A.D. often visited Almaty to lecture for the graduate students of the Physics and Mathematics Faculty of Al-Farabi Kazakh National University (then Kirov Kazakh University). Some of them, E. P. Sen′kin, V. V. Ovchinnikova, M. E. Kvachko, and Pogodina, arrived to Leningrad in the fall of 1951. The first three became A.D.'s postgraduate students. Three more, staying in Almaty, became his extramural postgraduate students. They were V. V. Strel′tsov (Head of the Geometry Department in the Border-Guard Military School), D. Sh. Yusupov (Head of the Geometry Department in the Female Pedagogical Institute) and later—A. A. Zil′berberg (she worked after her graduation as an associate professor in the Almaty Border-Guard Military School and then in Moscow in the Moscow Civil Engineering Institute.

Two of A.D.'s postgraduate students were from Ukraine: Sobchuk (an associate professor in Chernovtsy) and Starokhozyaev. Two others was a married couple, Herbert and Brigitta Frank, from the East Germany. They graduated from LSU in 1957; then were on probation in their country, and, over 1959–1962, stayed in Leningrad as A.D.'s postgraduates. Later, Herbert headed the Mathematical Section of Humboldt University in Berlin. All of them either became regular participants of A.D.'s seminar or arrived to the seminar many times.

When the postgraduates from Almaty settled in Leningrad, the relations in the seminar became closer. There were some collective trips to A.D.'s dacha or to a forest for mushrooming.

I remember a banquet on the rented yacht *Neva* after the public maintenance of Strel′tsov's Kandidat thesis. We sailed to Peterhof and back, and the voice of a singing Pogorelov sounded above the quiet Gulf of Finland.

The leader among those from Almaty was Sen′kin. Independent since his childhood (orphan), gifted, athletic, and perspicacious, he could say: "Why do you think now about such and such a thing?" and his guess would occur correct. After his postgraduate study, he became an associate professor in LSU. The head of the Analysis Department once said: "Sen'kin is the only one amongst you who can really teach." Sen'kin helped his wife to write her Kandidat thesis on the glass grinding. In 1964 he moved to Kharkiv as his wife was from there. He worked there in Pogorelov's Department and taught in KhSU. His postgraduate student A. A. Borisenko is now a corresponding member of the National Academy of Sciences of Ukraine. However, Sen′kin expected from himself a greater success than he actually could achieve. He got depressed, stopped working, and died at early age in 1981. He hardly understood to what extent his success in Leningrad was due to A.D.'s influence.

Postgraduate student Ovchinnikova left in a year for Moscow. A.D. gave her a generous gift: he published with her an outstanding paper on the causes of linearity of certain transformations in special relativity.

Among those who did not stay in the seminar for long, I remember A. D. Epifanov and V. Ya. Kreinovich.

In these years, we tried to imitate A.D. Professor V. A. Tartakovskii (1901–1973) once told me: "You violate the commandment: 'Thou shalt not make unto thee any graven image."' I still keep a photo of a group of the seminar participants made by Pogorelov in 1955. Standing are Borisov, Verner, Zalgaller, Sen'kin, A.D. himself, D. Yusupov, Reshetnyak, Bakel′man, Borovskii, and Pogodina. Reshetnyak made an inscription under the photo (expressed in verse):

They aren't coo-coos on a walk,
Not a company of crooks,
Not a group of alcoholics
Pooling money for little booze.
Neither dandies from a tailor,
Nor a patrol of NKVD.
This is just the school of hell
Headed by A.D. the Great.

(Here, NKVD is the KGB of Stalin's era. The Russian for "hell" is "ad.")

## 16. CONFERENCES AND GUESTS OF THE SEMINAR

From Moscow, N. V. Efimov (1910–1982) and his students É. G. Poznyak (1923–1993), V. T. Fomenko, É. R. Rozendorn, and I. Kh. Sabitov came often to A.D.'s seminar. From Kharkiv, Pogorelov came almost every year. Ya. P. Blank (1903–1988), L. A. Shor, Borisenko, and colleagues and postgraduate students of Pogorelov: A. D. Milka, Danilov, Lejbin, Danelich, Medyanik, and Sopov, also appeared in the seminar from time to time. The school of geometry "in the large" had been forming so under A.D.'s leadership. (This term emphasizes interest to

the global properties of geometric objects.) I do not remember how many All-Union symposia and conferences on geometry in the large took place. They were conducted almost each year and attracted many participants of the seminar and other mathematicians. I well remember the first symposium in Leningrad in 1958 and the third in Petrozavodsk (1967). (The venue was the same in 1969.) The second symposium was in Novosibirsk. Also I well remember a conference in Kiev in 1968. Other conferences took place in Kiev (1961), Novosibirsk (1963) and other cities approximately every other year. For the latest several years, they were organized in Rostov-on-Don.

In 1956, over the period of the III All-Union Mathematical Congress, among the guests of the seminar there was an outstanding German geometer W. Blaschke (1885–1962). Later A.D. was elected a member of the Accademia dei Lincei in Italy.

In 1961, a well-known American geometer H. Busemann (1905–1994) arrived at the IV All-Union Mathematical Congress; he also was a guest of the seminar. As a student, Busemann (almost the same age as A.D.) took part in preparation of the fundamental book by Bonnesen and Fenchel on convexity, and so he was an expert in A.D.'s area of research. He reviewed many publications of A.D.'s school for Mathematical Reviews. For the sake of that, he learned Russian and gave his talk at the Congress in Russian.

Later, many foreign geometers visited the seminar. Among them, I remember Wrubel (Poland), László Fejes Tóth (1915–2005, Hungary), Marie Moszyńska (Poland), and R. Sulanke (East Germany).

## 17. A.D. as RECTOR

A.D. was appointed the Rector of LSU by September of 1952. I remember he invited me then to his office and asked: "Do you think it makes sense for me to become the Rector?" I said no, you would waste your energy. A.D. responded: "But Lobachevskii was a rector!" He wanted to influence events and succeeded in many cases.

First of all, he supported genuine scientific schools in all faculties, promoted debunking false authorities, and nominated (even for administrative positions) real scientists. The Party leadership of LSU supported him in that. (The members of this Party committee loved A.D. and were proud of working with him after so many years.) A.D. also supported some arguable directions of research. He backed L. L. Vasil′ev (1891–1966) in parapsychology and L. N. Gumilev (1912–1992) in ethnic genesis.

There were two obstacles in the personnel area. From the Leningrad City Party Committee, he got politicized orders. A.D. stepped over them. Having that in mind, L. A. Verbitskaya (Rector of SPBU from 1994 to 2008), said at A.D.'s funeral that he had 4 Orders of the Red Banner of Labor and, also, 4 reprimands from the Party authorities (thus equalizing them to the decorations). Once, A.D. said about the Leningrad City Party Committee: "I warned them that they will have problems with me. For them, the university is buildings, but for me, it is professors." In spite of recommendations of the Leningrad City Committee, he invited to university positions some professors from the Academy, other institutes, and other cities. Also, there was a routine resistance from the bottom. Not everywhere we encountered professors ready to take part in renewing the academic life. I remember that, after

wasteful efforts to elect the Dean of Humanities where a bitter feud was in progress and nobody wanted to be dean, A.D. pronounced: "Dddamned professors... ."

A.D. took great care of true scientists as if he was responsible before history but authorities. Once he asked the University's photo laboratory to make portraits of two young mathematicians, Ladyzhenskaya and Pogorelov. Both of them became later members of the Russian Academy of Sciences.

Another of A.D.'s successes was a change in relations between the Rector and students. He refused to meet the parents of rejected high school graduates. But he willingly talked to these graduates themselves and, when he saw outstanding persons, arranged for their enrollment, sometimes even to the second year. Here are examples from the Mat-Mekh: G. Tsejtin (now a professor of the Faculty) and I. A. Ibragimov (an academician, Director of the PDMI from 2000 to 2007).

A.D. established wide contacts with the students, attended dormitories, gave talks at many faculties, and arranged question-and-answer meetings. He did not avoid sharp questions but answered in essence. Even to silly questions, he did respond (with humor). Once he was asked: "Who was the greatest scientist?" A.D. replied: "Well, in order to compare, we should choose a measure. Let, for instance, the measure of greatness be the number of years between the time of his discoveries and the time when they became commonly known. Then, Archimedes, probably, will be the greatest one. In essence, he could integrate. Though, if Jesus Christ was a real person, then that's him. He invented the morale of the communist society, but we are still far away from it."

Once he conducted an experiment with his postgraduates. By chance, he asked them questions on music, literature, the Bible, art, history, etc., and stated proudly: "Physicists answered the best!" As the Rector, he often traveled abroad and brought back excellent records of classical music. We often come to him after seminar to listen them. One of his big endeavors was not finished due to the economical situation in this country.

Around 1955, A.D. started his efforts in the most difficult task: arranging for a proper location of the University. Between the University and the Sjezdovskaya Line, there is a huge block of buildings of the Military Academy of the Rear and Transportation. The army was subject to a cut back that time, and the Academy was supposed to move outside of the city. A.D. achieved an agreement with the city authority that all these buildings would be occupied by the University. The faculties already started "division" of the buildings between themselves. But just at this time, the "Hungarian events" began. The government was scared, and the evacuation of the Military Academy was cancelled... At the same time, the Leningrad top architect (Kamenskij, I think) and his deputy Fomin, whom A.D. often contacted, worked on prospective planning of territories around the city. Fomin persuaded A.D. to accept the erection of new university buildings in Peterhof. "It will be like Cambridge. There will be dormitories for all students, including those living in Leningrad, good apartments for all university workers, fresh air, and their own yacht club. In 10 years, the city subway will go there." A.D., who had seen Cambridge, agreed. Unfortunately, the construction lasted many years, much longer than expected. The vegetable and oil storage-houses promised to be relocated were not moved. The number of apartments built for the University employees was insufficient. Moreover, half of them were given to the local people waiting for accommodation for a long time. Only 3 (out of 15) faculties could move to Peterhof. The dormitories were

too small, and more than half of the students had to spend daily 3 hours for their trip from Leningrad and back. The same was true of the employees. These days there is still no subway. Excellent class rooms are empty for the second half of the day: seminars have chosen to meet in Leningrad. Older professors leave teaching early. The number of high school graduates' applications per vacancy goes down. The Geology and Soil Faculty, whose building was under construction, refused to move to Peterhof...

Once, in the fall, visiting A.D., I told him how I waited for a train to Leningrad after my lecture on a very windy day. It did not stop because the train driver was catching up with his schedule. A.D., who was having his dinner, threw a plate on the floor and shouted: "You came to torment me, too!" His mother, still alive then, said: "Sashka, you are crazy!" He smiled, calmed down. In agitation, he repeated: "I believed them. I believed them."

A.D.'s research during his rectorship remained active. To gain more time, he left his headship of the Geometry Department. Though he skipped some seminars, he published in 1952–1963 thirty one original mathematical papers. (That is not counting abstracts of his conference talks, methodological and debatable articles, as well as many pieces of publicism.) Among these 31 papers there were two big series: "Uniqueness theorems for surfaces in the large" and "Research on the maximum principle." There were also a few articles on isometric deformations and a paper in German "On a generalization of Riemannian Geometry." The latter became a foundation of the theory of Alexandrov's spaces which is studied now by many geometers throughout the world. A.D. was a surprisingly industrial researcher, daily spending many hours at his desk. That time, I was finishing with him our *Two-dimensional manifolds of bounded curvature* and used his accurate diary notes of forties. Once, commenting on education, he told his students: "The personal example is an important tool of education. In my childhood, when I went to bed, I saw for a long time a ray of light from my father's office. It is this ray that educated me."

Let me tell two stories: on A.D.'s fight for genetics and on the destiny of a scientific school created due to A.D.'s support.

## 18. EPISODES OF FIGHT FOR GENETICS

After the war, there were many Party members among students. For a Party general meeting, a big hall on the Sjezdovskaya Line was rented. This was the hall where Lenin pronounced his sacramental phrase: "There is such a party." Dreaming about science, I was shocked by a squabble at this meeting. Professor I. I. Prezent (1902–1967) sharply criticized a dissertation. His main argument was: the bibliography contains more than a hundred foreign papers and only 14 Russian ones. Another professor objected: "Behind this dissertation there are 10 years of labor in the laboratory. While you, during these 10 years, didn't even come into the lab but intrigued only." Prezent shouted: "We will destroy you in any case. In our camp, we have Lysenko, whereas you have only Schmalhausen."

Soon after in 1948, the notorious session of the Lenin All-Union Academy of Agricultural Sciences took place (labeling genetics as pseudoscience).

A.D. refused to fire an excellent scientist Kolesnik who advocated genetics. (But Kolesnik had to leave his position of vice-rector for sciences.) Then A.D. organized the preparation of a textbook on genetics, authored by M. E. Lobashov(1907–1971), and published this text in the University. (For that, sneaking information against

A.D. was sent to the Central Committee of the Party.) A.D. defended genetics at academic meetings and conferences.

Under Khrushchev, Lysenko remained a favorite. But A.D. expelled Prezent from the university. He complained to the Ministry that A.D. fired him out by Anti-Semitic motives. The Ministry ordered A.D. to restore him. When Khrushchev arranged in Leningrad a meeting of the most active members of the Party emphasizing collective leadership, A.D. mentioned in the debates (as an example of his active position by the right of his knowledge of the University affairs) that he did not restore Prezent. Then Khrushchev asked a general nearby: "What do you do to those who refuse to follow the orders? – We execute them by a firing squad, Nikita Sergeevich. – You see..."

After that, A.D. restored Prezent but, in a month, there was a competition for his position, and he was not reelected.

For his support of genetics, A.D. got a reprimand that time and, much later, a decoration.

## 19. THE SCIENTIFIC SCHOOL OF V. A. ROKHLIN

Vladimir Abramovich Rokhlin (1919–1984) was born in a Caucasian region of Russia. His mother died in 1923, and he lost his father in 1937. In 1935, he entered Moscow State University (MSU). Many academicians consider him the most gifted mathematician of his generation. In 1941, he became a postgraduate student. Then he joined the Red Army as a volunteer. In an encirclement, he was wounded and underwent an operation on his legs. Germans took him in custody from a hospital. His fluent German allowed him to mislead Germans regarding his ethnicity. But then, he was arrested again. He was freed by our army and worked then as an interpreter in the army headquarters. After the war, he was sent to a detention camp for a "check up," where he was made not a detainee but a guard. In 1946, he was sent to Moscow where he worked in the LDMI and was a guide of (blind) Academician L. S. Pontryagin (1908–1988). He defended his thesis of Doctor of Sciences in 1951 and then was dismissed (probably due to his biography data). After that, he headed a department in Arkhangelsk, and then in Ivanov and Kolomna. From Kolomna, he went to Moscow every week to lead a seminar in MSU.

In fall 1960, on advice of Ladyzhenskaya, A.D. invited him to Leningrad. This bright scientist working in the fast developing Differential Topology and Ergodic Theory attracted like a magnet most gifted students and young faculties. An excellent lecturer on general and special topics, he expected from his graduate and postgraduate students exclusively hard work.

While the basic pedagogical method of A.D. was encouraging his students for early personal works with simple material, Rokhlin demanded that his students first mastered well a huge theoretical material. Only the strongest could bear such a load. For 26 years of teaching in Leningrad, Rokhlin had about 20 postgraduate students. Here are some whom I knew.

—Gromov who worked in the RIMM, then left refusing to work in Syktyvkar and emigrated. He is awarded with the Medal of the Best Mathematician of the USA, Wolf's prize in Israel (1993), the Lobachevskii medal (1997) in Russia, and the Abel Prize (2009). He is a leading professor of the IHES in France. He considers himself partly a student of A.D.

—Ya. M. Eliashberg is a professor in the USA.

—O. Ya. Viro is a laureate of an important Swedish prize. He heads a department in the USA and remains a member of the Scientific Council of the PDMI, where he once had headed a department.

—A. M. Vershik. He works in the PDMI and was the President of the St. Petersburg Mathematical Society for many years.

—N. Yu. Netsvetaev is today's head of the Geometry Department in SPSU.

—V. M. Kharlamov is a professor in France.

—N. V. Ivanov is a professor in France.

—V. G. Turaev is a professor in France.

—S. A. Yuzvinskij is a professor in the USA.

—S. M. Finashin is a professor in Turkey.

—I. V. Itenberg is a professor in France.

—Gluskina is an associate professor at Shipbuilding University.

—Abramov is an associate professor at the Economics Faculty of SPSU.

Education of many Petersburg mathematicians grew wider thanks to Rokhlin's transfer to Leningrad.

## 20. GROWTH OF THE UNIVERSITY

The example above shows the result of inviting just one scientist. But, under A.D, whole new parts of the University were created. At the Mat-Mekh there appeared the courses of improvement of mathematical education for engineers, a computer center, an Operation Research Department, and a Statistical Modeling Department. After moving to Peterhof, the RIMM grew considerably. Mechanics got spacious labs (only the wind-tunnel remained in the attic of a building on the Neva embarkment), and an observatory was built. The Physics Faculty, having left behind its crowded and mercury polluted rooms, got separate halls for the staff, the Physics Research Institute, and an experimental plant.

At the Economics Faculty there was created the Department of Economic Cybernetics. Before that, a one-time 6th year study (under Academicians Kantorovich and Yu. V. Linnik (1915–1972)) was arranged to prepare specialists in linear programming. This study yielded two academicians (A. I. Anchishkin (1933–1987) and S. S. Shatalin (1934–1997)) and a series of specialists who established appropriate programs in many economic universities. Similar changes took place under A.D. at many other faculties. Travels of professors to other universities for lecturing became a standard practice. Thus, in 1960, A.D. himself gave a "Rector's" course at Vladivostok University.

After A.D.'s departure for Novosibirsk, K. Ya. Kondrat′ev (1920–2006) became the Rector of LSU. In 1975 he was replaced by Corresponding Member of the Academy of Sciences V. B. Aleskovskii (1912–2006) who was a bureaucrat of science. First he ordered that no letter leave the University without his signature. That was just laughable: more than a thousand letters were sent each day. He had to cancel the order.

He brought to the agenda of the Scientific Committee a question of firing all professors older than 60. The Dean of the Mat-Mekh, Corresponding Member of the Academy of Sciences S. V. Vallander (1917–1975), vigorously objected to him. He called Vallander to his office, shouted at him and threatened to close the wind tunnel belonging to Vallander's Department. Vallander died of a stroke just after he

left Aleskovskii's office. The bright days of A.D.'s rectorship were over as standard bureaucratic authorities came to power.

## 21. PHILOSOPHICAL VIEWS OF A.D.

A.D. had been interested in Philosophy since 1946. After his death in 1999, I was startled with abundance of volumes of world philosophical literature in his office. These were volumes with many bookmarks. He worked hard.

The consistently materialistic views of A.D. on the science formed under influence of V. A. Fok. A.D. actively and fiercely defended relativity and quantum mechanics from the vulgar materialists of the type of A. A. Maksimov (1891–1976). A.D.'s views on the intrinsic connections of the distinct areas of science were wide and deep. When the second edition of the Great Soviet Encyclopedia was in preparation (and printed from 1949 to 1958), some draft articles on the issue were discussed. In Moscow (I think, in the Mathematical Society), A.D. gave a talk with subtle remarks on a draft article by Kolmogorov "Axiom." He did not get support from the audience. The listeners did not want to penetrate into the essence of the issue and perceived the remarks of young A.D. as a cock-fight attack on Kolmogorov "the Great." But Kolmogorov himself told A.D. later that, due to his criticism, he changed the article, and it became better.

When on a session of the Scientific Committee of the Faculty somebody Popov called on to restrict the teaching of such abstract subjects as Mathematical Logic, Topology, and Functional Analysis, then A.D. vigorously objected him. In these years in the USA there was published a book by R. Courant and H. Robbins "What is Mathematics?" It provoked certain methodological objections. The events like the notorious session of the Lenin All-Union Academy of Agricultural Sciences impended the Soviet mathematics. Instead of discussion, the Steklov Institute initiated the preparation of the book, *Mathematics, Its Content, Methods and Meaning*. (The plan was simple: while we write, things can change.) Of the three its scientific editors, A.D. was the most active. Not all academicians regarded seriously requests to write a chapter for the book. But A.D,'s chapters, "A General View of Mathematics" and "Abstract Spaces," and Malcev's chapter "Groups and Other Algebraic Systems" were very interesting. The work over the book was given a political touch: a draft edition (manuscript) was discussed in many seminars with mandatory shorthand record. The book (in 3 volumes) appeared in 1956, when, indeed, many things changed. Some chapters of this book have been translated into many languages.

In particular, the first chapter, by A.D., was translated into English 3 times (once in England and twice in the USA). These translations were sold out completely, and additional copies had to be printed.

The scientific community is inclined to form opposing camps. Once, in a philosophical seminar, Professor K. F. Ogorodnikov (1900–1985) responded to an opponent by the phrase which shocked me: "...but we will organize a proper social atmosphere and will not listen to your arguments." Phrases like that were difficult to pronounce in the presence of A.D. His speeches always directed the discussion towards the essence of the issue ignoring existence of any camps. This subjected A.D. to criticism from both sides, out of the argument: "those not with us are against us."

Once, such complete ignoring of the "camps" let him down. Normally, when he addressed the essence of a matter, this resulted in a support for a more progressive camp. But once, Academician B. M. Vul (1903–1985) asked him to speak in Moscow for physicists. A.D. was not warned about an atmosphere of expectation of an attack on Physics similar to that against Biology. Assuming that he spoke just to some interested specialists, A.D. ventured a criticism of the philosophical aspects of a posthumous volume of the works of Academician L. I. Mandel′shtam (1879–1944) edited by Corresponding Member of the Academy of Sciences S. M. Rytov (1908–1996). He also offended an interpretation of relativity by Ya. I. Frenkel (1894–1952). It looked as if he played into the hands of oppressors of physics. A.D. himself later explained that he was "provoked." Rytov achieved revenge many years later with his paper "The Cold Winter of 1953" in which he accused A.D. in preparing a background for an ideological attack on physics. Some physicists saw also an Anti-Semitism in his speech. Later, in Novosibirsk, members of the (nationalist) society "Memory" accused him similarly in Zionism because he defended Kolmogorov accepting the Wolf Prize, an international award granted by Israel, in 1980. That is what it could mean—to be above "two camps" and speak in essence.

## 22. DEPARTURE TO NOVOSIBIRSK

Ideologically, A.D. belonged to that part of the Russian intelligentsia which accepted the October revolution. In much the same way as Mayakovski who "plunged into Communism from the heights of poetry," A.D. accepted none but the ideals of communism. He did not become an "apparatchik"—a bureaucrat loyal to the authorities for material gains. For the latter, he was just a "fellow-traveler." His conflict with the Leningrad City Party Committee was unavoidable, and it started when the First Secretary of the Leningrad City Party Committee G. I. Popov chose to oppress the Leningrad science. A.D. wrote to the Central Party Committee about that. Just at that time, Popov was co-opted into the Central Committee. The consequences were prompt.

A.D. used to say: "There are 12 thousand people in the University. Hence, there is everything: talents, and criminals, and unavoidable emergencies." The authorities started to blame A.D. for these emergencies, and the file of his "mismanagement" grew thicker. I remember one of such cases. Students from Africa in a dormitory for men got angry because the dormitory keepers refused to let in their female friends in the evenings. They beat guards and a representative of the administration. A.D., by himself, went there and managed to stop the conflict. Nevertheless, he was blamed for this incident.

He was about to be removed from his position. In 1963, Corresponding Member of the Academy of Sciences of the USSR A. D. Alexandrov accepted the offer to fill the vacancy of a full member of its Siberian Division. He moved to Akademgorodok in Novosibirsk.

In Novosibirsk, A.D. worked for more than 20 years. Here, he created a large scientific school, and again was involved in Party activities. Once again, he fought the local Party committee and reactionaries, but I will let his Siberian students write about these years.

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# ON THE CENTENARY OF THE BIRTH OF ACADEMICIAN A. D. ALEXANDROV

YU. G. RESHETNYAK

Abstract.This is a short tribute to the memory of Aleksandr Danilovich Alexandrov (1912–1999) which was distributed at the Fourth Geometry Meeting Dedicated to the Centenary of A. D. Alexandrov in St. Petersburg, August 20–24, 2012.

The state of my health makes it obligatory for me to stay in Novosibirsk for some time. I asked Semën Samsonovich Kutateladze to undertake the task of rendering to your attention my thoughts about the occasion we celebrate.

I welcome all participants of the conference and hope that this meeting will be full success.

This is the centenary of the birth of Aleksandr Danilovich Alexandrov who was an outstanding mathematician and a brilliant and attractive person. We are all not eternal, and no one writes commendable mathematical papers if he is a centenarian. But my feelings are still full with some pain of the fact that A.D. has passed away. He possessed the unique understanding of life, had a far-sighted vision of the universe, and enjoyed the quality that was listed in the slang of the recent totalitarian past as an *active life standpoint*. These traits alongside with his buoyant temperament made the personality of A.D. enchanting.

My acquaintance with A.D. happened in the spring of 1948. Professor Dmitriĭ Konstantinovich Faddeev arranged a problem solving competition among the first and second year students of the Mathematics and Mechanics Department. The list of tasks contained a geometrical problem that was labeled as suggested by A.D. The statement of the problem was supplemented with the comment that the solution of the problem had been unknown. I think that this comment was some hoax of A.D.

Reshetnyak, Yu.G., On the Centenary of the Birth of Academician A. D. Alexandrov.

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since the problem had turned out rather simple and I had solved it quickly. Then I caught A.D. in a corridor of the department and showed my solution to him at a windowsill. "It's OK," he told me.

In September of the same year I began attending A.D.'s seminar. At that time the topic under study was A.D.'s book *Intrinsic Geometry of Convex Surfaces*. I had narrated the story elsewhere a few times, and so I will not dwell upon it now.

The happiest period of my life seems to be the span of time from 1949 to 1954 which embraces the last three years as a university student and the three years as a postgraduate. These years I had the opportunity to communicate with such a marvelous mathematician as A.D., I mastered mathematics, and the new results simply poured out of me.

I will tell the following story. It was in the spring of 1949 when Akram Yakubovich Yusupov, a student of A.D. from Bukhara, made a talk at A.D.'s seminar. A.D. had suggested in 1946 to construct some theory of curves that would embrace the theory of curves of the standard course of differential geometry, but would apply to the curves of a much more general nature. A.D. outlined a rough draft of the theory in his talk at a meeting of the Moscow Mathematical Society. A short notice of this appeared in *Russian Mathematical Surveys* in 1947. The task to be carried out by A. Ya. Yusupov was to make A.D.'s rough draft into a complete theory. Yusupov suggested a thought that was welcome in principle by A.D. But to perform A.D.'s task was unsurpassable for Yusupov for his lack of due education.

Yusupov gave a proof of one theorem of the theory of curves at one of the meetings of the seminar in the spring semester of 1949. After Yusupov had finished his talk, I suggested another proof of the theorem. "This is the real proof,"—said A.D. and continued, addressing me: "You will write an article with exposition of the theory of curves for *Russian Mathematical Surveys*!" I had started to fulfill this task right away, but I managed to cope with the arising difficulties only after the lapse of three years in the spring of 1952.

I had to pass an obligatory "Kandidat" examination in philosophy in June of 1952. I was very afraid of the exam and even panicked slightly. It was completely impossible to read all that was required. In dismay I decided to make a short break for mathematics. I happened to encounter an article by V. A. Zalgaller which proved some theorem about the length of a curve. Suddenly I asked myself whether there are similar theorems about other integral characteristics of a curve, namely, about integral curvature and integral torsion. The moment I posed the question I saw the answer. I reported my results at the nearest meeting of the geometrical seminar. After listening me, A.D. said: "How beautiful are these relations!"

Beautiful but completely useless formulas are galore in mathematics. In this case it turned out that my formulas enabled me to perform what I had tried in vain during three years and completed the theory of curves a lá Alexandrov. In particular, I had found easy solutions to some problems posed to me by A.D. I also reported this at a meeting of the seminar.

Some time after I observed that the relation for integral curvature which I discovered was already established independently by the two mathematicians: Ištvan Fari and John Milnor. Sixty years later I found out that the arguments of Fari had a serious flaw. He used the Dominated Convergence Theorem by Lebesgue. But Fari never justified the applicability of this theorem. It so happened that the Fari relation was fully proved by me. Milnor's reasoning was O.K. The original article

by Fari appeared in French which language I am insufficiently acquainted with. Recently I had found a translation of the Fari article in the Internet and everything became clear to me. But the difference in the classes these two mathematicians belonged to was evident from the very beginning.

Since I had mentioned the philosophy exam, it is in order to tell the end of the story. Recently I saw a video record of a sermon of some Christian priest Father Aleksandr. He explicated a new Christian view of miracle. Imagine that you walk to some place where you are expecting to have much trouble—*your face will meet a table*, as a Pidgin Anglo-Russian saying goes. But in fact you are welcome rather cordially with a candy. This is a real miracle by Father Aleksandr. Such a miracle happened to me: I got B for the exam! The result of the exam did not influence the amount of scholarship, but if one got C then the re-examination was obligatory. I disclose that the first question of the exam was about the content of the only lecture of philosophy for postgraduate students which I attended. So the outcome of B was impossible without some impact of sublime forces. For the sake of clarity, I must note that the event did not make me religious. By the way, the miracles in the sense of Father Aleksandr but with the opposite sign happened to me many times. It is not a rare event when you made something seemingly in the best of all possible ways after much effort to accomplish all perfectly. In result you receive no commendation you expected but your face meets a table once again. Forgive my little distraction.

I will not dwell upon the biographical data of A.D. since they are presented rather fully in the biobibliographical booklet printed this year and available in the Internet. In this regard, I want to acknowledge the efforts of Kutateladze whose energy and affection helped to compile the booklet.

The first scientific publications of A.D. were devoted to some problems of mathematical physics. But from 1936 on practically all of his scientific interests were in the realm of mathematics.

He was elected to a vacancy of a corresponding member of the Academy of Sciences of the USSR in 1946. From 1952 to 1964 Alexandrov was the Rector of Leningrad State University. In 1964 he was elected a full member of the Academy of Sciences and moved to Novosibirsk where he worked in the Institute of Mathematics (now the Sobolev Institute) and Novosibirsk State University. In 1986 Alexandrov left Novosibirsk and returned to Leningrad. He passed away in summer of 1997 and was berried at Bogoslov Cemetery in St. Petersburg.

Alexandrov owned the first class results in geometry, partial differential equations, real function theory, and mathematical crystallography. He paid much attention to the geometrical problems of foundations of relativity and achieved significant progress in this area.

Alexandrov's studies in geometry started within the theory of mixed volumes of convex bodies in which he significantly developed the results by H. Minkowski and other classics of this science.

One of the most brilliant results of Alexandrov is his solution of H. Weyl problem on realization of a convex surface with a given intrinsic metric. It is for this research that he was awarded with the Stalin Prize of the Second Degree in 1942.

The works of Alexandrov on the theory of irregular surfaces developed the geometrical conception of space, which makes them everlasting. These contributions

by Alexandrov deserve commendation along with the achievements of the best geometers such as N. I. Lobachevsky, B. Riemann, and É. Cartan.

A.D. was an extraordinarily gifted person. He raised to the summits of science and was an outstanding sportsman who received the title of the USSR Master of Sports in Mountaineering. A.D. was a capable polemicist and no one could beat him in any public dispute.

A few words are in order about A.D.'s stay in Novosibirsk. He was welcome and acclaimed at the beginning. Crowds of people visited his public lectures mostly on the general issues of life and science. But soon the bosses and their "dish-leasers" became envious of A.D.'s public influence. He encountered ribaldry, mockery, and even abuse. A.D. revealed stoicism of a warrior and overcame all attacks with dignity and honor.

The epoch of stagnation in the USSR was marked with a rather grim atmosphere of the intellectual life of the country. The then country was an instance of the realm of mediocrity. Chaps of no merits used their party connections to control practically all sides of academic life. The group of geometers together with a few allies from the other mathematical departments was a small detail besieged by adversaries. Only the broad back of A.D. made us stay in relative security.

I must emphasize that A.D.'s creative activity never declined in Novosibirsk. He wrote articles about science and morality, allotted much time and effort to improving education in high school, joined the biologists that were indignant of the publications of Academician N. P. Dubinin who denied the applicability of some laws of biology to humans, etc. His booklet on science and religion of 1972 is still actual now when the President of the Russian Academy of Sciences has proclaimed publicly that science needs an alliance with religion.

All his life up to his terminal day A.D. stood at the viewpoint of Communism. If asked whether he believed in Communism, he always answered that for him it is not a matter of belief but a matter of science. Also A.D. perfectly understood the rotten nature of the political system of the USSR and never concealed his negative attitude to the regime.

Academician Aleksandr Danilovich Alexandrov went through a long and exuberant life. He was a great citizen of his great country.

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